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Modal synthesis method for vibration analyses of damped mechanical systems

V. Zeman, Z. Hlaváč, Š. Dyk

NTIS – New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 301 00 Plzeň, Czech Republic

1. Introduction

Subsystems of flexible multibody systems are often described by models with many degrees of freedom (DOF) and with proportional damping. Nonlinear couplings between the subsystems make the vibration analyses too time consuming. Then it is desirable to apply one of the DOF number reduction of the whole system. A suitable and established methods is the modal synthesis method (MSM) [1, 3, 5]. The classical approach of the MSM is based on the reduction of the natural modes of conservative models of subsystems respected in dynamic response. The modal properties of damped subsystems are expressed by complex eigenvalues and complex eigenvectors. The main aim of this contribution is to present the new complex MSM with DOF number reduction of each proportionally damped subsystem. A variation of the proposed method modified for rotating mechanical systems with gyroscopic effects has been published in [7].

2. Mathematical model of the multibody system

Let us consider a multibody system composed of N subsystems linked by generally nonlinear couplings. Motion equations can be expressed in the matrix form

$$\boldsymbol{M}_i\ddot{\boldsymbol{q}}_i(t) + \boldsymbol{B}_i\dot{\boldsymbol{q}}_i(t) + \boldsymbol{K}_i\boldsymbol{q}_i(t) = \boldsymbol{f}_i^E(t) + \sum_{j=1, j\neq i}^{N} \boldsymbol{f}_{j,i}^C(\boldsymbol{q}_i, \boldsymbol{q}_j, \dot{\boldsymbol{q}}_i, \dot{\boldsymbol{q}}_j),\ i = 1, \ldots, N. \quad (1)$$

Let mass, damping and stiffness matrices $\boldsymbol{M}_i$, $\boldsymbol{B}_i$, $\boldsymbol{K}_i$ be symmetric of order n_i. In addition, let the damping matrices meet the proportionality conditions

$$(\boldsymbol{v}_\nu^{(i)})^T \boldsymbol{B}_i \boldsymbol{v}_\nu^{(i)} = 2D_\nu^{(i)}\Omega_\nu^{(i)},\ \nu = 1, \ldots, n_i,\ i = 1, \ldots, N\,, \quad (2)$$

where $\Omega_\nu^{(i)}$ are the eigenfrequencies and $\boldsymbol{v}_\nu^{(i)}$ are the eigenvectors of the conservative part of subsystem model i. These modal values satisfy the orthonormality conditions

$$(\boldsymbol{v}_\nu^{(i)})^T \boldsymbol{M}_i \boldsymbol{v}_\nu^{(i)} = 1,\ (\boldsymbol{v}_\nu^{(i)})^T \boldsymbol{K}_i \boldsymbol{v}_\nu^{(i)} = (\Omega_\nu^{(i)})^2,\ \nu = 1, \ldots, n_i,\ i = 1, \ldots, N. \quad (3)$$

Damping factors $D_\nu^{(i)}$ describe the proportional damping of subsystems. The time dependent vector $\boldsymbol{f}_i^E(t)$ expresses excitation of subsystem i. Nonlinear vectors $\boldsymbol{f}_{j,i}^C$ express the nonlinear forces—an action of subsystem $j \in \{1, \ldots, N\}$, $j \neq i$ on subsystem i in case of mutual contact.

The first-order formulation of the equation of motion (1) in the state space $\boldsymbol{u}_i = [\dot{\boldsymbol{q}}_i^T, \boldsymbol{q}_i^T]^T$ have the form

$$\boldsymbol{N}_i \dot{\boldsymbol{u}}_i(t) + \boldsymbol{P}_i \boldsymbol{u}_i(t) = \boldsymbol{p}_i,\ i = 1, \ldots, N, \tag{4}$$

where

$$\boldsymbol{N}_i = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_i \\ \boldsymbol{M}_i & \boldsymbol{B}_i \end{bmatrix},\ \boldsymbol{P}_i = \begin{bmatrix} -\boldsymbol{M}_i & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_i \end{bmatrix},\ \boldsymbol{p}_i = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f}_i^E(t) + \sum\limits_{j=1, j\neq i}^{N} \boldsymbol{f}_{j,i}^C(\boldsymbol{u}_i, \boldsymbol{u}_j) \end{bmatrix}. \tag{5}$$

