ABSTRACT

The unique characteristics of the arctic atmosphere make for very interesting effects that cannot be seen anywhere else in the planet. The extremely cold temperatures, along with the existence of inversion layers in the temperature gradients, make the medium highly inhomogeneous. Its properties, including the index of refraction that rules the behavior of light, are no longer constant. As a consequence, light rays get bent while traversing the atmosphere, and the result is some spectacular phenomena; several examples are the arctic mirages (that have probably given rise to numerous ghost ship legends), the Fata Morgana or the Novaya-Zemliya effect.

We present here an implementation of a ray tracer that can render all these effects, thus depicting phenomena never ray-traced before, as far as the authors know. We first build an accurate temperature profile for the arctic atmosphere, based on experimental data, then calculate the curved paths of the light rays as the index of refraction changes as a function of temperature, by solving the physically-based differential equation that describes their trajectory. The scenes are modelled using real data for the Earth and Sun dimensions and relative distance, thus maintaining accuracy in the results obtained.

Keywords
Curved ray tracing, inhomogeneous media, arctic atmosphere

1. INTRODUCTION

Refraction of sunlight due to cold dense polar air causes a number of spectacular effects almost impossible to see anywhere else in the planet. Probably the best-known is the arctic mirage, but even that one is constantly overshadowed by its hot-air equivalent, the inferior mirage (also known as desert mirage or the “water on the road” effect). Other phenomena such as the Fata Morgana or the Novaya-Zemliya remain mostly unknown.

We present here our ray tracing method to reproduce some of these spectacular phenomena. It is based on an accurate model of the temperature profile of the atmosphere, along with a physically-based resolution method that allows us to bend light correctly. The method is obviously valid for any inhomogeneous medium, but here we concentrate on the arctic atmospheres, where weather scientists have been gathering data for decades now.

The rest of the paper is organized as follows: section 2 offers an overview of previous works on curved ray tracing. Section 3 describes how light bends in an inhomogeneous atmosphere. Section 4 presents our rendered simulations, including the model of the atmosphere and the 3D scene settings. The last two sections discuss the conclusions and the future work.

2. PREVIOUS WORKS

There are several light effects in nature that are owed to the atmosphere being inhomogeneous, which causes the rays to travel in curved paths. There are few works which have already been published on the matter. In fact, none of them can handle generic situations where the index of refraction varies in an arbitrary way. They are either limited to a specific distribution of the index of refraction in the medium [Ber90] [Mus90] [Sta96], or they do not specify a...
direct relation between the curved paths of the rays and the index of refraction [Gro95]. In this paper, we use a general ray tracing method, based on Fermat’s Principle, which is valid for any inhomogeneous medium [Ser04].

3. LIGHT IN THE ATMOSPHERE

Fermat’s Principle

When a medium is inhomogeneous and the index of refraction changes continuously from point to point, the trajectory of light is affected at any differential step. The final result of this differential change is a curved path that distorts the normal view of a real scene, which cannot be calculated by traditional ray tracing. We apply Fermat’s principle to obtain the curved trajectory of the light rays, formulated as “light, in going between two points, traverses the route \( l \) having the smallest optical path length \( L \)”. From this Principle, the equation to obtain the trajectory of a light ray in an inhomogeneous medium with a known index of refraction is [Ser04]:

\[
\frac{d}{dl} \left( n \frac{dr}{dl} \right) - \nabla n = 0 \leftrightarrow \frac{d}{dl} \left( n \frac{dx_j}{dl} \right) - \frac{\partial n}{\partial x_j} = 0
\]

where \( l \) is the length of the arc, \( n \) is the index of refraction of the medium and \( r = x_j \) with (\( j=1,2,3 \)) are the coordinates of the point. If the index of refraction \( n \) is known for each point, the equation can be solved numerically by applying an embedded Runge-Kutta Dormand-Prince method.

Inversion layers

The atmosphere is not a homogeneous medium. Properties such as pressure, temperature or density change from point to point. It is the inversion layers the one that really explains the light effects that we show in this paper. Without inversion layers, none of the effects shown here can take place. In a normal atmosphere, temperature decreases with height. An inversion layer reverses the situation, having colder air below and warmer air higher up. This obviously occurs more easily in the polar regions, where the ice or the cold seawater chills the lowest air layers. Since cold air is heavier than warm air, they tend to be very stable, lasting even for hours.

4. RENDERED SCENES

The most important step in our simulations is to obtain a correct temperature profile, and to derive from it an accurate index of refraction profile for the whole atmosphere. That index alone is what will curve the rays as they traverse the medium. For modeling the atmosphere, we have used our previously published Atmosphere Profile Manager [Gut04].

