Repairing Heavy Damaged CAD-models

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ABSTRACT

The presented work is related to the problem of repairing incomplete reconstructed (damaged) CAD-models. To solute this problem, a general concept of the repairing that uses various types of mathematical fields is proposed. One method developed within the framework of this concept is described in details. As the base this method uses interpolation of a given successfully reconstructed surface to estimate the behavior of the corresponding missing one. Ability of the method to repair heavy damaged CAD-models has been proved. This method has a big potential for further development, because the main advantage of the presented concept is that its framework is open to adding various methods of missing surface estimation to supplement each other in the repairing process.

Keywords

Cloud of points, surface reconstruction, mesh repairing, repairing CAD-models.

1. INTRODUCTION

Creating a CAD-model on the base of a given cloud of points obtained by sampling the corresponding original object is widely used in science, culture and industry. But due to as physical so technical reasons such point cloud often contains as good sampled regions so regions with unsatisfactory allocation of points or without them. In this case using even a powerful and robust surface reconstruction algorithm often leads to absence of the resulting model surface in the badly sampled regions. So, the problem of repairing incomplete reconstructed (damaged) CAD-models is very challenging.

A majority of recently developed repairing methods can be related to two groups. Methods of the first one (let's call it the *rebuilding group*), in general, remake all a model to be repaired [TJ04, EBV05, ZJH07]. But it means that they ignore the most part of the previous work to create the model. Theirs processing cost doesn't depend essentially on the degree of damage of the model that leads to inefficient processing a little damaged models. Methods of the second group (let's call it the *template-warping group*) use warping of a suitable template from a database to repair a given damaged model region [ACP03, PMG05, SKR06]. They show impressive results, but are usable only for models corresponding to theirs template databases. Also, to provide a proper template fitting such methods require manually setting a certain number of point matches between the damaged model region and the used template.

In spite of the fact that even a heavy damaged CAD-model contains various kinds of information concerning its missing surface (the behavior of the reconstructed one, location of unused sampled points, an assumption of symmetry, etc.), existing repairing methods use only the corresponding restricted parts of it. The primary goal of the presented work is developing an approach that allows using such information as entirely as it is necessary. It is clear that developing one "universal" method with this property is impossible. Thus, we have to develop a concept that provides an interaction of various methods developed within its framework to supplement each other.

The presented paper is organized in the following way. In the next section a formalization of supposed input of our work is made. Then there is a description of two theoretical concepts. In section (3) the developed previously [EM04] *concept of bridges* is briefly explained. This concept considers reducing a heavy damaged model to a model that is easy to repair. In section (4) a general concept that is called a *missing surface field concept* is introduced. This concept makes the desired open framework that allows effective using various kinds of information about missing surface in a uniform way to achieve the best repairing result. One approach developed within this framework is described in section (5). This approach is based on interpolation of the behavior of existing surface along its boundary(ies) using a tensor field. In spite of that the both introduced concepts and the interpolation-based approach can be used independently, they naturally join with each other to be used together. A method based on such joined using is described in section (6). Some results and a discussion are adduced in section (7).

2. FORMALIZATION OF DAMAGED CAD-MODELS

Definition D2.1. Let's consider an unsuccessful result of work of some surface reconstruction algorithm that contains a partially reconstructed surface ($\mathbf{A}$), unused sampled points and does not contain invalid surface elements (edges, triangles, etc.). The remaining part of the surface ($\bar{\mathbf{A}}$) is missing. Let's call such result an *incomplete CAD-model* (*ICADM*).

Definition D2.2. In a given ICADM $\mathbf{A}$ can be represented by one or several isolated regions with possible holes inside them. When $\mathbf{A}$ is represented by more than one region, let's call such regions *islands*.

Definition D2.3. Let's consider a hole in a given surface. This hole is considered a *simple hole* if inside it $\bar{\mathbf{A}}$ can be reconstructed by an existing not so complicated method and a *complex hole* otherwise.

Definition D2.4. Each ICADM can be related to one of the following classes:

ICM1: $\mathbf{A}$ is represented by one region, that has only simple holes;

ICM2: $\mathbf{A}$ is represented by one region, but there are both simple and complex holes inside;

ICM3: $\mathbf{A}$ is represented by several islands, which have possible holes of the both types.

