Post- processing of 3D scanning data for custom footwear manufacture

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ABSTRACT

Footwear fitter has been using manual measurement of a long time, but developments of laser scanners in the last few years have now made automatic determination of footwear feasible. Despite the steady increase in accuracy, most available scanning techniques cause some deficiencies in the point cloud and a set of holes in the triangle meshes. Moreover, data resulting from 3D scanning are given in an arbitrary position and orientation in a 3D space. To apply sophisticated modeling operations on these data sets, substantial post-processing is usually required. In this paper, we described an algorithm for filling holes in triangle mesh. First, the advance front mesh technique is used to generate a new triangular mesh to cover the hole. Then, the Poisson equation is applied to optimize the new mesh; then the models are aligned by "Weighted" Principle Component Analysis.

Keywords

Advance front mesh, Poisson equation, "Weighted" Principle Component Analysis.

1. INTRODUCTION

Due to the rapid development of 3D scanning that can easily and quickly acquire enormous number of surface points from physical parts, reverse engineering (RE) and automatic alignment have become an important steps in the design, manufacturing of new products and shape analyses.

There has been a growing trend among shoe manufactures to introduce customized shoes to satisfy varying customer comfort needs. The design of new shoes starts with the design of the new shoe last. A shoe last is a wooden or metal model of human foot on which shoes are shaped.

Depending on the both of the complexity of the object and the adopted data acquisition technology some areas of the objects outer surface may never be accessible. This induces some deficiencies in the point cloud and a set of holes in the triangle mesh. Moreover, data resulting from 3D scanning are given in an arbitrary position and orientation in the space. Thus, substantial post-processing is usually required before taking these models to footwear application.

Various techniques have been proposed to fill holes in the mesh. Among non-geometric approaches, authors in [1] used a system of geometric partial differential equation derived from image inpainting algorithms for filling in the holes. Davis et al [2] used volumetric diffusion to fill the gaps. Considering the geometric approaches, Bareguet and Sharir [3], find a minimum area triangulation of a 3D polygon with dynamic programming method in order to fill holes. Authors in [4] applied a fairing technique based on solving a non-linear fourth order partial differential equation to fill holes.

Hole process that is implemented here is quite similar to [5]. The main stages of the method is applied in this paper are: covering the holes with advancing front mesh technique; modifying the triangles in initial patch mesh by estimating desirable normals instead of relocating them directly; solving the Poisson equation according to desirable normal and boundary vertices of the hole to optimize the new mesh. After obtaining complete 3D model, the result data must be generated and aligned before taking this models for shape analysis such as measuring similarity between foot and shoe last data base for evaluating footwear fit.

Principle Component Analysis (PCA), also called Karhunen-Loeve transform, aligns a model by considering its center of mass as the coordinate system origin, and its principle axes as the coordinate axes. The purpose of the PCA applied to a 3D model is to make the resulting shape feature vector independent to translation and rotation as much as possible. In analysis, instead of applying the PCA in a classical way (sets of 3D point-clouds) in order to account different sizes of triangle, Paquet and Rioux [6], established weights associated to center of gravity of triangles and Varanic et al [7], used weighting factors associated to vertices. We used the "Weighted" PCA analysis for alignment of 3D models.

This paper is structured as follows: in section 2 and 3 we introduced terminology, hypothesizes and background while in section 4 filling hole in triangle mesh for building complete model is presented and alignment of 3D models based on the "weighted" PCA is described in section 5. Finally, conclusion and remarks are summarized in Section 6.

2. Terminology and hypothesize

A triangle mesh is defined by a set of oriented triangles joining a set of vertices. Two triangle are adjacent if they share a common edge.

A boundary edge is adjacent to exactly one triangle. A boundary vertex is a vertex used to define a boundary edge. Thus, a closed cycle of boundary edges defines a hole. A given hole is assumed to have no island and all mesh models are oriented and manifold. A boundary triangle is a triangle that own one or two boundary vertices.

1-ring triangles of vertex are all triangles that share one common vertex. 1-ring edges of vertex are all edges that share one common vertex and all vertices on 1-ring edges of a vertex (expect itself) are called 1-ring vertices of the vertex.

3. Background

The Poisson equation has been used extensively in computer vision [8]. It arises naturally as a necessary condition in the solution of certain variational problems.

