

# 3D Modeling of Free-Form Object (Interpolation, Visualization and Volume Estimation)

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## ABSTRACT

This work describes an efficient method of 3D object modeling. The method is applied for solving biomedical engineering tasks. The method is based on interpolation model creation. The model is built using Coons surface basis. For further implementation of the model the Coons surface basis is transformed to Bezier surface model. This work also describes a method of volume estimation of the reconstructed 3D model. Example of practical implementation is solving various biomedical tasks.

## Keywords

Volume Estimation, Interpolation, Visualization, 3D Model.

## 1. INTRODUCTION

The problem of free – form objects 3D visualization and their further analysis is a very interesting task of computer graphics. Also, the solving of this task is very actual in practical implementation for biomedical engineering. This work describes a new method of free-form objects reconstruction, with a medical object used as an example. The object is visualized and its volume is estimated (a very actual task for physicians). The known methods of object volume estimation do not take into consideration the 3D structure of the object (this leads to miscalculations in volume estimation), meaning that the method of objects volume estimation using reconstructed 3D model is a more precise approach.

The proposed methods and algorithms can be used in various spheres (scientific, medical). To show practical use of the proposed methods, medical images are used in this work.

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## 2. PROPOSED METHODS

### Segmentation

The goal of image segmentation is to categorize the pixels (or voxels) of an image into a certain number of regions, which are homogenous with respect to some characteristics (for example: intensity, color, texture, etc.). In order to estimate the volume of the object and to visualize it, image segmentation should be done first.

Because practical implementation of our method is biomedical engineering tasks, a method of segmentation was developed, that was specifically designed for medical image segmentation, taking into account the special features of medical images.

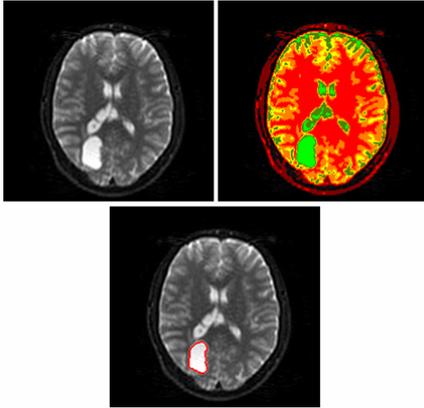
Standard medical imaging software provides manual image segmentation, which, although is most accurate if done by expert physician is also very time-consuming. On the other hand, automatic image segmentation is often doubted by the physician, because a computer could never replace a doctor in diagnostics. Therefore, a simple and quick semi-automatic segmentation algorithm is proposed [1], with the possibility to also manually adjust the region of interest.

The proposed method combines the advantages of grey-level thresholding, region based and edge based segmentation. Segmentation algorithms are combined with developed image enhancement algorithm that is based on image histogram

enhancement and existing segmentation result post-processing algorithm.

The possibility of manual adjustment of ROI (Region of Interest) borders minimizes the risk of imprecise segmentation that automatic segmentation could provide.

The proposed method was tested on medical images of a brain acquired with use of computer tomography. Figure 1 shows the results of this method.



**Fig. 1 Segmentation results. Top left: initial image. Top right: segmentation map. Bottom: extracted pathology zone (no manual adjustment)**

The algorithm automatically removes the background from the medical image and segments the data, providing a segmentation map of the image. From this point, the physician can select any region of interest and then manually adjust borders of the region until the result is satisfactory.

### Mathematical model for rendering 3D objects

The input data for developing the mathematical model of a 3D object is an array of points, obtained from segmented regions. The task of image visualization in this case can be reduced to the task of creating an array of patches of the modeled surface. We use patches that are described with Coons surface to build the resulting surface. Coons surface in matrix form can be expressed as follows:

$$Q_{Coons}(u, v) = [U] \cdot [N_C] \cdot [P_{Coons}] \cdot [N_C]^T \cdot [V], \quad (1)$$

where  $[U]$ ,  $[V]$  – matrix of parameters,

$[N_C]$  – matrix of basis function coefficients,

$[P_{Coons}]$  – array of control points.

In this work, it is proposed to use bi-cubic Coons surfaces for modeling the resulting surface. In this

case the array of points for one separately taken patch is calculated as follows:

$$[P_{Coons}] = \begin{bmatrix} P(i, j) & P(i, j+1) & P(i, j) & P(i, j+1) \\ P(i+1, j) & P(i+1, j+1) & P(i+1, j) & P(i+1, j+1) \\ P_u(i, j) & P_u(i, j+1) & P_{uv}(i, j) & P_{uv}(i, j+1) \\ P_u(i+1, j) & P_u(i+1, j+1) & P_{uv}(i+1, j) & P_{uv}(i+1, j+1) \end{bmatrix} \quad (2)$$

where  $P(i, j)$  – control points in the corners of the patch (the points from the initial array of data),

$P_u(i, j)$ ,  $P_v(i, j)$  – tangent vectors in parametric directions  $u$  and  $v$ ,

$P_{uv}(i, j)$  – twist vectors.