Modal properties of each individual subsystem i are expressed by the complex diagonal spectral matrix

$$\boldsymbol{\Lambda}_i = \mathrm{diag}[\lambda_1^{(i)}, \ldots, \lambda_{n_i}^{(i)}, \lambda_1^{(i)*}, \ldots, \lambda_{n_i}^{(i)*}] = \mathrm{diag}[\overline{\boldsymbol{\Lambda}}_i, \overline{\boldsymbol{\Lambda}}_i^*] \in C^{2n_i, 2n_i} \tag{6}$$

and the complex modal matrix

$$\boldsymbol{U}_i = [u_1^{(i)}, \ldots, u_{n_i}^{(i)}, u_1^{(i)*}, \ldots, u_{n_i}^{(i)*}] = \mathrm{diag}[\overline{\boldsymbol{U}}_i, \overline{\boldsymbol{U}}_i^*] \in C^{2n_i, 2n_i}\,. \tag{7}$$

Complex eigenvalues $\lambda_\nu^{(i)}$ with a positive imaginary part and corresponding eigenvectors $\boldsymbol{u}_\nu^{(i)}$ can be expressed in terms of eigenfrequencies $\Omega_\nu^{(i)}$ and eigenvectors $\boldsymbol{v}_\nu^{(i)}$ of the conservative part of subsystem model i in the form

$$\lambda_\nu^{(i)} = -D_\nu^{(i)}\Omega_\nu^{(i)} + \mathrm{i}\Omega_\nu^{(i)}\sqrt{1-(D_\nu^{(i)})^2},\ \boldsymbol{u}_\nu^{(i)} = \begin{bmatrix} \lambda_\nu^{(i)}\boldsymbol{q}_\nu^{(i)} \\ \boldsymbol{q}_\nu^{(i)} \end{bmatrix}, \nu = 1, \ldots, n_i, i = 1, \ldots, N\,. \tag{8}$$

Eigenvectors $\boldsymbol{q}_\nu^{(i)}$ in the original space of generalized coordinates of the subsystems can be quickly calculated from the relation

$$\boldsymbol{q}_\nu^{(i)} = \frac{1}{\sqrt{\mathrm{i}}} \cdot \frac{\boldsymbol{v}_\nu^{(i)}}{\sqrt{2\Omega_\nu^{(i)}\sqrt{1-(D_\nu^{(i)})^2}}},\ \nu = 1, \ldots, n_i,\ i = 1, \ldots, N\,, \tag{9}$$

where i is the imaginary unit.

3. Complex modal synthesis method with DOF reduction

Complex MSM with DOF reduction is based on an incomplete transformation of state vectors $\boldsymbol{u}_i(t)$ in (4) into the vectors of complex modal coordinates $\boldsymbol{x}_i(t)$ of the subsystems in the form

$$\boldsymbol{u}_i(t) =^m \boldsymbol{U}_i \boldsymbol{x}_i(t) = \sum_{\nu=1}^{m_i} (\boldsymbol{u}_\nu^{(i)} x_\nu^{(i)} + \boldsymbol{u}_\nu^{(i)*} x_\nu^{(i)*}),\ m_i \leq n_i,\ i = 1, \ldots . N\,. \tag{10}$$

By using orthogonality conditions [1, 4]

$${}^m\boldsymbol{U}_i^T \boldsymbol{N}_i\, {}^m\boldsymbol{U}_i = \boldsymbol{E}_{2m_i},\ {}^m\boldsymbol{U}_i^T \boldsymbol{P}_i\, {}^m\boldsymbol{U}_i = -{}^m\boldsymbol{\Lambda}_i \tag{11}$$

and the form of eigenvectors $\boldsymbol{u}_\nu^{(i)}$ in (8), equations (4) can be written as

$$\dot{\boldsymbol{x}}_i(t) -^m \boldsymbol{\Lambda}_i \boldsymbol{x}_i(t) =^m \boldsymbol{Q}_i^T \left[\boldsymbol{f}_i^E(t) + \sum_{j=1, j\neq i}^{N} \boldsymbol{f}_{j,i}^C(\boldsymbol{u}_i, \boldsymbol{u}_j) \right],\ i = 1, \ldots, N\,. \tag{12}$$

