

An Image Matching Using Critical-Point Filters and Level Set Analysis

Kittipong Thanasoontornlerk
Department of Computer Engineering
Faculty of Engineering
Chulalongkorn University
4870221721@student.chula.ac.th

Pizzanu Kanongchaiyos
Department of Computer Engineering
Faculty of Engineering
Chulalongkorn University
pizzanu@cp.eng.chula.ac.th

ABSTRACT

Critical-Point Filters (CPFs) are nonlinear filters which preserve intensity and location of each critical point in the image and reduce the resolution without any prior knowledge. Although CPFs can avoid blurred intensity and ambiguous location problem of previous linear filters, its computational cost is still expensive due to its complexity. We propose an enhancement of the CPFs algorithm for image matching using level set analysis. An image is analyzed and transformed to hierarchical level sets of pixel having same intensity. Connectivity of the level sets represents the image contrast invariant features. Between the corresponding level sets of the input images, two pixels are mapped based on their energy and bijectivity conditions. Finally, less computational time with precise mapping is shown in the experimental result.

Keywords

Image morphing, Feature measurement, Level set, Critical-point filter

1. INTRODUCTION

Image matching is an important problem in many researches such as computer graphics, computer animation and computer vision [BJNS03, Jcu02a, Jcu03a, Jdu02c]. However, there isn't efficient algorithm for matching images because algorithms for image matching usually trade off between complex computation and accuracy. Image Filter is a favored method for matching because the principle of image filter is the enhancement and attenuation of image attributes such as edges or the intensity of the image. Afterwards, Images are matched using image attributes. Filters are classified into two groups: linear filters and non-linear filters. The Linear filters are not useful because of its blurred intensity and ambiguous location. For example, the stereo photogrammetry methods use the edge detection which is high-pass filter. This method suffers from noise because high-pass filters can also detect noise. [Chr96a, YT98].

Multiresolutional Critical Point Filters (CPFs) are non-linear filters which effectively and accurately match two images. This filter extracts a maximum, a minimum, and two types of saddle point of pixel intensity for every small region which the results are four multiresolution hierarchies. The matching of the images is computed in each hierarchy, each resolution, from coarse level to the fine level.

CPFs are the same concept as an image pyramid [Fra01a, RR02a]. Image pyramid is multiple resolution of the original image which is reduced by a factor of 2. Figure 1 and Figure 2 show examples of image pyramid.

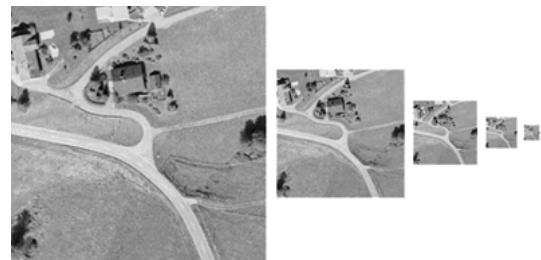


Figure 1. Example of an image pyramid [Chr96a]

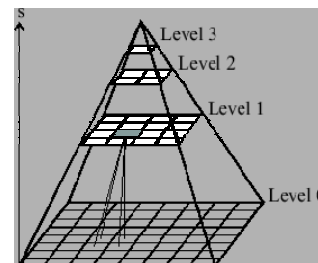


Figure 2. Example hierarchy of image pyramid [Fra01a]

The structure of an image pyramid is determined by 2 directions: The horizontal relations between neighborhoods in the same level and, the vertical

relations; parent-child relations, between adjacent levels. Image pyramid combines the advantages of both high and low resolutions of image following table 1.

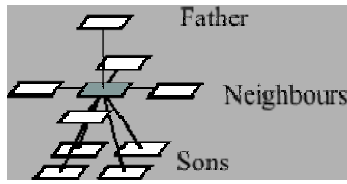


Figure 3. Example structure of image pyramid [Fra01a]

	High resolution	Low resolution
Data amount	huge	small
detail	rich and many	very few
overview	bad	Low
precision	high	Low

Table 1. Qualities of images at different resolutions [Fra01a]

However, matching two images by using CPFs requires complicated computation and redundancy of image data because of hierarchy of multiresolution.