3. CONCEPT OF BRIDGES

In accordance with D2.3 and D2.4 obtaining a correct CAD-model from a given ICADM of class ICM1 is not a problem. Let's use the fact that to reduce an ICADM of class ICM3 to an ICADM of class ICM2 and then to an ICADM of class ICM1 it is enough to reconstruct $\bar{\mathbf{A}}$ only in properly chosen local regions. In the simplest case such region is a curve connecting two specified boundary points of $\mathbf{A}$. At each point of it a unit vector is defined. This vector lies in the curve normal plane at the point and is assumed as the normal vector of $\bar{\mathbf{A}}$ at the point (figure F3.1). Let's call such curve a *bridge*.

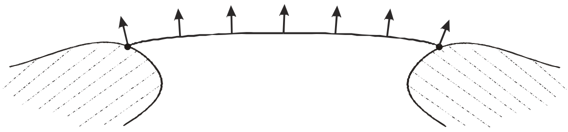

Figure F3.1

Using bridges we initially reduce $\bar{\mathbf{A}}$ between islands of an ICADM of class ICM3 (figure F3.2, left) to a set of regions of $\bar{\mathbf{A}}$ bounded by the corresponding contours with the defined normal vectors along them. Each contour is formed by the corresponding bridges and fragments of island boundaries. Such bounded region of $\bar{\mathbf{A}}$ can be considered as a hole, and, therefore, the model is reduced to class ICM2 (figure F3.2, middle). Then using bridges again each complex hole of the ICADM is recursively decomposed until a set of only simple holes is obtained, that is the criteria of class ICM1 (figure F3.2, right).

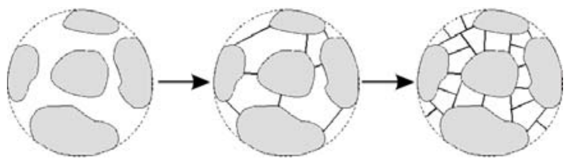

Figure F3.2

In this way bridges create an irregular mesh approximating $\bar{\mathbf{A}}$. Inside each cell of such mesh $\bar{\mathbf{A}}$ can be reconstructed by a simple existing method. This concept minimizes using costly reconstruction methods and therefore, essentially reduces the repairing cost.

4. MISSING SURFACE FIELD CONCEPT

4.1 Scalar missing surface field

Definition D4.1.1. Let's call a field of some nature that contains some information about $\bar{\mathbf{A}}$ a *missing surface field* (*$\bar{\mathbf{A}}$F*).

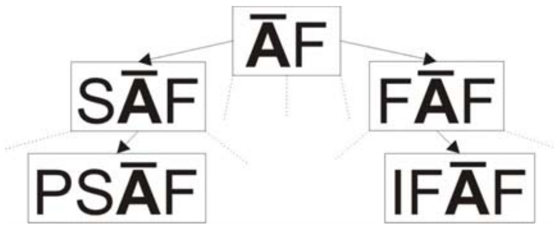

Figure F4.1.1

This paper introduces several particular cases of $\bar{\mathbf{A}}$F at various levels of abstraction. The relationship between them is shown in figure F4.1.1. The cases of $\bar{\mathbf{A}}$F shown in this scheme are defined in this and the next sections.

 ISBN 978-80-86943-93-0

Definition D4.1.2. For a given $\bar{\mathbf{A}}$F let's call the corresponding set of indices which characterize this $\bar{\mathbf{A}}$F at a point the *tension* of the $\bar{\mathbf{A}}$F at the point.

Definition D4.1.3. Let's call the estimated in some way probability of that $\bar{\mathbf{A}}$ passes through a specified point the *missing surface potential* (*MSP*, $\bar{a}$, $\bar{a} \in [0,1]$) at the point.

Definition D4.1.4. Let's suppose that the MSP is defined at each point of a certain space area surrounding a given ICADM. So, it can be considered that in this area the corresponding scalar field of the MSP is defined. Let's call such field *scalar missing surface field* (*S$\bar{\mathbf{A}}$F*). The MSP is directly the tension of it.