The aim of this method is solving an unknown target mesh with known topology and unknown geometry (vertex coordinate). This equation is able to reconstruct a scalar function from a guidance vector filed and boundary condition.

Consider an unknown scalar function, f, the Poisson equation with Dirichlet boundary condition is given by:

$$\nabla^2 f = \nabla . w \qquad \text{over } \Omega \text{ , with} \qquad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega} \quad (1)$$

where w is Guidance Vector Field, $\nabla . w = \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z}$ is the divergence of $w=(w_x, w_y, w_z)$, f* provides the desirable values on the boundary $\partial\Omega$, and Laplacian operator is $\nabla^2 = (\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2})$. Thus it can be defined as least-squares minimization problem:

$$\min_f \iint_\Omega |\Delta f - w| \qquad \text{with} \qquad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega} \quad (2)$$

A discrete vector field on a triangle mesh is defined to be a piecewise constant vector function whose domain is the set of points on the mesh surface. A constant vector is defined for each triangle, and this vector is coplanar with the triangle. For a discrete vector filed w on the mesh, its divergence at vertex v_i can be defined with:

$$(divw)(v_i) = \sum_{T_k \in N(i)} \nabla B_{ik} . w |T_k| \quad (3)$$

where $|T_k|$ is the area of triangle T_k, N_i is the 1-ring vertices of v_i and $\nabla B_{i,k}$ is the gradient vector of B_i within T_k. The discrete gradient of the scalar function f on a discrete mesh is expressed as:

$$\nabla f(v) = \sum_i f_i \nabla \phi_i(v) \quad (4)$$

with $\phi_i(.)$, the piecewise linear basis function valued 1 at vertex v_i and 0 at all other vertices begin and f_i begins the value of f at v_i and it is one of the coordinate of v_i . The discrete Laplacian operator can determine as:

$$\Delta f(v_i) = \sum_{v_j \in N_i} \frac{1}{2}(\cot \alpha_{i,j} + \cot \beta_{i,j})(f_i - f_j) \quad (5)$$

where $\alpha_{i,j}$ and $\beta_{i,j}$ are the two angles opposite to edge in the two triangles sharing edge (v_i and v_j) and N_i is the set of the 1-ring vertexes of vertex v_i, see Figure1.

Finally discrete Poisson equation is expressed as follows: $\nabla^2 f \equiv div(\nabla f) = \nabla w$

Discrete Poisson equation with known boundary condition can be defined by a linear system as it results:

$$Ax=b \quad (6)$$

Furthermore, the coefficients matrix A is a symmetric positive definite matrix. So the solution of Poisson equation is reduced to solve the sparse linear system where the coefficients matrix A is determined by Eq.5 and the vector b is determined by Eq.3 and unknown vector x is the coordinate of all vertices on the patch mesh.

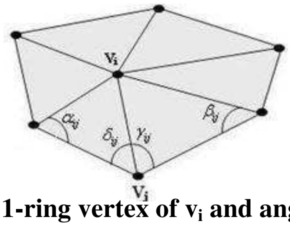

Figure 1. 1-ring vertex of v_i and angles opposite to edge v_iv_j

4. Hole filling algorithm

4.1 Identification and hole triangulation

Given the previous hypothesizes, the hole contour can be automatically identified while looking for a closed cycle of boundary edges. The Advance front mesh technique applied over the hole to generate an initial patch mesh is as follow:

First, the angle θ between two adjacent boundary edges at each vertex v_i on the front should be calculated. Next, depending on the angle between e_i and e_{i+1}, the new triangles on the plane should be built, See Figure 2. Then, the distance between new vertex and related boundary vertices is calculated; if distance between them is less than given threshold they should merge. Finally, the front should be updated and the algorithm will be repeated until the hole is patched with new triangle.

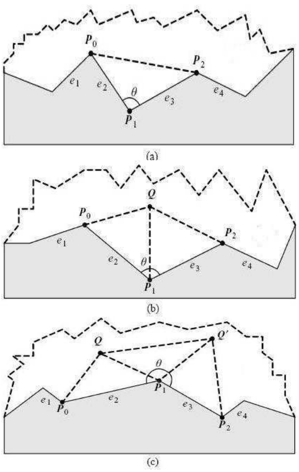

Figure 2. Initial patch mesh generation.

a) $\theta \le 75°$; b) $75° \le \theta \le 135°$; $\theta \ge 135°$

4.2 Harmonic normal computation

The most important task of the discrete harmonic function is to map a given disk like surface $S \subset R^3$ into the plane S^*.