In this research, level set analysis [PAD02a] is used for creating regions in each hierarchy. The matching of two images uses iteratively searching corresponding region. The image level set is a hierarchy of the gray intensity level image. An image will be transformed to image level sets. Connected components in each level are found. The components which have the size of connected component less than a pre-defined value will be removed.

Section 2 introduces the definition of CPFs, and Section 3 introduces the definition of image level set. Correspondence level set presents in Section 4. The results of experiment are shown in Section 6. The last section is the conclusion and future work.

2. THE MULTIREOLUTIONAL CRITICAL POINT FILTER

Multiresolutional Critical Point Filters (CPF) are introduced by Shinagawa and Kunii [YT98]. CPFs are non-linear filters that extract the critical point of the image in each resolution and do not destroy the essential structures of the critical points in image. The height and the width of the image are equal, power of two. The matching method uses an energy equation; based on pixel intensity and location, to search point correspondence between two images which have minimized energy.

2.1 Critical Point Filters

CPFs create four multi-resolution hierarchies that represent minimum, two saddles and maximum point of image; following equation 1 to equation 4. In other words, CPFs extract a maximum, a minimum, and two types of saddle points of intensity pixel for every 2×2 (horizontal \times vertical) pixels. Then, an image of low level of resolution is generated from an image of high level of resolution for each type of a critical point. Let width and height of image be N and M , respectively. Thus, $N = M = 2^n$. A pixel of image at position (i, j) is defined by $p_{(i,j)}^{(m,l)}$, where m represents the level of the hierarchy.

$$p_{(i,j)}^{(m,0)} = \min(\min(p_{(2i,2j)}^{(m+1,1)}, p_{(2i,2j+1)}^{(m+1,1)}), \min(p_{(2i+1,2j)}^{(m+1,1)}, p_{(2i+1,2j+1)}^{(m+1,1)})) \quad (1)$$

$$p_{(i,j)}^{(m,1)} = \max(\min(p_{(2i,2j)}^{(m+1,1)}, p_{(2i,2j+1)}^{(m+1,1)}), \min(p_{(2i+1,2j)}^{(m+1,1)}, p_{(2i+1,2j+1)}^{(m+1,1)})) \quad (2)$$

$$p_{(i,j)}^{(m,2)} = \min(\max(p_{(2i,2j)}^{(m+1,1)}, p_{(2i,2j+1)}^{(m+1,1)}), \max(p_{(2i+1,2j)}^{(m+1,1)}, p_{(2i+1,2j+1)}^{(m+1,1)})) \quad (3)$$

$$p_{(i,j)}^{(m,3)} = \max(\max(p_{(2i,2j)}^{(m+1,1)}, p_{(2i,2j+1)}^{(m+1,1)}), \max(p_{(2i+1,2j)}^{(m+1,1)}, p_{(2i+1,2j+1)}^{(m+1,1)})) \quad (4)$$

where

$$p_{(i,j)}^{(n,0)} = p_{(i,j)}^{(n,1)} = p_{(i,j)}^{(n,2)} = p_{(i,j)}^{(n,3)} = p_{(i,j)}$$

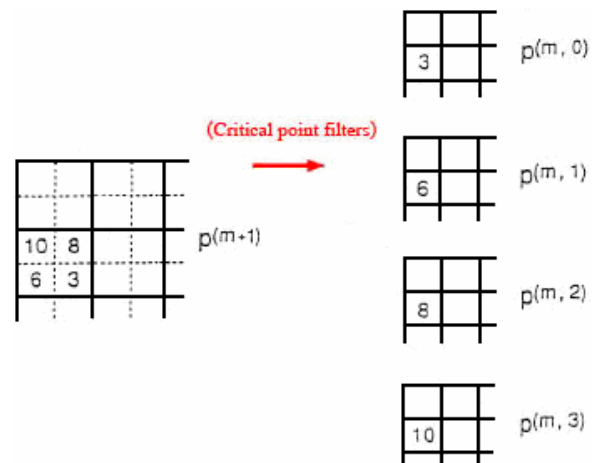


Figure 4. Procedure of CPFs

The four images are called as subimage having size $\frac{1}{4}$ of the higher level. Figure 4 shows the example of CPFs, which a value in the picture is an intensity value of image.

