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Propensity assessment of tensile surface structures to water ponding

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1. Introduction

Water accumulation on a tensile surface structure is a well-known phenomenon called the ponding effect. The risk of structure damage or failure is increased due to an accumulated load. Distinguishing whether the structure is sufficiently designed could be a highly demanding task even for professionals. The research published in [4] concluded that the accumulated load on a tensioned structure is more dangerous than on an inflatable structure. Hence, the ponding effect occurrence must be avoided.

In contrast with conventional buildings, periodic variable loading may affect the prestress level and thus change a force distribution in the structure [2]. Another reason for regular maintenance is the possibility of drainage blockage. The actual failures on structures caused by accumulated load were studied in [5]. The authors of research [1] summarized the recommendations for the basic shapes of tensile surface structures to avoid the undesired stress increase in the structure.

2. Methodology

To assess the structure of water accumulation propensity the algorithm for the lowest point and its catchment area detection was developed. The input for the algorithm is an arbitrary number of discretized surfaces, a specific weight of liquid and an amount of precipitation if required.

Firstly, the local lowest points on the finite element mesh are found. Each point is surrounded by its catchment area defined by the concentric slope. This area is found to determine the amount of water that may accumulate in this particular area called the pond. The whole process is based on physical laws and fluid behaviour. Two ponds are suitable for merging if they share a drainage point. Such a point is defined as the lowest point on the edge of the pond where water drains.

The level of the pond surface bounds the liquid volume. This volume could be calculated as a numerical integration over the region Ω. The region Ω is defined by the pond surface. The discretization of the volume is determined by the finite element mesh of the surface. Then the volume of each element is calculated as $dV_i = A_{p,i} \cdot dz$, where $A_{p,i}$ is the projected area of the finite element on the horizontal XY plane and dz is the height of the water column above the centre of gravity of the element. The total volume of water in the pond $V_{tot} = \sum_{i=1}^{n} dV_i$, where n is the number of flooded elements.

The real structure must withstand the always-changing climate loading. To assess the structure in real conditions every possible risk state of the load must be verified. That is possible only if the amount of water applied to the structure is controlled.

The amount of precipitation p consideration is essential because it affects the stress state in the structure significantly. Hence, it is the crucial input parameter of the algorithm and it is specified as a volume of water per collection area. The algorithm compares the total volume of the pond and the possible volume considering the precipitation. The subsequent process determines the level of the pond considering the possible amount of water on the structure. So the surface level of the pond is found iteratively by the bisection method.

The tool to facilitate a tensile surface structure design was developed and implemented in the finite element solver by FEM consulting, s.r.o. The basics of the algorithm described above were presented at the 20th International Conference of Numerical Analysis and Applied Mathematics at Heraklion, Crete. However, the complexity of the issue was high enough to initialize further development thus the algorithm was improved and new features developed.

The most important task to deal with was the pond regions merging. Subsequent development focused on the algorithm implementation, utilization and results evaluation.

The last but foremost issue to deal with was a proper load calculation and application to the finite element model. The principle of virtual work states that the variation of the external virtual work δW^{ext} equals the variation of the internal virtual work δW^{int}.

The virtual work of external forces $\mathbf{R}$ in the new configuration at time $(\mathrm{t} + \Delta\mathrm{t})$ assuming the linearized equillibrium equations

$$ {}^{\mathrm{t+\Delta t}}\mathbf{R} = \int_{{}^{\mathrm{t}}\mathrm{V}} {}^{\mathrm{t+\Delta t}}\mathbf{f}^{\mathrm{V}}\ \delta\mathbf{u}^{\mathrm{V}}\ {}^{\mathrm{t}}\mathrm{dV} + \int_{{}^{\mathrm{t}}\mathrm{S}} {}^{\mathrm{t+\Delta t}}\mathbf{f}^{\mathrm{S}}\ \delta\mathbf{u}^{\mathrm{S}}\ {}^{\mathrm{t}}\mathrm{dS} + \sum {}^{\mathrm{t+\Delta t}}\mathbf{F}\delta\mathrm{u}, \tag{1} $$

where ${}^{\mathrm{t}}\mathrm{V}$ is the volume and ${}^{\mathrm{t}}\mathrm{S}$ is the surface area, $\delta\mathbf{u}$ is the nodal displacement variation and ${}^{\mathrm{t+\Delta t}}\mathbf{f}^{\mathrm{V}}$, ${}^{\mathrm{t+\Delta t}}\mathbf{f}^{\mathrm{S}}$, ${}^{\mathrm{t+\Delta t}}\mathbf{F}$ represents volume, surface and nodal forces. The integral $\int_{{}^{\mathrm{t}}\mathrm{S}} {}^{\mathrm{t+\Delta t}}\mathbf{f}^{\mathrm{S}}\ \delta\mathbf{u}^{\mathrm{S}}\ {}^{\mathrm{t}}\mathrm{dS}$ represents variable nonlinear external load dependent on deformation [3].