A S$\bar{\mathbf{A}}$F is defined by the corresponding function of one vector argument that is the coordinates of a specified point (X):

$$\bar{a} = f(X) \quad \text{(E4.1.1)}$$

In any ICADM boundaries of $\mathbf{A}$ can be considered as degenerated to lines equipotent surfaces of a specified S$\bar{\mathbf{A}}$F with $\bar{a} = 1$. Also, it is obvious, that at points which belong to $\mathbf{A}$, except the boundaries, $\bar{a} = 0$.

For example, if a given ICADM contains a sufficient number of unused sampled points, we can define a S$\bar{\mathbf{A}}$F assuming these unused points and boundary points of $\mathbf{A}$ as its elementary sources (let's call such field *point-based missing surface field, PS$\bar{\mathbf{A}}$F*). The MSP created by each such source is defined by a function of the distance ($r, r \in [0,\infty)$) to it: $\bar{a} = p(r)$, $p'(r) < 0$, $p(0) = c$, $p(\infty) = 0$, where $c \in (0,1]$ is the confidence value of the point (figure F4.1.2).

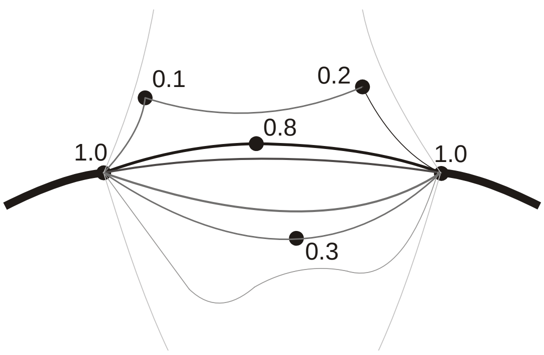

Figure F4.1.2: Side-view of a surface gap

Definition D4.1.5. A S$\bar{\mathbf{A}}$F directly defines the corresponding vector field formed by its gradient vectors. Let's call vector lines of this vector field *force lines* of the corresponding S$\bar{\mathbf{A}}$F (several such force lines are shown in F4.1.2).

Definition D4.1.6. Let there is an ICADM with a defined S$\bar{\mathbf{A}}$F. Consider a boundary point of $\mathbf{A}$. Among all force lines outgoing from the point let's select the line with the highest average value of the MSP along it. Let's call such force line the *principal force line* (in F4.1.2 the principal force line passes through the point with the confidence value 0.8).

Basic assumption. Let's assume that the principal force line of a boundary point of $\mathbf{A}$ belongs to $\bar{\mathbf{A}}$.

Therefore, by defining a proper S$\bar{\mathbf{A}}$F for a given ICADM we can reconstruct $\bar{\mathbf{A}}$ by tracing principal force lines of the S$\bar{\mathbf{A}}$F (figure F4.1.3).

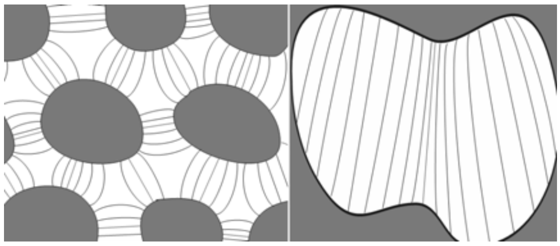

Figure F4.1.3

So, a S$\bar{\mathbf{A}}$F represents $\bar{\mathbf{A}}$ by a set of its traced principal force lines. This set doesn't contain any explicit information about the normal vectors of $\bar{\mathbf{A}}$ that requires from it to be quite dense. The described concept of bridges can't be directly applied because of this fact as well.

4.2 Function missing surface field

At the same time $\bar{\mathbf{A}}$ can be represented by a set of bridges. Such representation requires less density of sampling, because in this case it can be considered that $\bar{\mathbf{A}}$ is approximated by arbitrary narrow surface strips. Using in this case the bridge concept is obvious choice.