Tangent vectors are calculated as follows:

$$P_u(i, j) = \frac{P(i, j+1) - P(i, j-1)}{2} \quad (3)$$

$$P_v(i, j) = \frac{P(i+1, j) - P(i-1, j)}{2},$$

Twist vector is calculated by tangent vector multiplication:

$$P_{uv}(i, j) = P_u(i, j) \times P_v(i, j), \quad (4)$$

The twist vector can be used as a zero-vector in further patch calculation. The direction of twist vector is used for polygonal visualization of the model.

### Transformation of the mathematical model for use with standard computer graphics library

To visualize the 3D object by standard means of computer graphics it is necessary to transform Coons surfaces to Bezier surfaces. Bezier surface in matrix form can be expressed as follows:

$$Q_{Bezier}(u, v) = [U] \cdot [N_B] \cdot [P_{Bezier}] \cdot [N_B]^T \cdot [V], \quad (5)$$

where  $[U]$ ,  $[V]$  – matrix of parameters,

$[N_B]$  – matrix of basis function coefficients,

$[P_{Bezier}]$  – array of control points.

To transform the Coons surface to Bezier surface we use the relation between  $Q_{Bezier}(u, v)$  and  $Q_{Coons}(u, v)$ .

As described by Rogers et. al. [2], the relation between Coons surface and Bezier surface can be expressed by equating the two surface equations (1) and (5). In matrix form this can be described as follows:

$$[U] \cdot [N_B] \cdot [P_{Bezier}] \cdot [N_B]^T \cdot [V] = [U] \cdot [N_C] \cdot [P_{Coons}] \cdot [N_C]^T \cdot [V], \quad (6)$$

where  $[U]$ ,  $[V]$  – matrix of parameters,

$[N_B]$ ,  $[N_C]$  – matrix of coefficients,

$[P_{Bezier}]$ ,  $[P_{Coons}]$  – matrix of control points.

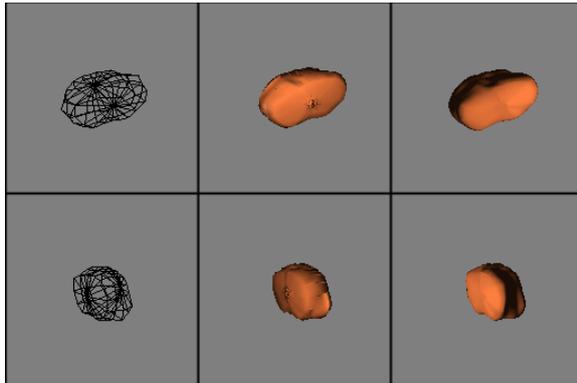
The array of control points for equivalent Bezier surface is calculated using the following formula:

$$[P_{Bezier}] = [N_B]^{-1} \cdot [N_C] \cdot [P_{Coons}] \cdot [N_C]^T \cdot [N_B]^T, \quad (7)$$

In case of bi-cubic surfaces this formula assumes the following form:

$$[P_{Bezier}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P(i, j) & P(i, j+1) & P_v(i, j) & P_v(i, j+1) \\ P(i+1, j) & P(i+1, j+1) & P_v(i+1, j) & P_v(i+1, j+1) \\ P_u(i, j) & P_u(i, j+1) & P_{uv}(i, j) & P_{uv}(i, j+1) \\ P_u(i+1, j) & P_u(i+1, j+1) & P_{uv}(i+1, j) & P_{uv}(i+1, j+1) \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (8)$$

In the end, the mathematical model of the 3D object is an array of Bezier patches. This allows using standard methods of computer graphic visualization. In this work, we use standard computer graphics library – OpenGL [3] and rendering by ray tracing. The method of visualization by ray tracing is described in [4]. Figure 2 shows the results of visualization using standard graphic library OpenGL and ray tracing.



**Fig. 2 Left column: mesh of the model. Middle column: models obtained using standard OpenGL library. Right column: models obtained by ray tracing.**

## Volume Estimation

Volume estimation algorithms were also tested on medical images (pathology zone extraction and its

volume estimation). There are various approaches to volume estimation in medical images. The most common methods are the Trapezoidal estimation and Cavalieri's method, both described in [5]. These methods use the 2D data obtained from segmenting medical images. Unfortunately, this could lead to miscalculations in volume estimation, because such methods have no information about the form of the pathological region. This means there is no information about the pathology zone between the medical slices. Therefore, in this work it is proposed to use the reconstructed 3D model of the pathology zone.

