

# Optimization of a single-phase capacitor induction motor by applying a field-circuit model

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**Abstract**— The paper presents optimization of the single-phase capacitor induction motor by 2D field-circuit model combined with response surface methodology. Developed field-circuit model was implemented for simulation using Flux2D software package. The dimensions of rotor slot and capacitor capacitance are treated as input variables whereas functions describing motor performance are outputs (objective and constraints). The optimization goal is to find a motor configuration with maximal efficiency and starting torque. The model is suitable for study the effect of parameters of the motor on starting and running performance under desired load. For this purpose, the sensitivity analysis of chosen parameters had been performed before the optimization problem was formulated. Some results of motor efficiency optimization are presented in the paper.

**Keywords**—single-phase, capacitor, induction motor, optimization, efficiency.

## I. INTRODUCTION

Single-phase capacitor induction motors are commonly used as an electric drive for various appliances such as fans, blowers, pumps and compressors. By using the capacitor connected in series with the auxiliary stator winding, the auxiliary winding current leads the main winding current by somewhat less than 90 electrical degrees. The auxiliary winding and the capacitor usually are designed for better operation of the motor (e.g. to obtain high efficiency and power factor or low torque pulsations) at some desired load (a specific operating point) [1]. For this purpose the optimization methods combined with the two-dimensional field-circuit model of induction motor has been implemented in Flux2D/GOT-it environment.

## II. FIELD-CIRCUIT MODEL OF THE CAPACITOR MOTOR

For steady-state investigation of the single-phase 2 poles capacitor induction motor the 2D field-circuit model has been applied (Fig. 1). The model takes into account nonlinear material of laminated core and eddy currents induced in rotor bars. In the stator there are two windings: main ( $B_M$ ) and auxiliary ( $B_A$ ) which are distributed in 18 slots [2]. Capacitance of run capacitor placed in series with auxiliary winding is selected to provide a good performance of the motor in both conditions, starting and rated load.

### A. THE MODEL FORMULATION

For investigation of a capacitor induction motor a steady-state AC Magnetic application of Flux2D was used. In finite element 2D domain an equation of complex vector model is solved:

$$\nabla \times (\nu_0 [\nu_r] \nabla \times \underline{A}) + [\sigma] (j\omega \underline{A} + \nabla V) \quad (1)$$

where  $\underline{A}$  is a complex vector of magnetic potential,  $[\nu]$  and  $[\sigma]$  are reluctivity and conductivity of the medium,  $\nu_0$  is a reluctivity of vacuum,  $V$  is electric scalar potential.

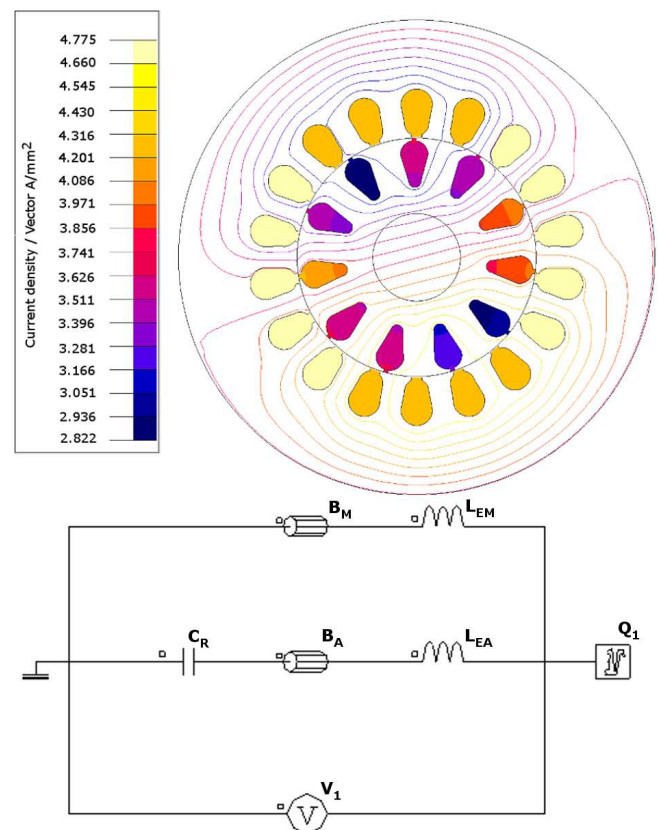


FIGURE 1. TWO-DIMENSIONAL MODEL OF TESTED MOTOR

The active power absorbed by the motor is as follows:

$$P_1 = \text{Re}(\underline{U} \cdot \underline{I}^*) \quad (2)$$

The stator Joule losses are a sum of Joule losses of main ( $B_M$ ) and auxiliary ( $B_A$ ) windings:

$$P_{J1} = R_M I_M^2 + R_A I_A^2 \quad (3)$$

Then the rotor Joule losses can be expressed with:

$$P_{J1} = s(P_1 - P_{J1}) \quad (4)$$

Motor output power can be written:

$$P_2 = P_1 - P_{J1} - P_{J2} - P_m \quad (5)$$

Then the output torque is:

$$T_m = \frac{60P_2}{2\pi n} \quad (6)$$

Since in calculation of magnetic field in the field model iron ( $P_{FE}$ ) and mechanical losses ( $P_M$ ) are not included - these losses are evaluated in post-processing when calculating output power and efficiency of the motor. Finally motor efficiency can be written as follows [3]:

$$EF = \frac{P_2}{P_1 + P_{FE} + P_m}, \quad (7)$$

and power factor of the motor is:

$$PF = \frac{(P_1 + P_{FE} + P_m)}{\sqrt{(P_1 + P_{FE} + P_m)^2 + Q_1^2}} \quad (8)$$