Definition D4.2.1. In the rest of this paper objects which are aggregates of a specified number of scalar and tensor values are used. At any place when such aggregate is implied, it is denoted by enumeration of its components in braces. For example $\{a, \boldsymbol{b}\}$ is an aggregate consisted of a scalar and a vector.

Definition D4.2.2. Let's call an aggregate of a point and a unit vector at this point an *oriented point*.

Definition D4.2.3. Each oriented point defines the corresponding plane. Let's call such plane the *tangent plane* of a given oriented point.

The bridge-based representation of $\bar{\mathbf{A}}$ can be provided by another concept of $\bar{\mathbf{A}}$F. In this concept a field expresses the probability of the tangent plane of a specified oriented point to be the approximation of $\bar{\mathbf{A}}$ at the point. Such field is defined by the corresponding function of the two vector arguments:

$$\bar{a} = g(X, \boldsymbol{n}^X) \quad \text{(E4.2.1)}$$

 ISBN 978-80-86943-93-0

where X is the coordinates of a specified point; $\boldsymbol{n}^X$ is a specified normal vector at the point.

Thus, at each point where such field is defined, there is the corresponding function that takes as the argument a specified normal vector ($\boldsymbol{n}$) at the point and returns the corresponding MSP value:

$$\bar{a} = u(\boldsymbol{n}) \quad \text{(E4.2.2)}$$

Let's call such kind of **Ā**F *function missing surface field* (F**Ā**F).

Function E4.2.2 binds this field with the introduced above concept of S**Ā**F. On the base of this relationship let's consider what force lines in the case of F**Ā**F are. It is clear, that concerning a defined field of normal vectors a F**Ā**F defines the corresponding S**Ā**F. Let's suppose that there is such field of normals that exactly determines normal vectors of **Ā**. This field with the given F**Ā**F define the corresponding S**Ā**F, whose principal force lines belong to **Ā**. Assuming **Ā** smooth enough, we can determine this field of normals and the corresponding S**Ā**F along such force lines during tracing them, that is sufficient from the point of view of our task. The tracing is performed in the following way. Let it starts from a boundary point of **A**. On the base of the assumption of smoothness it can be considered that in a close neighborhood of this point the normal vector to **Ā** has the same direction like the normal vector to **A** at the point. Thus, it can be considered that in this neighborhood the required field of normals and the corresponding S**Ā**F are defined. By determination of the S**Ā**F gradient in the given neighborhood the next point of the traced force line can be obtained. At this next point the unit vector that maximizes function u (E4.2.2) is assumed as the normal.

The following tracing steps can be performed in the same way.

This tracing process description illustrates that in the case of F**Ā**F the required in the tracing process MSP gradient vector at a specified point is, like the MSP, a function of the normal vector:

$$\nabla(\bar{a}) = v(\boldsymbol{n}) \quad \text{(E4.2.3)}$$

So, to construct a convenient to use F**Ā**F, at each considered point as function u (E4.2.2) so function v (E4.2.3) should be defined.

4.3 Using several missing surface fields

In many cases we can define for a given ICADM more than one **Ā**F. To supplement each other in the repairing process, at each considered point theirs MSPs and gradient vectors should be summed with the corresponding positive weights (w^i):

$$\bar{a} = \sum_i w^i \overline{a^i} \quad \text{(E4.3.1a)}$$

$$\nabla(\bar{a}) = \sum_i w^i \nabla(\overline{a^i}) \quad \text{(E4.3.1b)}$$

Each weight is the relative confidence value of information contained in the corresponding **Ā**F. They, in general, should not be constant everywhere, but the following condition should be satisfied:

$$\sum_i w^i = 1 \quad \text{(E4.3.2)}$$

5. INTERPOLATION MISSING SURFACE FIELD

5.1 Interpolation concept

In this section a particular case of F**Ā**F that uses interpolation of **A** as the MSP estimation method (let's call it *interpolation missing surface field*, *IF**Ā**F*) is formulated. To estimate the MSP boundaries of **A** are considered mainly, because just the behavior of **A** along the boundaries in a majority of cases has the most influence on **Ā**.

