

# The shrink-fit using the rotation heating

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**Abstract**—Hard-coupled model of rotation heating of a ferro-magnetic clamping head is presented. The problem is described by three coupled partial differential equations whose coefficients are temperature-dependent functions. The system is solved numerically in the monolithic formulation and results from different software are compared together. The methodology is illustrated by a concrete example which was built and measured. The results are evaluated and discussed.

## I. INTRODUCTION

Hot pressing belongs to widely spread industrial technologies used in a great number of industrial applications [1], [2]. The same system is used in a case of clamping the tools [6], [7]. The process of heating is mostly realized by gas or induction. Induction heating is characterized by an easy control of the intensity of heating and its local distribution, no chemical changes in the surface layers of the processed material and no products of combustion.

In case of assembly of a shrink fit the clamping head must be heated as long as the dilatation of its bore allows inserting the internal part (shank of a machine tool) into it. The system (particularly its heated part) is then cooled, which produces the shrink fit. Its purpose is usually to transfer a prescribed mechanical torque. The process is schematically depicted in Fig. 1.

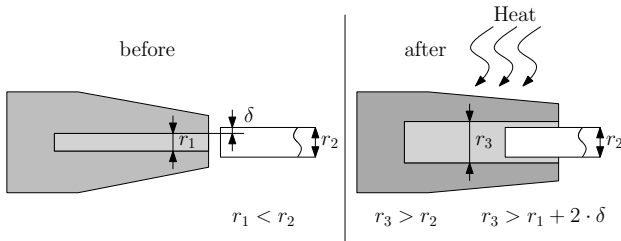


Fig. 1. Schematic view of manufacturing a shrink fit.

Manufacturing of shrink fits often requires a lot of energy. In order to minimize its consumption the authors try to develop new systems of heating instead of the induction one. They used their knowledge with the rotation heating and tried to combine this kind of heating with the thermoelasticity principle of the tools clamping.

## II. FORMULATION OF THE PROBLEM

Consider an arrangement of the system in Fig. 2. The rotation of the clamping head in the magnetic field generated

by permanent magnets induces in the head the eddy currents whose leads to the Joule losses. Those are the source of heating.

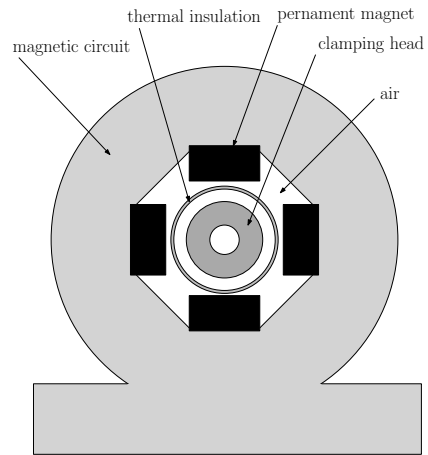


Fig. 2. An arrangement of the whole system.

## III. CONTINUOUS MATHEMATICAL MODEL

The continuous mathematical model of the problem consists of three partial differential equations providing the distribution of magnetic field, temperature field, and field of thermoelastic displacements in the system. The first one, describing the distribution of the magnetic vector potential, may be written in the form

$$\text{curl} \left( \frac{1}{\mu} \text{curl} \mathbf{A} - \mathbf{H}_c \right) - \gamma \mathbf{v} \times \text{curl} \mathbf{A} = 0. \quad (1)$$

The relevant physical parameters of materials involved in the system (magnetic permeability, electric conductivity) are nonlinear functions of temperature  $T$ . All equations are explained in full paper more in detail. The sufficiently distant artificial boundary is characterized by the Dirichlet condition  $\mathbf{A} = 0$ .

The temperature field in the system is described by the heat transfer equation in the form [3]

$$\text{div} (\lambda \text{grad} T) = \rho c_p \cdot \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \text{grad} T \right) - p_J. \quad (2)$$

Equation (2) is supplemented with the correct boundary condition (respecting generally both convection and radiation).

All parameters of materials included in the system are again nonlinear temperature-dependent functions.

Finally, the field of thermoelastic displacement is described by the Lam equation in the form [4]

$$(\varphi + \psi) \cdot \text{grad} (\text{div} \mathbf{u}) + \psi \cdot \Delta \mathbf{u} - (3\varphi + 2\psi) \cdot \alpha_T \cdot \text{grad} T + \mathbf{f} = 0. \quad (3)$$

#### IV. NUMERICAL SOLUTION

The numerical solution of the problem is realized by a fully adaptive higher-order finite element method [5], whose algorithms are implemented into codes Hermes [8] and Agros2D [9]. Both codes have been developed in our group for a couple of years.