2.2 Hierarchies

Four multiresolution hierarchies at depth d of each input image are computed, where the size of each image at m^{th} level is $2^m \times 2^m$ ($0 \leq m \leq n$) pixels. Figure 5 shows an example of three hierarchies which original image is 8×8 pixels.

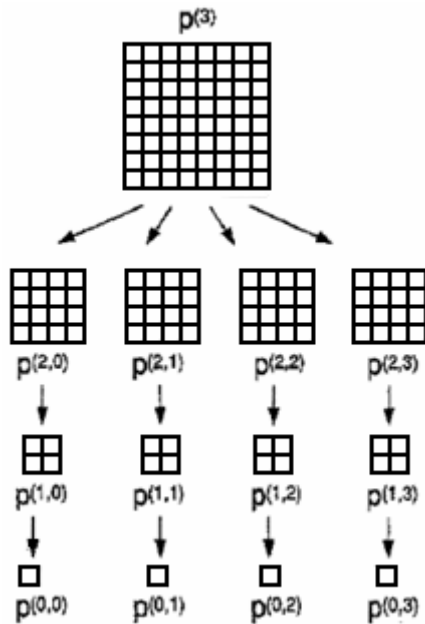


Figure 5. Multiresolutional hierarchy

Figure 6 shows the result of CPFs, Original image is eight-bit gray-scale image having size 256×256 pixels. Subimages are size 64×64 pixels at 6 levels of hierarchies. Both original image and subimages shows same resolution. In each subimage, checkerboard pattern (or aliasing) happens around the borders of object [RR02a].

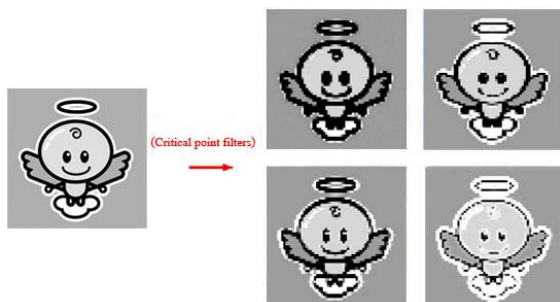


Figure 6. The Original image and subimages at level 6 of the hierarchy (original image is size 256×256 pixels and subimage is 64×64 pixels)

2.3 Image matching

In matching two images, the first image is called the *source image* and the second is called the *destination image*. Let $p_{(i,j)}^n$ be the pixel of the source image at the position (i, j) , and $q_{(k,l)}^n$ be the pixel of the destination image at the position (k, l) .

Parent-child relationships are defined by: $p_{(i',j')}^{(m-1,s)}$ at level $m-1$ is called *parent* of $p_{(i,j)}^{(m,s)}$ at level m where $(i', j') = (\lfloor \frac{i}{2} \rfloor, \lfloor \frac{j}{2} \rfloor)$. Conversely $p_{(i,j)}^{(m,s)}$ at level m is called *child* of $p_{(i',j')}^{(m-1,s)}$ at level $m-1$. The function $parent(i, j)$ is defined as

$$parent(i, j) = (\lfloor \frac{i}{2} \rfloor, \lfloor \frac{j}{2} \rfloor)$$

Point p in source image will be mapped to point q in destination image at level m . Point a, b, c and d are 4 nearest diagonal neighbors of point p and they are children of A, B, C and D at level $m-1$. Point A, B, C and D of the source image are mapped to A', B', C' and D' of the destination image at level $m-1$. The pixel p should be mapped to the pixel q inside of inherited quadrilateral $A'B'C'D'$.