Scenes setup

We have designed scenes with very simple geometry, but with very accurate atmospheric profiles and using real-world distances and dimensions. The Earth is always modeled as a sphere of 6371 kilometers; the atmosphere is 40-kilometer high; the features to appear as mirages in the horizon are textured cards placed up to 26 kilometers away from the camera; the camera is finally placed ten meters above the Earth’s surface. For the Novaya-Zemlya effect, the sun is a circle with a 1392000-kilometer diameter, placed 150 million kilometers away from the camera, and the camera has been placed 100 meters above sea level. All the inversion layers will have a thickness of fifteen meters, whereas their location and their gradients will be specified for each effect.

The simplicity of the whole setup is irrelevant, though, since it is the curvature of the ray traversing the medium the only factor that matters. This correct curvature is guaranteed by a) a precise atmosphere profile, b) using real geometric dimensions and c) accurately solving the physically-based equation that governs the paths of the light rays.

The images were rendered at 400x200 pixel resolution, one ray per pixel, on a Pentium IV at 2.8Ghz, and each one took between two and two and half minutes to render. The error tolerance for the numerical resolution of the equation was one centimeter.
Results

When the temperature of the lower atmosphere rises at a rate of 11.2°C per 100 meters, the light rays bend in an arc exactly equal to the curvature of the Earth, and the horizon will appear flat, like an infinite plane. If the inversion gradient becomes even stronger, the light rays exceed the curvature of the Earth, and the horizon will appear to rise vertically from the flat position. This way the distortion of the horizon can be seen. Figure 1 shows the scene rendered without curvature of the rays, and therefore the horizon appears undistorted. Figure 2 shows how the horizon gets distorted when taken into account the real paths that light travels in a polar atmosphere.

The arctic mirage occurs when there is an increase of temperature with height near the ground. As a consequence, the light rays approaching the ground are curved downward. Figures 3 and 5 show real pictures of arctic mirages while figures 4 and 6 show the respective rendered simulations. For both simulations, an inversion layer was placed centered at 100 meters, with a temperature gradient of 10°C.

The inferior mirage occurs when there is a strong decrease of temperature with increasing height. As a consequence, the light rays approaching the ground are curved upwards, generating an inverted image of the object in the ground. Although less frequent, inferior mirages can sometimes be seen also in arctic latitudes. Figure 7 shows a picture of a polar inferior mirage, and figure 8 shows our rendered simulation of the phenomenon, with an inversion layer at ground level and a 5°C gradient.

The mirage known as Fata Morgana occurs when there are several alternating cold and warm layers of air near the ground or the sea, causing a multiple concatenation of inferior and superior mirages, that result in a complicated superposition of images of the same object (figure 9). The number of inversion layers usually varies between two and five, with the images alternating between upright and inverted. Figure 10 shows our ray-traced image, with two inversion layers: one centered at 80 meters with a 5°C gradient, and the second one at 120 meters with a steeper, 15°C gradient.
The Novaya-Zemlya mirage happens when celestial objects, such as the sun, can be seen even when they are situated below the horizon. Steep arctic temperature inversions trap the sunrays, making them travel within an inversion layer for hundreds of kilometers (a phenomenon known as ducting), so the light bends following the curvature of the Earth over that long distance, effectively showing up above the horizon and distorting the sun into an unusual rectangular shape. Figures 11 and 12 show a real Novaya-Zemlya effect and our ray-traced image. The inversion layer at ground level has a gradient of 11.2°C per 100 meters.

5. CONCLUSIONS
We have presented here a work that overcomes the limitations of previous works (section 2), by accurately modeling a profile of the index of refraction in the atmosphere, based on real inversion layers data obtained by scientists doing climate research in the poles. Using also real-world dimensions for our 3D scenes, we obtain the curved paths of the rays by solving the differential equation that describes their trajectory, based on Fermat’s Principle. We have simulated several of the most spectacular effects that are owed to light traveling in curved paths: the distortion of the horizon, the arctic mirage, the inferior mirage, the Fata Morgana and the Novaya-Zemlya.

6. FUTURE WORK
Still lots of work lay ahead. The number of light effects owed to inhomogeneous media does not end with the ones presented here. Adding polarization effects to our system, we could also aim at obtaining halos, sun dogs, glories... We also want to produce animations of the effects, so we need further research to find out how the index of refraction changes with time. Finally, we would like to be able to speed up the rendering times, either by solving Fermat’s Principle with a faster numerical method or by implementing a parallel ray tracer.

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8. REFERENCES