Let V be set of vertices, S_T be a piecewise linear surface that has a boundary, V_B be the set of vertices lying on the boundary of S_T, V_I be the set of interior vertices of S_T. The goal is to find a suitable (polygonal) domain $S^* \subset R^2$ and a suitable piecewise linear mapping f: $S_T \longrightarrow S^*$. Such a mapping is uniquely determined by the images $f(v) \in R^2$ of the vertices $v \in V$.

Finite element method based on linear elements is one of the earliest methods for mapping disk-like surface into the plane to approximate a harmonic map. This methods is based on fixing the boundary mapping and minimizing the Dirichlet energy for internal vertices.

$$E_D = \frac{1}{2}\int_{S_T} \left\| grad_{S_T} f \right\|^2 \tag{7}$$

The advantage of current method includes a quadratic minimization and solving linear system of equation. For the triangle T={v_1, v_2, v_3 }in the surface S_T. The Dirichlet energy can be expressed as follow:

$$\begin{aligned} 2\int_T \left\| grad_T f \right\|^2 &= \cot\theta_3 \left\| f(v_1) - f(v_2) \right\|^2 \\ &+ \cot\theta_2 \left\| f(v_1) - f(v_3) \right\|^2 \\ &+ \cot\theta_1 \left\| f(v_2) - f(v_3) \right\|^2 \end{aligned} \tag{8}$$

The normal equations for the minimization problem can be expressed as the linear system of equations

$$\sum_{v_j \in N_i} w_{i,j}(f(v_j) - f(v_i)) = 0 \qquad v_i \in V_I \tag{9}$$

where

$$w_{i,j} = \cot\alpha_{i,j} + \cot\beta_{i,j} \tag{10}$$

The angles α_{ij} and β_{ij} are shown in the Figure 1. N_i denotes to 1-ring vertices of vertex v_i. The associated matrix is symmetric and positive, so the linear system is uniquely solvable with sparse and iterative methods such as conjugate gradients methods. Note that the system has to be solved three times, once for the x-, once for the y-, and once for the z-. Now the desirable normal of all vertices in initial patch mesh is obtained.

4.3 Utilizing the Poisson equation to optimize the new mesh

In this section, we imply the Poisson equation according to the desirable normals and the boundary vertices of the hole to approximate the missing geometries more accurately.

Poisson equation requires a discrete guidance field, defined on the triangles of the patch mesh. We applied the local rotation to each triangle of initial patch mesh in order to construct guidance vector field in Poisson equation.

Local rotation can be obtained by rotating original normal of each triangle in patch mesh to new normal of triangle around center of the triangle. After triangle rotation is performed all triangles on the patch mesh turn to a new direction. So triangles on the patch mesh are not connected anymore and these torn triangles are used to construct a guidance vector filed for the Poisson equation, See Figure 3. Once a discrete guidance filed is given, its divergence at the vertex can be computed.

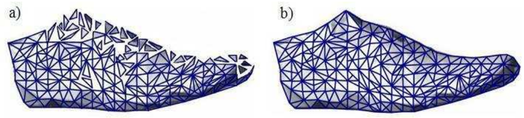

Figure 3. a) Each triangle is locally rotated and triangles becomes disconnected. b) The Poisson equation sticks together new triangles for obtaining smooth triangle mesh

The disconnected triangle are stuck and smooth and accurate patch mesh is reconstructed base on Poisson equation as follow:
First, for each new vertex on their adjacent triangles compute gradient using Eq.4. Next, the divergence of every boundary vertex by using Eq.3 should be calculated. Then, the coefficient matrix A by Eq.5 should be determined and vector b in this equation is determined by using divergence of all boundary vertices. Finally, the new coordinate of all vertices of patch mesh by solving the Poisson equation will be obtained.

5 Alignment of 3D model

The obtained data from 3D scanning are given in arbitrary position and orientation in the space. However for evaluating footwear fit, there is a need for measuring similarity of 3D foot model with shoe last and selecting shoe last from the shoe last data base. Thus, 3D foot model must be properly positioned and aligned before shape analyses such as measuring similarity. We assume the all shoe last in data base are aligned parallel with x-y space from heel to toe. Thus, eigenvectors of all models in shoe last data base are in the same position. see Figure 4.