5.2 Basic definitions

Definition D5.2.1. For a surface (and a plane, as a particular case), let's assume that the normal vector at a point of the surface defines its external side.

Definition D5.2.2. For a closed surface boundary let's assume as the positive direction the counter-clockwise direction, if we look to the external surface side.

Definition D5.2.3. Let's consider a surface boundary, a point (O) on it and the tangent line to the boundary at this point (figure F5.2.1). Let's call the unit vector ($\boldsymbol{\tau}^O$) on the tangent line with the origin at O and directed in accordance with the positive direction of the boundary the *tangent vector* to the boundary at the given point.

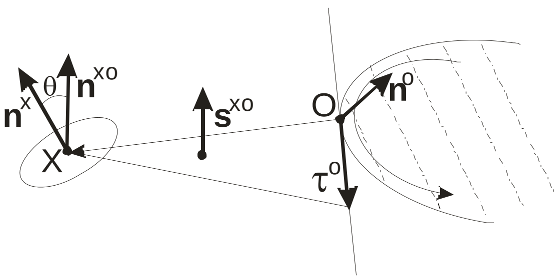

Figure F5.2.1

In this section let's assume that the term "*boundary point*" implies the aggregate of a point on a surface boundary (O), the normal vector to the surface at the point ($\boldsymbol{n}^O$) and the tangent vector at it ($\boldsymbol{\tau}^O$):

$\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$. A boundary point within an arbitrary small its neighborhood represents the surface by its tangent plane and the boundary by its tangent vector.

Definition D5.2.4. Let's call that the tangent plane (α, see figure F5.2.2) of a given oriented point ($\{X, \boldsymbol{n}^X\}$) has the *non-twisted connection* with the tangent plane of a given boundary point ($\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$) if the line of intersection of these planes is the tangent line of the boundary point with condition, that the plane normals are concerted, in other words, if we walk from X to O on the external side of the tangent plane of $\{X, \boldsymbol{n}^X\}$, we reach the external side of the surface.

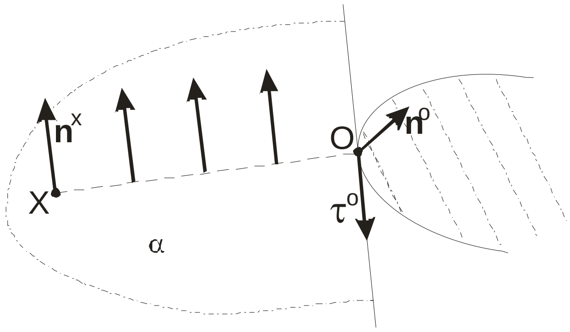

Figure F5.2.2

Definition D5.2.5. From the properties of the vector cross-product follows (see figure F5.2.1), that the normal vector ($\boldsymbol{n}^{XO}$) at a given point (X) that provides the non-twisted connection of the tangent plane of $\{X, \boldsymbol{n}^{XO}\}$ with the tangent plane of a given boundary point ($\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$), is defined by the following equation:

$$\boldsymbol{n}^{XO} = \frac{\boldsymbol{s}^{XO}}{|\boldsymbol{s}^{XO}|} \quad \text{(E5.2.1a)}$$

where

$$\boldsymbol{s}^{XO} = \boldsymbol{d}^{XO} \times \boldsymbol{\tau}^O \quad \text{(E5.2.1b)}$$

$$\boldsymbol{d}^{OX} = \frac{\overrightarrow{OX}}{|\overrightarrow{OX}|} \quad \text{(E5.2.1c)}$$

Let's call such vector the *non-twisting normal* at a point concerning a given boundary point,

Definition D5.2.6. Let there is an oriented point ($\{X, \boldsymbol{n}^X\}$, see figure F.5.2.1), a boundary point ($\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$) and the non-twisting normal at X concerning $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ ($\boldsymbol{n}^{XO}$). Let's call the angle (θ) between $\boldsymbol{n}^X$ and $\boldsymbol{n}^{XO}$ the *twist angle* of $\{X, \boldsymbol{n}^X\}$ concerning $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$.

