SHAPE MATCHING USING SET OF CURVE GEOMETRIC INVARIANT POINT

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ABSTRACT

We introduce a non-iterative geometric-based method for shape matching using a novel set of geometric landmarks residing on a 2D contours. These landmarks are intrinsic and are computed from the differential geometry of the curve. We exploit the invariant properties of geometric landmarks that are local and preserved under the affine and some perspective transformation. Geometric invariant exploits coplanar five-point invariant and ration of area constructed from a sequence of consecutive landmarks. These invariants are preserved not only in affine map but weak perspective map as well. To reduce the sensitivity of the landmarks to noise, we use a B-Spline surface representation that smoothes out the curve prior to the computation of the landmarks. The matching is achieved by establishing correspondences between the landmarks after a conformal sorting based on derived absolute invariant and registering the contours. The experiments have shown that the purposed methods are robust and promising even in the presence of noise.

Keywords Geometric Invariant, Affine Invariant, B-Spline

1. INTRODUCTION

Shape matching is a central problem in visual information system, computer vision, pattern recognition image registration, and robotics. Application of shape matching includes image retrieval, industrial inspection, stereo vision, and fingerprint matching. The term shape is referred to the invariant geometrical properties of the relative distance among a set of static spatial features of an object. These static spatial features are known as shape features of the object. After extracting the shape features for a model and a scene, a similarity may be used to compare the shape features. The similarity measure is referred to as a shape measure.

The shape measure should be invariant under certain class of geometric transformation of the object. In the simple scenario, shape measures are invariant to translation rotation and scale. In this case, the shape measures are invariant under similarity transformation. When included the invariance of shape measures to shear effect, the shape measures are said to be invariant under affine transformation. Finally in the complicated case, shape measures are invariant under perspective transformation when included the effect caused by perspective projection.

There are many techniques available to shape matching which can be classified mainly into two main categories; a global method and a local method. The global method works on an object as a whole; while the local method on a partially visible object or occlusion. The well-known global method includes Wavelet transform [Job95a, Wan97a], Moment-based approach [Hu62a], Fourier descriptors [Bri68a, Zah72a, Ott92a] and Median Axis Transformation [Blu67a, Blu78b, Pel81a]; while the well-known local method exploits intrinsic properties of the shape which includes geometric invariant [Bes88b, Gov99a, Mun92a], curvature extrema point [Bol82a, Mil89a, Chi89a, Rao94a], zero-curvature points.
As a result, \( r(1)(t) \) and \( r(2)(t) \) are parallel at the or the points at which

\[
\text{transformed inflection points of the original curve}
\]

or the points at which

points of the affine transformed curve are the

Inflection points on the affine transformed curve are

inflection points, the curvature or affine curvature is

which are extracted from the

perspective) invariants which are extracted from the

local geometric projective (affine and weak

landmark. We then establish the correspondence

registration. We use the set of fiducial points as a

feature. The maximum order of derivative of curve

affine invariant and the other three features

of which associate with points of minimum relative

threshold is selected. As a result, maximum affine

maximum affine curvature points are more robust to

noise. Moreover, threshold of affine curvature can be

set such that only the maximum affine curvature

point of which its affine curvature exceeding the

threshold is selected. As a result, maximum affine

curvature points caused by local disturbance are

excluded.

The same concept can be applied to derive a set of
derivative of curve. Table 1 shows a set of geometric

feature. Table 1 shows a set of geometric

features used as the candidates for curve matching

[Wal98a].

If we take derivative of (2.1), we have

\[
\begin{align*}
\dot{x}(t)\ddot{y}(t) + \dot{y}(t)\ddot{x}(t) - \ddot{x}(t)\dot{y}(t) - \ddot{x}(t)\dot{y}(t) &= (2.3) \\
\dot{x}(t)\ddot{y}(t) - \ddot{x}(t)\dot{y}(t) &= r(1)(t) \times r(2)(t) = 0
\end{align*}
\]

which is the point at which \( r(1)(t) \) and \( r(3)(t) \) are parallel and hence affine invariant. This point is the point at which the affine curvature is maximum. We call this point the maximum affine curvature point. Compared with zero affine curvature points, the maximum affine curvature points are more robust to noise. Moreover, threshold of affine curvature can be

set such that only the maximum affine curvature

point of which its affine curvature exceeding the

threshold is selected. As a result, maximum affine

curvature points caused by local disturbance are

excluded.

The same concept can be applied to derive a set of
derivative of curve. Table 1 shows a set of geometric

feature. The maximum order of derivative of curve

used in the computation of relative affine invariant is

limited at two. In such case, we have 6 features; three

of which associate with points of minimum relative

affine invariant and the other three features

associated with points of maximum relative affine.