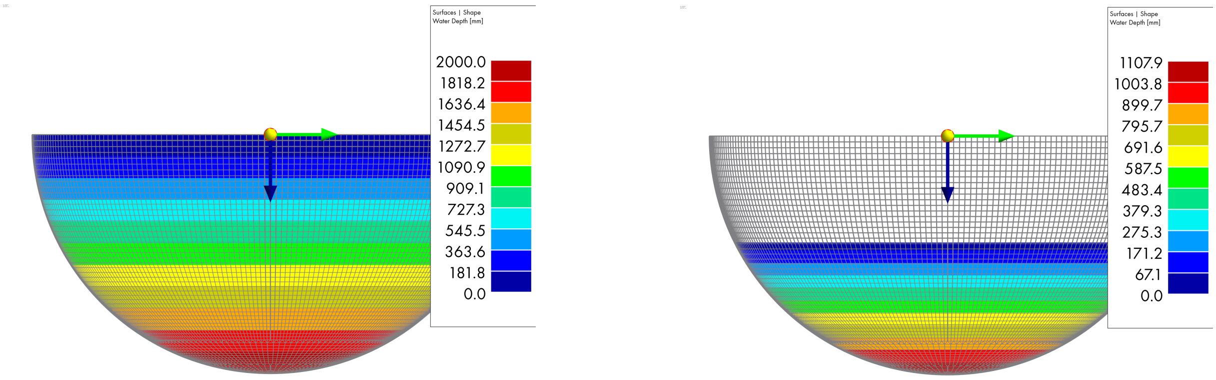
3. Results and discussion

Firstly, the algorithm was verified on a hemisphere filled with water. The specific weight of water γ was $10.0\,\mathrm{kN/m^3}$. The radius r of the rigid structure was $2.0\,\mathrm{m}$. The volume of the hemisphere can be calculated analytically as $V = 0.5 \cdot \frac{4}{3} \cdot \pi \cdot r^3 = 16.755\,\mathrm{m}^3$. Therefore, the sum of forces in the vertical direction Z is $F = V \cdot \gamma = 167.6\,\mathrm{kN}$. The analytically calculated force corresponds with the force numerically obtained. The water depth in the hemisphere filled with water is in Fig. 1a.

The amount of precipitation p was $0.5\,\mathrm{m}^3$ of water per square meter to verify the precipitation algorithm. The volume of water on the structure $V_w = p \cdot A = p \cdot (\pi \cdot r^2) = 2\pi\ \mathrm{m}^3$, where A is the hemisphere area projected in global Z. The volume of a spherical cap can be calculated as $V = \frac{\pi \cdot v^2}{3} \cdot (3 \cdot r - v)$, where v is the height of the spherical cap. Analytical solution equals $v = 1.11\,\mathrm{m}$ and corresponds to the results numerically obtained in Fig. 1b.

Secondly, a membrane surface in the shape of a hyperbolic paraboloid in Fig. 2 was studied. The shape was chosen due to its popularity and common problems with insufficiently designed structures. Despite the anticlastic curvature, the load applied to the structure may cause the change of the geometry to synclastic curvature [2].