Due to the structure of the reduced (master) spectral and modal matrices of the subsystems in the form

$$ {}^{m}\boldsymbol{\Lambda}_i = \mathrm{diag}[\lambda_1^{(i)}, \ldots, \lambda_{m_i}^{(i)}, \lambda_1^{(i)*}, \ldots, \lambda_{m_i}^{(i)*}] = \mathrm{diag}[{}^{m}\overline{\boldsymbol{\Lambda}}_i, {}^{m}\,\overline{\boldsymbol{\Lambda}}_i^{*}] \in C^{2m_i,2m_i}\,, \tag{13} $$

$$ {}^{m}\boldsymbol{Q}_i = [\boldsymbol{q}_1^{(i)}, \ldots, \boldsymbol{q}_{m_i}^{(i)}, \boldsymbol{q}_1^{(i)*} \ldots, \boldsymbol{q}_{m_i}^{(i)*}] = [{}^{m}\overline{\boldsymbol{Q}}_i, {}^{m}\,\overline{\boldsymbol{Q}}_i^{*}] \in C^{n_i,2m_i} \tag{14} $$

and vector of complex modal coordinates in the form

$$ \boldsymbol{x}_i = [x_1^{(i)}, \ldots, x_{m_i}^{(i)}, x_1^{(i)*}, \ldots, x_{m_i}^{(i)*}]^T = \begin{bmatrix} \overline{\boldsymbol{x}}_i \\ \overline{\boldsymbol{x}}_i^{*} \end{bmatrix}\,, \tag{15} $$

equations (12) can be divided to

$$ \dot{\overline{\boldsymbol{x}}}_i(t) - {}^{m}\,\overline{\boldsymbol{\Lambda}}_i\overline{\boldsymbol{x}}_i(t) = {}^{m}\,\overline{\boldsymbol{Q}}_i^{T}\left(\boldsymbol{f}_i^{E}(t) + \sum_{j=1,j\neq i}^{N}\boldsymbol{f}_{j,i}^{C}\right)\,, \tag{16} $$

$$ \dot{\overline{\boldsymbol{x}}}_i^{*}(t) - {}^{m}\,\overline{\boldsymbol{\Lambda}}_i^{*}\overline{\boldsymbol{x}}_i^{*}(t) = {}^{m}\,\overline{\boldsymbol{Q}}_i^{*T}\left(\boldsymbol{f}_i^{E}(t) + \sum_{j=1,j\neq i}^{N}\boldsymbol{f}_{j,i}^{C}\right)\,. \tag{17} $$

The global form of these equations is

$$ \dot{\overline{\boldsymbol{x}}}(t) - {}^{m}\,\overline{\boldsymbol{\Lambda}}\overline{\boldsymbol{x}}(t) = {}^{m}\,\overline{\boldsymbol{Q}}^{T}(\boldsymbol{f}^{E}(t) + \boldsymbol{f}_C)\,, \tag{18} $$

$$ \dot{\overline{\boldsymbol{x}}}^{*}(t) - {}^{m}\,\overline{\boldsymbol{\Lambda}}^{*}\overline{\boldsymbol{x}}^{*}(t) = {}^{m}\,\overline{\boldsymbol{Q}}^{*T}(\boldsymbol{f}^{E}(t) + \boldsymbol{f}_C)\,, \tag{19} $$

where

$$ {}^{m}\overline{\boldsymbol{\Lambda}} = \mathrm{diag}[{}^{m}\overline{\boldsymbol{\Lambda}}_1, \ldots, {}^{m}\,\overline{\boldsymbol{\Lambda}}_N] \in C^{m,m}\,,\ {}^{m}\overline{\boldsymbol{Q}} = \mathrm{diag}[{}^{m}\overline{\boldsymbol{Q}}_1, \ldots, {}^{m}\,\overline{\boldsymbol{Q}}_N] \in C^{n,m}\,, \tag{20} $$

$$ \overline{\boldsymbol{x}}(t) = \begin{bmatrix} \overline{\boldsymbol{x}}_1(t) \\ \vdots \\ \overline{\boldsymbol{x}}_N(t) \end{bmatrix}\,,\ \boldsymbol{f}_E(t) = \begin{bmatrix} \boldsymbol{f}_1^{E}(t) \\ \vdots \\ \boldsymbol{f}_N^{E}(t) \end{bmatrix}\,,\ \boldsymbol{f}_C = \begin{bmatrix} \sum_{j=2}^{N}\boldsymbol{f}_{j,1}^{C}(\boldsymbol{u}_1, \boldsymbol{u}_j) \\ \vdots \\ \sum_{j=1}^{N-1}\boldsymbol{f}_{j,N}^{C}(\boldsymbol{u}_N, \boldsymbol{u}_j) \end{bmatrix}\,. \tag{21} $$