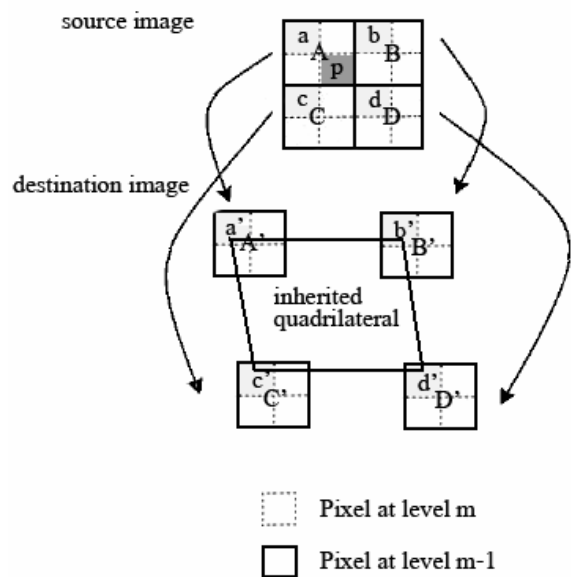


Figure 7. The inherited quadrilateral of pixel p

Finding corresponding point between two images use an energy function determined by the difference in the intensity of the pixel of the source image and its corresponding. Points are mapped recursively by computing and comparing the energy of candidate points located in the inherited quadrilateral

2.4 Energy of mapping

Let $p_{(i,j)}$ be the point to map and $q_{(k,l)}$ be the point to test within the quadrilateral inheritance. Between two points is computed using energy, find minimum energy, for each subimage and at each level of the hierarchy

2.4.1 Cost related to the pixel intensity

The energy determines difference of intensity in source image pixel and its corresponding pixel of the destination image.

$$C_I = \left| V(p_{(i,j)}) - V(q_{(k,l)}) \right|^2$$

where $V(p_{(i,j)})$ and $V(q_{(k,l)})$ are the intensity values of the pixel $p_{(i,j)}$ and $q_{(k,l)}$, respectively.

2.4.2 Cost related to the locations of the pixel for smooth mapping

This energy is used to prevent a pixel from being mapped too far away and to ensure the smoothness of mapping. D is determined by the value difference locations.

$$D = \eta E_0 + E_1$$

where $\eta \geq 0$ is a real number and

$$E_0 = \left\| (i, j) - (k, l) \right\|^2$$

$$E_1 = \sum_{i=i-1}^i \sum_{j=j-1}^j \left\| ((k, l) - (i, j)) - ((k', l') - (i', j')) \right\|^2 / 4$$

where $\left\| (x, y) = \sqrt{x^2 + y^2} \right\|$. (i', j') is defined to be zero for $i' < 0$ or $j' < 0$.

E_0 prevent a pixel being mapped to a pixel too far away. E_1 ensure the smoothness of the mapping.

2.4.3 Total energy of the mapping

The total energy of the mapping is defined as $\lambda C_f^{(m,s)} + \gamma D_f^{(m,s)}$. The matching finds corresponding points which have minimum energy.

$$\min \lambda C_f^{(m,s)} + \gamma D_f^{(m,s)}$$

Where λ and γ are constants (0.1 and 0.01 respectively [Jcu02a])

3. IMAGE LEVEL SET

Let Ψ be the set of all intensity values in gray-scale image. In 8-bits gray-scale image will be $\Psi = \{0, 1, 2, \dots, 255\}$. Let $Z_m = \{1, 2, \dots, m\}$ by M be height of the image, and $Z_n = \{1, 2, \dots, n\}$ by N be width of the image. Let λ be real function represents the spatial information of the image; $\lambda : Z_m \times Z_n \rightarrow \Psi$. If z be a pixel in the image, $z \in Z_m \times Z_n$, and the corresponding intensity value $\zeta \in \Psi$, then we can define $\lambda(z) = \zeta$.

3.1 Level set

Level sets [PAD02a] and its properties are defined as following;

Definition: For $\zeta \in \Psi$, consider the following sets

ω is a set of intensity which has value more than or equal to ζ . Define by

$$\omega_{\zeta \in \Psi} = \{x \in \Psi : x \geq \zeta\}$$

θ is a set of pixel which has intensity value equal to y . Define by

$$\theta_{\zeta \in \Psi} = \{z \in Z_m \times Z_n : \exists y \in \omega_{\zeta \in \Psi} \ni$$

$$\lambda(z) = y\}$$

Consider the function $L_\zeta : Z_m \times Z_n \rightarrow \{0, 1\} \ni$

$$L_\zeta(z) = \begin{cases} 1 & \text{if } z \in \theta_{\zeta \in \Psi} \\ 0 & \text{otherwise} \end{cases}$$

The matrix representation of L_ζ is binary image which is called a level set and ζ is corresponding level (see Fig.8).