The codes written in C++ are intended for monolithic numerical solution of systems of generally nonlinear and nonstationary second-order partial differential equations whose principal purpose is hard-coupled modelling of complex physical problems. While Hermes is a library containing the most advanced procedures and algorithms for the numerical processing of the task solved, Agros2D represents a powerful preprocessor and postprocessor. Comprehensive information about them can be found on webpages [9]. Both codes are freely distributable under the GNU General Public License.

#### V. EXPERIMENTAL MEASUREMENT

Experimental measurement was carried out at the thermoelastic clamping head used for the conventional induction heating. For rotating induction heating it was necessary to design an appropriate magnetic circuit according to the available permanent magnets and geometrical dimensions of the clamping head. The circuit was designed for 4 permanent magnets which corresponds with the 2mm air gap in the base of the head.

The clamping head was fixed with an interface on the shaft of a 4-pole asynchronous engine placed on a stand with designed magnetic circuit, see Fig. 3. The nominal speed of the engine with the frequency of power supply 50Hz corresponds to 1500rpm. The asynchronous engine was supplied from the frequency converter. It was possible to adjust the speed of rotation to the double value from the nominal engine speed, i.e. 3000rpm.

#### VI. OBTAINED RESULTS

The obtained results from the measurement was compared with results from the mathematical model (see Fig. 4). This was made in two different programs, mentioned Agros2D and the professional software COMSOL 3.5. The numerical model was necessary to improve and correct due to the simplification made in them according the measurement.

#### VII. CONCLUSION

The presented results can be used like the basic step for the numerical model of this coupled problem. Now it is necessary to improve the temperature measurement, correct the model material coefficients to respect all made simplifications. The 3D model is prepared for the results verification.

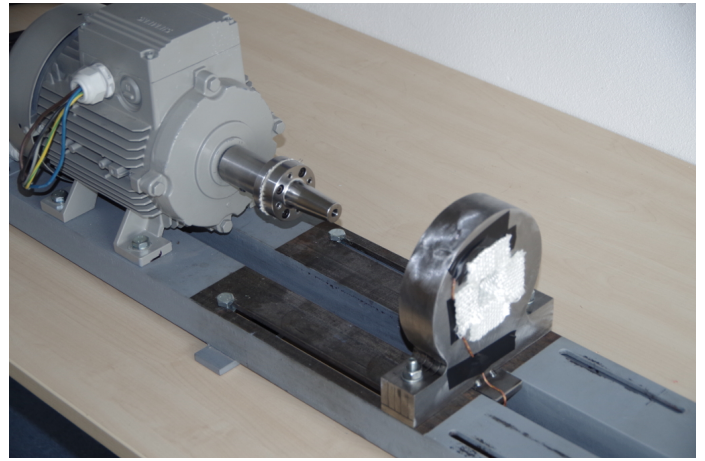


Fig. 3. Device for experimental measurement.

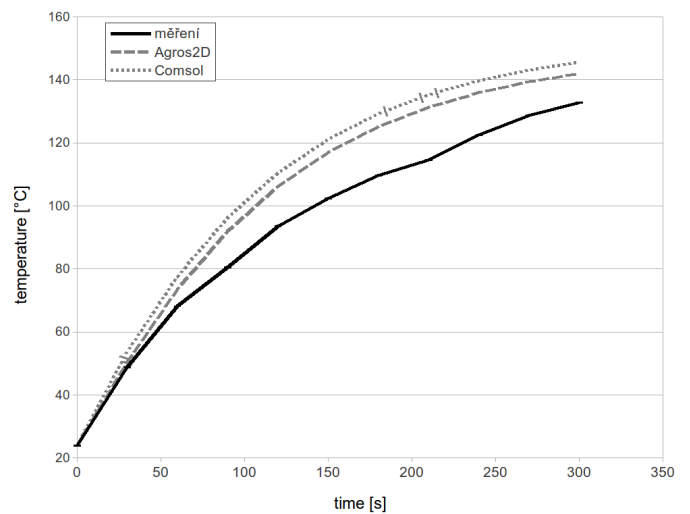


Fig. 4. Temperature evolution - from the inner radius of the clamping head

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