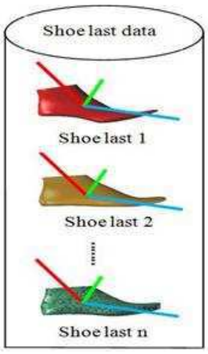

Figure 4. All the models are aligned, parallel with x-y space form heel to toe. Red, green, blue lines are eigenvectors that are in equal positions for all the models

Let $T=\{t_1, \ldots, t_n\}$ ($t_i \subset R_3$) be a set of "triangle mesh", $V=\{v_1, \ldots, v_n\}$ ($v_i= (x_i, y_i, z_i) \in R^3$) be a set of "vertices" associated to triangle mesh, matrix O_m be the position of eigenvetors of shoe last as column and c_{om} be the "center of gravity of a foot model". The main steps of "weighted" PCA is described the following steps:

Step 1. The translation invariance is accomplished by finding the center of gravity of a model and forming the point set $I=\{v_1- c_{om}, \ldots, v_n- c_{om}\}$.

Step2. The covariance matrix C (type 3*3) can be determined by:

$$C = \begin{bmatrix} \mathrm{cov}_{xx} & \mathrm{cov}_{xy} & \mathrm{cov}_{xz} \\ \mathrm{cov}_{yx} & \mathrm{cov}_{yy} & \mathrm{cov}_{yz} \\ \mathrm{cov}_{zx} & \mathrm{cov}_{zy} & \mathrm{cov}_{zz} \end{bmatrix} \quad (11)$$

$$\mathrm{cov}_{xx} = \sum_{i=0}^{n} \frac{A_i (ct_i x - ctx)(ct_i x - ctx)}{A}$$

Where A be total sum of the areas of all triangles in the mesh, A_i be the area of triangle i within the mesh, let ct_i be "center of gravity of each triangle" and ct the total sum of "center of gravity" of all of triangles in mesh.

Matrix C is a symmetric real matrix, therefore its eigenvalues are positive real numbers. Then, we sort the eigenvalues in the non-increasing order and find the corresponding eigenvectors. The eigenvectors are scaled to the Euclidean unit length and we form the rotation matrix R, which has the scaled eigenvectors as rows. We apply this matrix to I and we set a new vertex sets called: I′

Step 3. Let matrix N_m be transpose of a matrix R. The alignment is accomplished by constructing a rotation matrix R′ through the following formula:

$$R' = O_m \times N_m$$

Step 4. We apply this matrix to I′ and calculate new point set .
Figure 5 shows the steps of the current method.

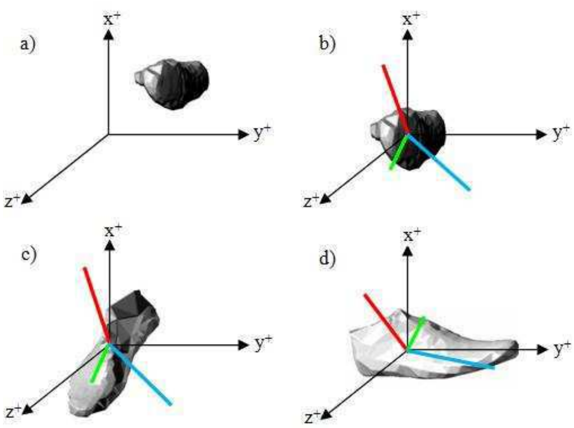

Figure 5. a) Initial Steps of the method. a) Input 3D model. b) Translated center of gravity to the origin. c) Rotated 3D model. d) Alignment of 3D foot with shoe last data base

6 Conclusions

This paper has proposed post-processing steps of 3D scanning data before taking 3D models to the real application.

We proposed a robust and an efficient algorithm for filling holes in triangle mesh. For this purpose, we used the advance front mesh technique to generate initial patch mesh. Then the desirable normals based on the Harmonic function is calculated for modifying initial patch mesh. Finally an accurate and smooth triangle mesh is generated by solving the Poisson equation according to desirable normals and boundary vertices of hole. After obtaining complete 3D models, we applied the "Weighted" Principle Component Analysis technique for alignment of 3D foot with shoe last data base.

After these substantial post processing methods, the 3D foot model is ready for sophisticated modeling operations.

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