### III. NUMERICAL OPTIMIZATION OF MOTOR EFFICIENCY

After coupling the Flux 2D model with GOT-it optimization tool, all parameters and functions (the objectives and constraints) of the optimization are determining. As the variable parameters are chosen dimensions of rotor slots and capacitor capacitance, which values and limits are listed in table 1.

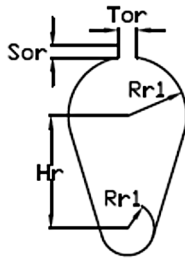


FIGURE 2. SHAPE OF ROTOR SLOT

The objective function is determined as follows:

$$\text{Min} (-EF) + 1000 (\text{Max} (0, J_{s\max}) + \text{Max} (0, P_{2\min})) \quad (9)$$

where  $EF$  is the motor efficiency and  $J_s$  is the total stator winding current density. Since the optimization is performing for one value of slip some constrains for output power were imposed ( $P_2 \geq 85W$ ).

TABLE 1 VARIABLES OF OPTIMIZATION PROBLEM

Variable	Value	Range	Step
$C_r$	3	2-6	0.5 $\mu F$
$H_r$	4.75	3.5-7.5	0.005 mm
$R_{R1}$	2.55	1.15-2.7	0.005 mm
$R_{R2}$	1.15	0.75-2.7	0.005 mm
$T_{OR}$	0.8	0.2-2	0.005 mm
$S_{OR}$	0.5	0.25-1	0.005 mm

For solving the optimization problem, the sequential surrogate optimization (SSO) procedure was applied, which uses a simplified function to find the minimum of objective function [4-6]. Basic motor geometrical and performance parameters and optimized motor parameters are listed in table 1. Inside the SSO algorithm the Chaining Optimizer, which is a combination of Genetic algorithm and Sequential quadratic programming algorithms was applied. The latter algorithm is used to find the best solution. After the optimization process, the model was solved for all range of the motor slip. Afterwards, the efficiency of optimized

motor was compared with the nominal one (Fig. 3.). Basic and optimized motor geometrical and performance parameter are listed in table 2.

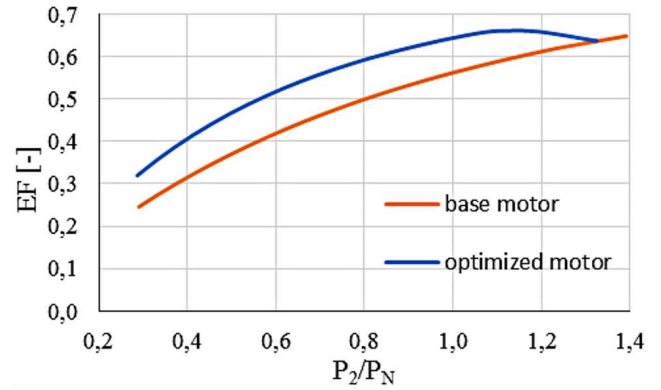


FIGURE 3. EFFICIENCY OF A BASE AND OPTIMIZED MOTOR

The rated efficiency of the optimized motor design was increased by 8.2% due to reducing power consumption of stator windings with the capacitance of the run-capacitor reduced to about 2  $\mu F$  and the height of rotor slot to 3.5 mm. Area of the rotor slot was decreased from 30.6 to 22 mm<sup>2</sup>. The rated speed of the optimized motor was established at 2823 rpm and at the same time the starting torque was reduced by 18%.

TABLE 2. BASIC AND OPTIMISED PARAMETERS OF THE MOTOR

A) PARAMETERS					
$C_r$	$H_r$	$R_{r1}$	$R_{r2}$	$T_{or}$	$S_{or}$
$\mu F$	Mm				
3	4.85	2.55	1.15	0.8	0.5
2.25	3.5	2.45	0.75	0.61	0.28

B) RATED LOAD PERFORMANCE								
EF	PF	I	$P_1$	$P_{11}$	$P_{12}$	$J_s$	$T_m$	$P_2$
%	[-]	A	W	W	W	A/mm <sup>2</sup>	N·m	W
56	0.83	0.81	153	45.6	5.3	3.38	0.303	90
64.2	0.86	0.61	140	36.5	5.9	2.59	0.305	90.5

### IV. CONCLUSION

The implemented 2D field-circuit model of the single-phase capacitor induction motor was used to optimize motor efficiency. The optimization shows that variables, objective and constraints were properly chosen. Main advantage of this optimization procedure is utilization of the SSO (sequential surrogate optimization) algorithm exploiting a polynomial surrogate factory and genetic algorithm to find minimum of the objective function. Some drawback of the optimization procedure is that it allows solving the numerical model for only one value of slip and it is necessary (in post-processing) to check the model performance in the whole range of motor speed to validate the result.

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