5.3 Formalization of the field

Here the introduced above IFĀF has been defined. Initially, several basic dependencies of the MSP in the simplest case are formulated. Then two indices at a point which are called an *interpolation quality* and a *force vector* correspondingly are defined. These indices aren't exactly the MSP and its gradient but have a relationship with them. After formalization of the field using these indices this relationship is highlighted. It binds the formalized field with the introduced above general concept of ĀF.

So, in the beginning let's formalize the basic dependencies of the MSP at a given oriented point ($\{X, \boldsymbol{n}^X\}$), located somewhere outside **A**, for the simplest case, when **A** and its boundary are locally represented by the tangent plane and the tangent line of a given boundary point ($\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$). Of course, such formalization depends on the context of a concrete task, but in a majority of cases the adduced below dependences are found true.

Concerning the location of point X, it is clear that $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ doesn't have the same influence on the behavior of **Ā** everywhere. Let's assume that the MSP at X:

1) decreases with increasing the distance (r) between X and O in accordance with some distance function $func\_range(r)$;
2) decreases with decreasing the angle between $\overrightarrow{OX}$ and the tangent line of $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ (φ, $0 \le \varphi \le \pi/2$) in accordance with a function $func\_face(\varphi)$.

Concerning the direction of $\boldsymbol{n}^X$, let's assume that the MSP at X:

3) decreases with increasing the angle between $\boldsymbol{n}^X$ and $\boldsymbol{n}^O$ (η, $0 \le \eta \le \pi$) in accordance with a function $func\_normal(\eta)$;
4) decreases with increasing the twist angle of $\{X, \boldsymbol{n}^X\}$ concerning $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ (θ, $0 \le \theta < \pi$) in accordance with a function $func\_twist(\theta)$.

Conditions (1, 3) have a valuable dependence on a concrete task. In a majority of cases only extreme large values of r and η lead the MSP to 0. For clarity of the further statement let's assume that the values of $func\_range$ and $func\_normal$ are restricted by $[0,1]$.

Conditions (2, 4) have a weak task dependence. Assuming $func\_face(\varphi) \equiv \sin(\varphi)$ and $func\_twist(\theta) \equiv \cos(\theta)$, we can define the introduced above interpolation quality (q) of

 ISBN 978-80-86943-93-0

$\{X, \boldsymbol{n}^X\}$ concerning $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ in the following way:

$$q = m \sin(\varphi)\cos(\theta) \quad \text{(E5.3.1)}$$

where $m = func\_range(r)\, func\_normal(\eta)$.

This quality expresses the degree of interpolation of the surface represented by $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ by the tangent plane of $\{X, \boldsymbol{n}^X\}$ at X. From this formalization it follows that the interpolation quality is equal to 0 if X locates on the tangent line of $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ and is equal to 0 or even negative if the tangent planes of $\{X, \boldsymbol{n}^X\}$ and $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$ are "overtwisted" concerning each other. On the base of definition D5.2.4 a negative value of the quality can be interpreted in the following way: if we walk from X to O on the external side of the tangent plane of $\{X, \boldsymbol{n}^X\}$, we reach the internal side of the surface instead the external one. Also we can see, that for any mutual location of X and O except cases when X is on the tangent line of $\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$, the quality value can be made positive by proper choosing the direction of $\boldsymbol{n}^X$.

So, let's formulate the field. As an elementary source of it a boundary point of **A** is assumed. The tension of the field created by such source ($\{O, \boldsymbol{n}^O, \boldsymbol{\tau}^O\}$) at a given point ($X$) is represented by the following aggregate: $\{\boldsymbol{q}^X, \boldsymbol{H}^X\}$ where $\boldsymbol{q}^X$ is the *quality vector* and $\boldsymbol{H}^X$ is the *force matrix* at X:

$$\boldsymbol{q}^X = m\boldsymbol{s}^{XO} \quad \text{(E5.3.2a)}$$

$$\boldsymbol{H}^X = m\begin{bmatrix} d_x^{OX} s_x^{XO} & d_x^{OX} s_y^{XO} & d_x^{OX} s_z^{XO} \\ d_y^{OX} s_x^{XO} & d_y^{OX} s_y^{XO} & d_y^{OX} s_z^{XO} \\ d_z^{OX} s_x^{XO} & d_z^{OX} s_y^{XO} & d_z^{OX} s_z^{XO} \end{bmatrix} \quad \text{(E5.3.2b)}$$

where $\boldsymbol{s}^{XO}$ and $\boldsymbol{d}^{OX}$ are defined by E5.2.1b,c; m is the same like in E5.3.1.