<table>
<thead>
<tr>
<th>Relative affine Invariant</th>
<th>Feature related to the zero relative affine Invariant</th>
<th>Feature related to the maximum of Relative affine Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1(t) = \frac{r(1)(t) \times r(2)(t)}{</td>
<td>r(1)(t)</td>
<td>} )</td>
</tr>
<tr>
<td>( r(t) \times r(1)(t) )</td>
<td>( r(t) \times r(2)(t) )</td>
<td>( r(t) \times r(3)(t) )</td>
</tr>
<tr>
<td>( k_2(t) = \frac{r(1)(t) \times r(2)(t)}{</td>
<td>r(1)(t)</td>
<td>} )</td>
</tr>
<tr>
<td>( f_3(t) \equiv f_3(t) \equiv 0 )</td>
<td>( r(t) \times r(2)(t) )</td>
<td>( r(t) \times r(3)(t) )</td>
</tr>
<tr>
<td>( k_3(t) = \frac{r(1)(t) \times r(2)(t)}{</td>
<td>r(1)(t)</td>
<td>} )</td>
</tr>
<tr>
<td>( r(1)(t) \times r(2)(t) )</td>
<td>( r(1)(t) \times r(3)(t) )</td>
<td>( r(2)(t) \times r(3)(t) )</td>
</tr>
</tbody>
</table>

Table 1. Set of Geometric Feature of Curve

3. CONSTRUCT ABSOLUTE INVARIANT

Geometric invariants are shape descriptors that
remain unchanged under geometric transformations
such as perspective and affine transformation that
will be used mainly as object recognition and/or
image registration. A new approach of obtaining the invariants has been developed and used in the curve matching and recognition. A feature point explained in section 3 which is the local and invariant intrinsic property of the curve can be uniquely identified both before and after transformation. The identified feature points then are used to compute the geometric invariants which serve as the curve signature for the curve matching and recognition. The well-known perspective invariant is five-point coplanar.

Any five nonlinear points in the plane, namely \( P_1, \ldots, P_5 \) can also form perspective invariant-the so-called five-point coplanar [Eam91a] with their image, \( P'_1, \ldots, P'_5 \),

\[
\begin{align*}
\left| \begin{array}{ll}
m_{532} & m_{521} \\
m_{421} & m_{531}
\end{array} \right| &= \left| \begin{array}{ll}
m'_{532} & m'_{521} \\
m'_{421} & m'_{531}
\end{array} \right| \tag{3.1}
\end{align*}
\]

where \( m_{jk} = (P_j, P_k, P_3) \) with \( P_j = (x_j, y_j, 1)^T \), \( m'_{jk} = (P'_j, P'_k, P'_3) \) with \( P'_j = (x'_j, y'_j, 1)^T \) and \( |m| \) is the determinant of \( m \).

Since the invariant relationship in equation (3.1) holds under a perspective transformation, a set of perspective invariant can be constructed by considering 4 consecutive maximum-curvature points and a centroid which is also preserved under an affine and perspective transformation. For a curve with \( n \) maximum-curvature points, there are \( n \) set of geometric invariants denoted as \( I_k \) for \( k = 1, 2, \ldots, n \).

In the absence of noise and/or occlusion, each of \( I_k(j) \) in the affine-transformed curve (3.1) will have a counterpart \( I_l(i) \) on the original curve with \( I_k(j) = I_l(i) \), with that counterpart easily determined through a circular shift involving \( n \) comparison where \( n \) is the number of invariant. In the presence of noise, occlusion and non-linear transformation, we allow smaller error percentage between counterpart invariant. We adopted a run length method to decide on the correspondence between the two ordered set of maximum curvature point. For every starting point on the transformed, this run length method computes a sequence of consecutive invariant that satisfies \( |I_l(i) - I_k(j)| < 0.05|I_l(i)| \) and declares a match based on the longest string. Once this correspondence is found, these matched landmarks are used to estimate the transformation matrix.

4. EXPERIMENT

To demonstrate the capabilities of the shape matching exploiting a set of geometric invariance curve, we applied the algorithm for shape classification thirteen contours of fish which have the various patterns. Each contour has approximately 400 points of coordinates. The third order B-Spline with control points of 30 is employed to fit the contours of fish. To find the fish which matches with the 0th fish, a feature point \( f_1 \) is determined.

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{alignment}
\caption{The alignment of contour of fish 0th (dark contour) against thirteen contours of fish (light contour) using feature \( f_1 \) and five-point coplanar invariance}
\end{figure}

5. DISCUSSION AND CONCLUSION

This paper we explore the geometric invariant points. The fact that the geometric invariant points are local and preserved under an affine transformation makes them feasible to use for matching and classification of curve. By exploiting the absolute affine and perspective invariant using the feature points, the correspondence between the feature points on the original and the transformed curve is established, the transformation parameters are computed and the transformed curve is aligned against an original.

6. REFERENCES