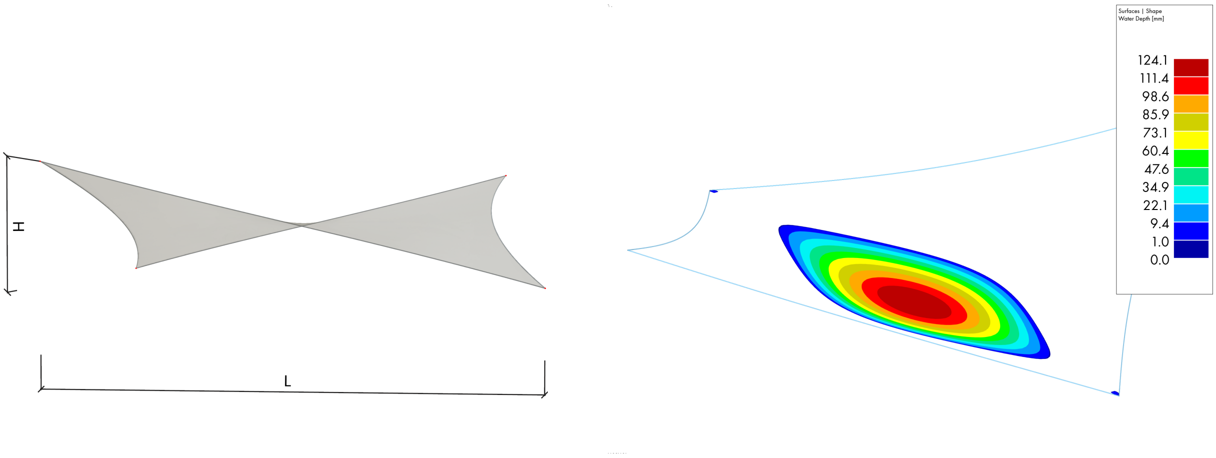
Multiple computational models were created and analyzed to highlight the issue of poorly constructed or maintained structures. The corner nodes of the membrane surface form a square of the side length L equal to $5.0\,\mathrm{m}$. The height changes from $0.4\,\mathrm{m}$ to $1.2\,\mathrm{m}$. The fabrics



(a) Full volume

(b) $p = 500\,\mathrm{mm/m}^2$

Fig. 1. Water depth in hemisphere filled with water



(a) Hyperbolic paraboloid dimensions

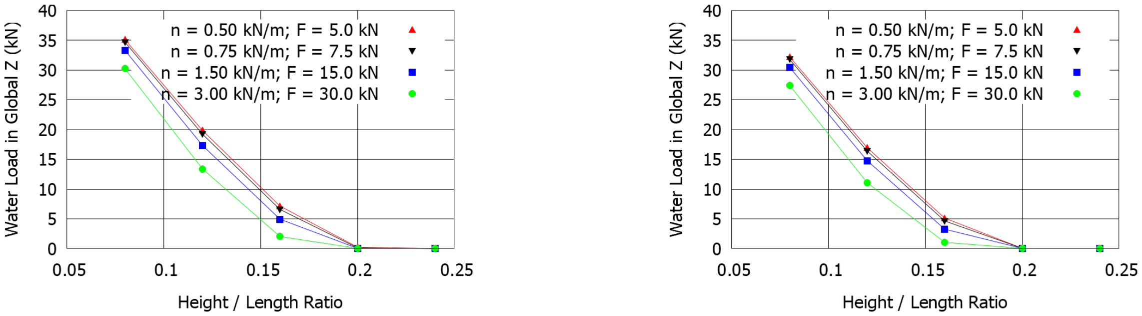
(b) Accumulated water on membrane surface

Fig. 2. Water load accumulated on the membrane structure

coated with PVC type I and III for membrane surface represent common materials used for the tensile surface structures. Recommendations for the prestress level in the surface were intentionally violated to examine insufficiently designed structures. The level of prestress in the membrane changes from $0.50\,\mathrm{kN/m}$ to $3.00\,\mathrm{kN/m}$. There are prestressed cables on the edge of the membrane made of steel S235. The prescribed forces for the form-finding process vary from $5.0\,\mathrm{kN}$ to $30.0\,\mathrm{kN}$ to ensure the same curvature radii on the edge of the surface. To initialize the deformation the surface force $2.0\,\mathrm{kN/m}^2$ was applied.

There are sums of water loads accumulated on the structure in Fig. 3. The results imply the importance of the initial design of the support points. More prestressed structures do not show significantly higher resistance to water ponding. The amount of accumulated water on the surface was approximately the same in case of the same height-to-length ratio. As anticipated, the models with higher curvature in both directions were more resistant to load accumulation than those with flatter surfaces.

The influence of the fabric type is another important result of the case study. The stronger material may imply higher resistance against the ponding effect occurrence. However, the surfaces with the same topology show almost the same inclination to load accumulation. Stronger materials must be prestressed to higher levels so the supporting structure must withstand higher forces. Therefore, the financial costs may be reduced by using a cheaper material with a lower prestress level on the properly designed structure.



(a) PVC type I

(b) PVC type III

Fig. 3. Water load accumulated on the membrane structure

4. Conclusion

In conclusion, the usage of the algorithm for ponding effect detection and membrane surface evaluation was presented. The study focused on the proper structure dimensions determination. The influences on the tendency to load accumulation were evaluated in the case study.

The results imply that the initial dimensions affect the tendency for water accumulation the most. Hence, the proper structure dimensions may lower the internal forces and thereby reduce financial costs.

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