Using the quality vector and the force matrix at X we can define the two following functions at the point:

$$q^X = \frac{\boldsymbol{q}^X}{|\boldsymbol{q}^X|}\boldsymbol{n} \quad \text{(E5.3.3a)}, \quad \boldsymbol{f}^X = \boldsymbol{H}^X \boldsymbol{n} \quad \text{(E5.3.3b)}$$

where q^X is the interpolation quality of the oriented point $\{X, \boldsymbol{n}^X\}$; $\boldsymbol{f}^X$ is the force that acts at X if $q^X \neq 0$. If $q^X > 0$, $\boldsymbol{f}^X$ "attracts" X to O and "repulses" it away otherwise.

Definition D5.3.1. Let $\boldsymbol{q}$ is the quality vector at a point. Then $|\boldsymbol{q}|$ is the highest possible quality value at the point. Let's call such value the *principal quality* at a point.

It is clear, that the made formalization provides the property of superposition of IF**Ā**F. Thus all the considerations and conclusions can be generalized to the case when there are k elementary sources of the field:

$$\{\boldsymbol{q}^X, \boldsymbol{H}^X\} = \sum_{i=1}^{k} \{{}^i\boldsymbol{q}^X, {}^i\boldsymbol{H}^X\} \quad \text{(E5.3.4)}$$

and to the case when as a source of the field a continuous surface boundary segment is considered:

$$\{\boldsymbol{q}^X, \boldsymbol{H}^X\} = \int_l \frac{d\{\boldsymbol{q}^X, \boldsymbol{H}^X\}}{dl} dl \quad \text{(E5.3.5)}$$

and a set of such boundary segments as well.

At it has been mentioned above, the interpolation quality and the force vector have a relationship with the MSP and its gradient. Assuming $\nabla(\bar{a}) = \mu\boldsymbol{f}$, where μ is a positive constant, it can be considered that $\bar{a} = \alpha(q)$, where $\alpha(q) \in [0,1]$, $\alpha'(q) > 0$. This relationship is sufficient to obtain the same result, if in the force line tracing process we substitute $\bar{a}$ by q and $\nabla(\bar{a})$ by $\boldsymbol{f}$. Also, in practice, to use in the sum operations defined by E4.3.1, function $\alpha(q)$ can be approximated by $((q+1)/2)^k$, where k is a positive constant. At the same time, exact values of $\bar{a}$ and $\nabla(\bar{a})$ at each point can be obtained using known distribution of q and $\boldsymbol{f}$, if it is necessary.

6. IMPLEMENTATION

On the base of the adduced above theoretical statement the described in this section repairing method has been developed and implemented. It uses the properties of IF**Ā**F as the base. Initially, boundaries of **A** of a given ICADM are considered as IF**Ā**F sources. Then on these boundaries we determine points corresponding to local maximums of the principal quality on a specified little distance from the boundaries. From these points the corresponding principal force lines are traced. They connect isolate islands of the ICADM with each other and divide complex holes inside the islands to more simple ones. Then, in accordance with the concept of bridges, from the obtained topology the created set of holes is extracted. Each hole is decomposed by the tracing IF**Ā**F principal force lines in a similar way that in the previous step, with the difference that only the hole boundary is considered as an IF**Ā**F source.

If the processed ICADM contains a valuable number of unused sampled points, then within the framework of the **Ā**F concept IF**Ā**F is supplemented by defined in subsection (4,1) PS**Ā**F. Because the confidence values of such points, as a rule, are low, in the sum operations defined by E4.3.1 the weight of PSĀF is relative small, so this field plays only a perturbing role during the IFĀF force line tracing.