Basic properties of the level sets are also defined as the following :

Theorem 1. The total pixel-wise sum of all the level sets is equal to the image.

Lemma 1: $\forall z \in Z_m \times Z_n, \forall \alpha, \beta \in \Psi : \alpha > \beta$ then $L_\alpha(z) = 1 \Rightarrow L_\beta(z) = 1$ but not conversely.

Lemma 2: $\forall z \in Z_m \times Z_n, \forall \alpha, \beta \in \Psi : \alpha > \beta$ then $L_\beta(z) = 0 \Rightarrow L_\alpha(z) = 0$ but not conversely

Lemma 3: $\forall z \in Z_m \times Z_n, \lambda(z) = \alpha \in \Psi$, then $L_\alpha(z) = 1$ and $L_{(\alpha+1)}(z) = 0$

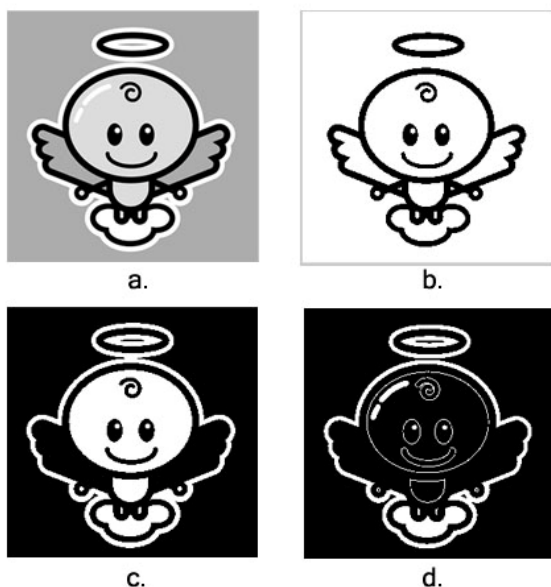


Figure 8. Level sets of an image (a) Original image (b) Level set at intensity 64 (c) Level set at intensity 192 and (d) Level set at intensity 224

A Level set of 8-bits image can be reachable to 255 levels. In Figure 8, three examples of level sets of image at 64, 192 and 224 are shown respectively. Each figure is a binary image; white represents pixel valued one, and black represent pixel valued zero.

In our method, the corresponding level sets are used for matching instead of corresponding pixel having minimum value of intensity. The period of intensity is considered from histogram of image. The detail is described in Section 4.

4. CORRESPONDANCE LEVEL SET

This section explains how to use level set for reduce computational time of the original CPFs. The level set is initially bounded by a range of intensity whereas its threshold is defined from the histogram of the input image. Figure 9 shows an example image and its histogram which we can classify level of the image into 7 levels concerning about its histogram.

The computation of four multiresolution hierarchies of image following equation 1 to equation 4 is then composed. For each resolution, level set and connecting components of an input image are extracted at the same time.

Matching between corresponding components is then computed by comparing between level sets at each hierarchy of the two images from minimum level to the maximum level. The corresponding level set is then mapped. The mapping algorithm is as follows:

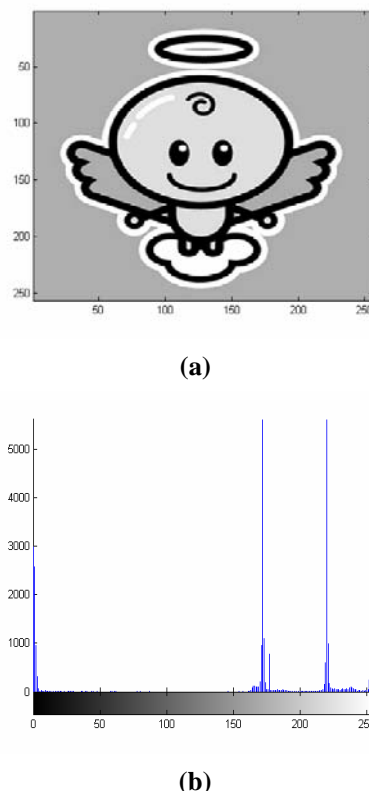


Figure 9. Computing histogram of an image (a) Original image (b) Histogram of original image