Reduced DOF number $m = \sum_{i=1}^{N} m_i$ corresponds to the dimension of vector $\overline{\boldsymbol{x}}(t)$ and full DOF number $n = \sum_{i=1}^{N} n_i$ correspond to the dimension of vectors $\boldsymbol{f}^{E}(t)$ and $\boldsymbol{f}_C$. According to (10) and the structure of eigenvectors $\boldsymbol{u}_\nu^{(i)}$ in (8), the dynamic response of the arbitrary subsystem i in the original generalized coordinates is real in the form

$$ \boldsymbol{q}_i(t) = {}^{m}\,\overline{\boldsymbol{Q}}_i\overline{\boldsymbol{x}}_i(t) + {}^{m}\,\overline{\boldsymbol{Q}}_i^{*}\overline{\boldsymbol{x}}_i^{*}(t) = 2\mathrm{Re}[{}^{m}\overline{\boldsymbol{Q}}_i\overline{\boldsymbol{x}}_i(t)]\,, \tag{22} $$

$$ \dot{\boldsymbol{q}}_i(t) = {}^{m}\,\overline{\boldsymbol{Q}}_i\,{}^{m}\overline{\boldsymbol{\Lambda}}_i\overline{\boldsymbol{x}}_i(t) + {}^{m}\,\overline{\boldsymbol{Q}}_i^{*}\,{}^{m}\overline{\boldsymbol{\Lambda}}_i^{*}\overline{\boldsymbol{x}}_i^{*}(t) = 2\mathrm{Re}[{}^{m}\overline{\boldsymbol{Q}}_i\,{}^{m}\overline{\boldsymbol{\Lambda}}_i\overline{\boldsymbol{x}}_i(t)]\,. \tag{23} $$

The presented method is illustrated by numerical experiments on the impact-vibration of two nuclear fuel assemblies in the reactor core excited by coolant pressure fluctuations. The model of the single FA (see Fig. 1) is created using simple beam-type finite elements [6] and the computational model of FAs interaction is described in detail in [2].

4. Conclusions

The new modal synthesis method enables dynamic analysis of the large multibody systems composed from linear damped subsystems mutually coupled by nonlinear discrete couplings. The method is suitable especially for dynamic analysis of the systems with clearances between subsystems characterised by impact and friction forces in contact surfaces.

Consideration of the chosen master complex natural modes of each subsystem improves approximation of the reduced model in comparison with classical approach of the MSM. Calculation of complex modal values of the subsystems models with proportional damping based on real modal values of their conservative part and the damping ratios greatly speeds up the calculation time. These facts have been illustrated by means of numerical experiments with the nuclear fuel assemblies in mutual interactions excited by the coolant pressure pulsations. The concept of fuel assembly modelling is detailed in [2].

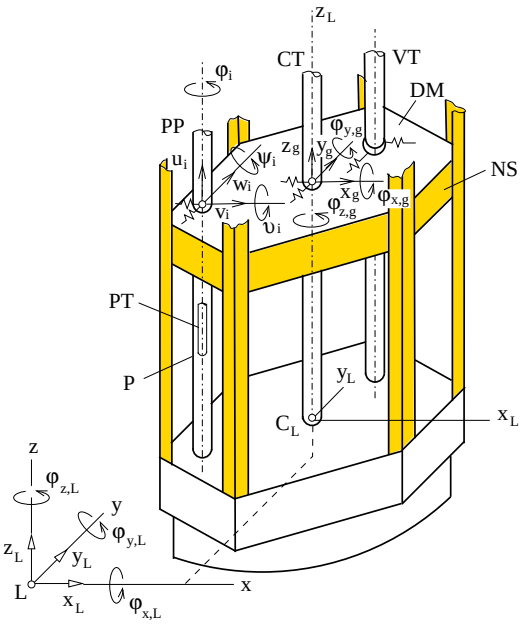

Fig. 1. Detailed FA model

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