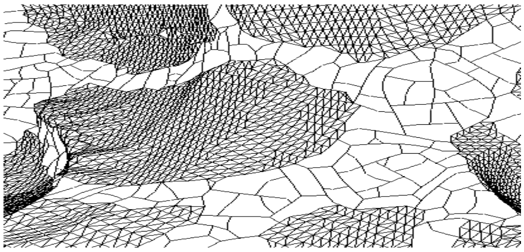

Figure F6.1

Because the presented method works directly in regions of damage, its cost does not essentially depend on the square of **A** and as a consequence on the number of existing edges, triangles and points used in **A**. To estimate the cost, let's take into account that traced force lines finally forms a mesh approximating **Ā** (figure F6.1) where each cell can be considered as a simple hole. Assuming for simplicity that all the force lines are traced with the same step and cells of the mesh have the same square we can conclude that the number of points, at which the tension of the both fields should be calculated is proportional to the total square of **Ā**. So, the cost behavior can be approximately expressed by the following equation:

$$t = kSquare(\overline{A}) \quad \text{(E6.1)}$$

where k is a positive constant.

In the current state of development, the method works correctly if a given ICADM adequately represents topology of the corresponding original object. Using the given **A** and unused sampled points it provides obtaining a closed surface. If the original object has holes, theirs contours should be explicitly given.

7. TESTS AND COMPARISON

An example of a heavy damaged ICADM obtained in practice of surface reconstruction is shown in figure F7.1. This ICADM has been generated from a cloud of 228741 sampled points. A valuable part of its surface is missing due to undersampling. **A** is represented by 32 islands which have 306 holes. During the repairing (71 sec. for 3~GHz Pentium-4) the tension of the both fields (IFĀF and PSĀF) at 123295 points has been calculated and 5173 force lines of the combined ĀF (IFĀF + PSĀF) have been traced. The reconstruction result is shown in figure F7.2. Such result is comparable with results providing by the considered methods of the template warping group, but it has been obtained without any template database and manual management. Of course, the template warping methods can provide the super-resolution effect of reconstructed **Ā** due to theirs database knowledge, but a similar knowledge expressed by a proper ĀF can be in principle used and in the presented method.

During the done tests a direct comparison with a method of the rebuilding group has been made. This method is similar to [EBV05] and models the behavior of a shrinking elastic membrane surrounding an ICADM to be repaired. To make the experimental comparison more clear, three test ICADMs (the same well known "Bunny" artificially damaged in a different degree) have been processed by the both methods. The results of this processing are shown in table T7.1. They, in general, are in agree with the theoretical estimations and E6.1. The cost of the warping method weakly depends on the model damage degree and the presented method has a superiority, which, although, decreases when this degree increases. In general, methods of the rebuilding group, which use warping, have a greater theoretical robustness, than the presented one. They can reconstruct a closed surface from an extremely damaged ICADM, but practical usability of such repairing result is questionable.

8. CONCLUSION

The presented method using only the two implemented ĀFs (IFĀF and PSĀF) has proved ability to repair heavy damaged CAD-models without any manual management. The method has good potential for further development, which is caused by using the introduced ĀF concept. This concept provides the open framework to use together various methods of missing surface estimation. And many of existing ones can be embedded in this framework by defining corresponding ĀFs. As a particular, it seems perspective to supplement the two implemented ĀFs by an ĀF that expresses a property of symmetry of an original object. The method has very good parallelization abilities, because each force line can be traced independently on tracing other ones and the tension of a given ĀF at a point can be calculated independently on calculating the tension of other ĀFs at the same point as well.

9. ACKNOWLEDGMENTS

This work is supported by the Russian Foundation for Basic Research, projects 08-07-00362, 08-07-00399.

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Figure F7.1

Figure F7.2

ICADM:				
damage degree	Square($\bar{A}$)/Square(A+$\bar{A}$)	0.11	0.29	0.38
	relative	1.0	2.6	3.5
warping method cost	absolute (sec)	27	34	32
	relative	1.0	1.3	1.2
IF$\bar{A}$F-based method cost	absolute (sec)	4.2	12	16
	relative	1.0	2.9	3.8

Table T7.1

 ISBN 978-80-86943-93-0