1. Filter the input images with critical point filters following equation 1 to equation 4.

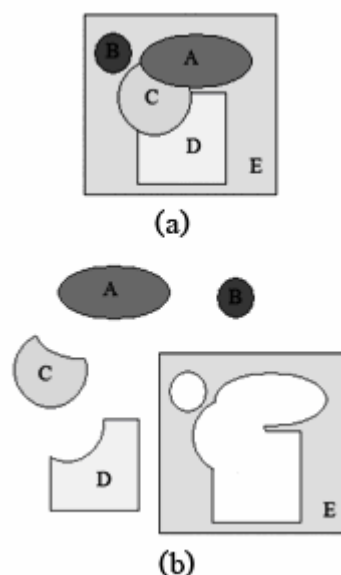


Figure 10. Example of extracting image components

2. Find connecting components in each level set of image for every level of hierarchies. Figure 10 shows an example of input image having five level sets, A B C D and E, which are divided to five connecting components.

3. Map between connecting components of source image and destination image in corresponding level set and level of hierarchies having shortest average distance between corresponding pixels in connect components according to parent – child relationship starting from coarse level to the fine level.

5. Map between pixels in corresponding connecting components by using energy of intensity and energy of location. Figure 11 displays an example of mapping at a fine level having three level sets.

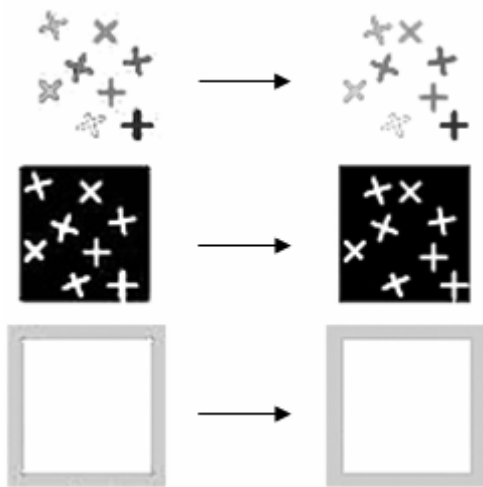


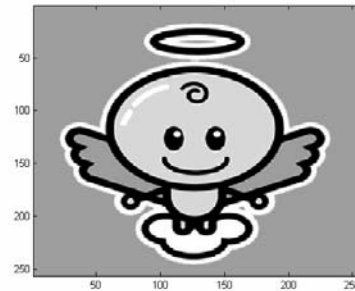
Figure 11. Example of mapping in fine level

5. TIME COMPLEXITY

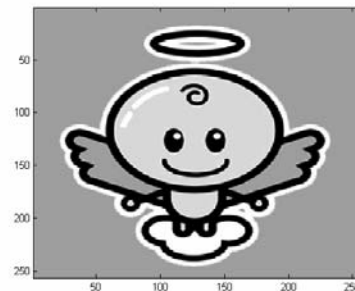
The time complexity of filtering of the image size ($n*n$) is $\theta(n^2)$, while mapping between input images can be reduced to mapping between two graphs having an edge between each connecting component in each level set and hierarchical level represented as a node. Although the subgraph isomorphism is categorized in the complexity class *NP-complete*, the number of nodes here depends on the similarity of the input image and the size of level set which is adjustable. Fastest known subgraph isomorphism can test most graphs of less than 100 vertices in well under a second [BD81]. The worst case happens when the size of the graph grows up to $O(n^2)$ which equals to the original CPFs.