The input data for the volume estimation algorithm is an array of Bezier patches (mesh), which were obtained using segmented data. The patches are aligned so that the z axis is in the center of the medical object. Based on this, the volume estimation formula is described as a sum of patch volumes:

$$V = \sum_{i=1}^P V_i, \quad (9)$$

where  $V_i$  – patch volume,

$P$  – number of patches.

To calculate the volume of each patch a surface integral is used:

$$V = \frac{1}{2} \iint_s |\vec{r}| \cdot \cos(\vec{r}, \vec{p}) \cdot ds, \quad (10)$$

where  $\vec{r}$  – vector from the patch to Z axis,

$\vec{p}$  – a perpendicular vector to patch.

To successfully implement the surface integral in a program, first it is transformed to a double integral. Because the surface of the model is given in parametric form, after the transformation, the integral (10) assumes the following form:

$$V = \frac{1}{2} \int_0^1 \int_0^1 |\vec{r}| \cdot \frac{r_x p_x + r_y p_y + r_z p_z}{|\vec{r}| |\vec{p}|} \cdot \sqrt{A^2 + B^2 + C^2} du dv, \quad (11)$$

where

$r_x, r_y, r_z$  – are vector's  $\vec{r}$  coordinates

$p_x, p_y, p_z$  – are vector's  $\vec{p}$  coordinates

$$A = \begin{vmatrix} \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix} \quad B = \begin{vmatrix} \frac{\partial z}{\partial u} & \frac{\partial x}{\partial u} \\ \frac{\partial z}{\partial v} & \frac{\partial x}{\partial v} \end{vmatrix} \quad C = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} \quad (12)$$

Since

$$p = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{dx}{du} & \frac{dy}{du} & \frac{dz}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} & \frac{dz}{dv} \end{vmatrix}, \quad (13)$$

the double integral (11) can be transformed to the form

$$V = \frac{1}{2} \int_0^1 \int_0^1 x(u, v) \cdot A(u, v) + y(u, v) \cdot B(u, v) du dv, \quad (14)$$

where  $x(u, v)$ ,  $y(u, v)$  – surface point coordinates,

To calculate this integral in a program, graphic integration is used. The integral (14) assumes the following form:

$$V \approx \frac{1}{2nm} \cdot \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} x(u_i, v_j) \cdot A(u_i, v_j) + y(u_i, v_j) \cdot B(u_i, v_j), \quad (15)$$

where  $n, m$  – are the number of steps in parametric directions  $u$  and  $v$ .

Since the proposed method uses the 3D model of the pathological region, it has information about the form of the pathology zone, meaning the method can estimate the region even between the slices. This is more correct than the usual methods that use information only from 2D medical images.

The experiments were conducted on an ideal model object (cylinder and frustum combination) and a medical object.

Table 1 shows results on pathology zone's volume estimation, obtained by the proposed method and trapezoidal method.

	Model object	Difference from proposed, %	Medical object	Difference from proposed, %
Proposed method, mm <sup>3</sup>	90598 2	-	3475	-
Trapezoidal method, mm <sup>3</sup>	90421 6	0,19%	3508	3,1%
Cavalieri method, mm <sup>3</sup>	90421 6	0,19%	3539	4,0%

**Table 1 Results**

As can be seen from table 1, the results on 2D trapezoidal method and the proposed method are

similar when calculating the volume of a model object, but are also quite different when the object is irregular. Additional research has to be conducted, to test whether the reconstructed 3D model corresponds ideally to the real object.

### 3. CONCLUSIONS

Several methods are proposed for segmentation, visualization and volume estimation of the 3D objects.

The proposed semi-automatic medical image segmentation algorithm combines the advantages of grey-level thresholding, region based and edge based segmentation. It is fast and efficient, while allows manual adjustment to the region of interest, if necessary.

3D object visualization is based on standard graphic library OpenGL. This reduces application's requirements for computer resources and allows real-time rendering of the modeled object. The alternative method of visualization is based on ray tracing. In this case the surface of the modeled object is visualized with more detail. But the time of rendering does not allow using this method in real time.

The method of volume estimation uses 3D object model as initial data. It is more correct than the known methods that are based on 2D image data, because these methods have no information about the object between slices. Since the proposed method has information about the form of the medical object, it can correctly estimate the volume even between slices, where there is no information about the object. Although additional research has to be conducted to test whether the reconstructed model correctly describes the real object.

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