6. EXPERIMENTAL RESULTS

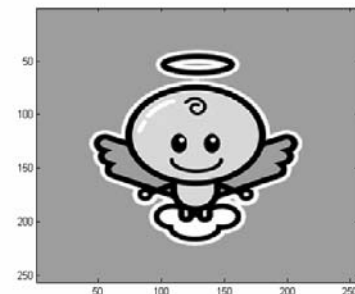
The experiment of image matching based on the proposed algorithm is conducted by MATLAB 7.0. In the experiment the destination images are applied by affine transformations. Figure 12 shows examples of source image and destination image in our experiment. Figure 13 shows a result from our method. Blue pixels in source image are mapped to blue pixels in destination image properly. Figure 14 and 15 graph present correspondence of point between source image and destination image.



(a)

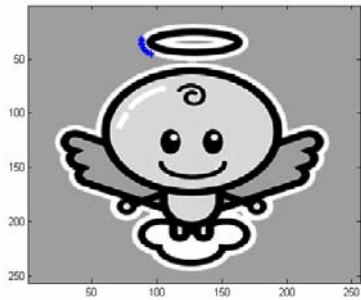


(b)

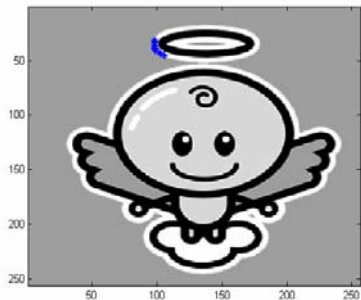


(c)

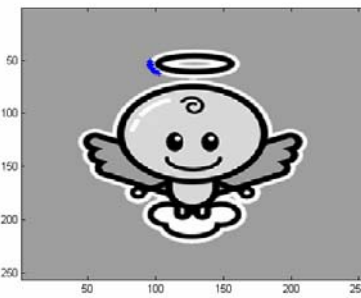
Figure 12. Input images a) source image (b) destination image shifted by 10 pixels in x axis (c) destination image reduce size 20 % of source image



(a)



(b)



(c)

Figure 13. Result of mapping (a) source image (b) destination image shifted by 10 pixels in x axis and (c) destination image reduce size 20 % of source image

7. CONCLUSION AND FUTURE WORK

We propose an enhancement of the critical point filters algorithm using level set analysis. Our method can reduce redundancy of an image data in matching image, find corresponding pixels by original CPFs comparisons. The experiment, our method has high accuracy in matching image. This method can apply affine transformation image. Our method is work properly with clearly image, source image and destination image have the same detail in the image, and width and height of image must have equal size and power of two.

More efficient algorithms can be developed based on both faster subgraph isomorphism algorithm and faster level set method. Both matching for animating image and more complex image seem different properties or resolution is also challenged.

In the future work, we will solve imperfection of our method; matching ambiguously image, and improve efficiency of the algorithm. We will develop our method for color image.

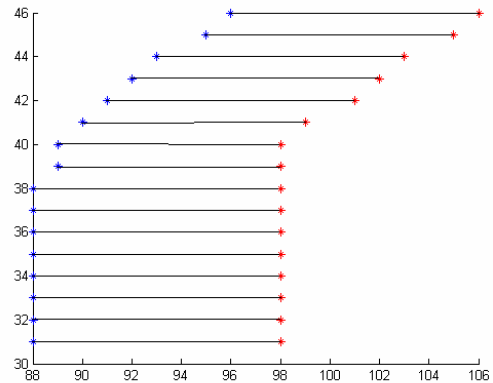


Figure 14: Points mapped from source to destination shifted by 10 pixels in x axis

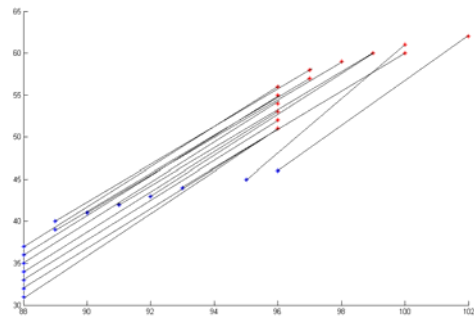


Figure 15: Points mapped from source to destination image reduce size 20 % of source image

8. REFERENCES

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