

## Analysis of HF half-bridge matrix converter

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### Anotace:

Článok sa zaoberá DC/HF\_AC/2AC meničovým systémom s ortogonálnym dvojfázovým výstupom a premenlivým napätím i frekvenciou. Navrhnutý systém s vysokofrekvenčným striedavým medziobvodom v porovnaní s bežne používanými systémami, používa dva jednofázové polomostové maticové meniče, ktoré sú ovládané bipolárnou šírko-impulzovou moduláciou PWM. Výhodou takéhoto systému je potom menší počet polovodičových súčiastok. Fourierova transformácia bola vykonaná pre jednofázový a dvojfázový ortogonálny systém. Elektrický motor bol nahradený R-L záťažou a indukované protinapätie záviselo od otáčok motora. Modelovanie a výsledky simulačných experimentov polomostového maticového meniča pre ustálené a prechodné stavy sú v článku uvedené. Výsledky potvrdzujú veľmi dobre časové priebehy prúdu uvažovaného dvojfázového AC motora.

### Annotation:

The paper deals with DC/HF\_AC/2AC converter system which can generate two-phase orthogonal output with both variable voltage and frequency. The proposed system with HF AC interlink in comparison with currently used conventional systems uses two single phase half-bridge matrix converters operated with the bipolar PWM. The advantage of such a system is then less number of semiconductor devices. The Fourier transformation has been considered for both single- and two phase orthogonal systems under substitution of the equivalence scheme of the electric motor by R-L load and back EMF voltage depended on the motor speed. Modelling and simulation experiment results of half-bridge matrix converter for both steady- and transient states are given in the paper. The results confirm a very good time-waveform of the considered two-phase AC motor current.

## INTRODUCTION

In the very early days of commercial electric power, some installations used two-phase four-wire systems for motors. Next two-phase systems have been replaced with three-phase systems. However, some applications of two-phase systems have been produced, especially with 90 degrees between phases. In these days new fields of application come up – in industrial or transport area [1]- [4].

A two-phase supply can be derived from three-phase system using a Scott-connection [5]. It can be also easily created using power electronic converters e.g. from battery supply, with two-phase transfer of energy for zero distance. DC/2AC and DC/HF\_AC/2AC converter system can generate two-phase orthogonal output with both variable voltage and frequency [6]-[8]. Ones of the possible schemes with full bridge and half-bridge converter on second stage and two-phase motor load are depicted in Fig.1. [4]

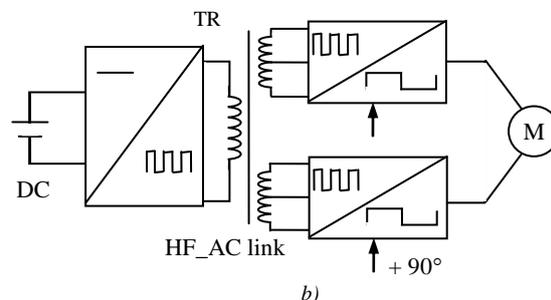
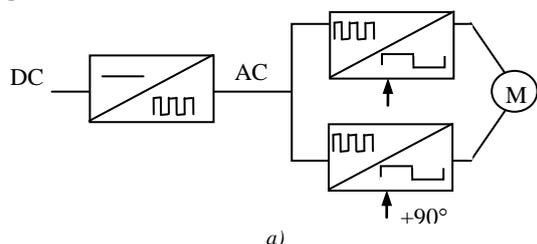


Fig.1. Principle diagram of full bridge (a) and half-bridge converter system (b) with second phase shifted by 90 degrees

The Fourier transformation can be considered for both single- and two phase orthogonal systems under substitution of the equivalence scheme of the electric motor [9], [10].

## TWO-STAGE DC/AC/AC CONVERTER FOR TWO-PHASE SYSTEM

DC/2AC and DC/HF\_AC/2AC converter system can generate two-phase orthogonal output with both variable voltage and frequency. Such a system usually consist of single-phase voltage inverter, AC interlink, HF transformer, 2-phase converter and 2-phase AC motor.

Due to AC interlink direct converter (cyclo-converter or matrix converter) is the best choice. Each matrix- or cyclo-converter can be connected as

1. full bridge converters connection (Fig.1a), 2. two half bridge one with central point of the source using HF transformer (Fig.1b, 2a) or 3. half-bridge one with central points of the motor load (Fig. 2b).

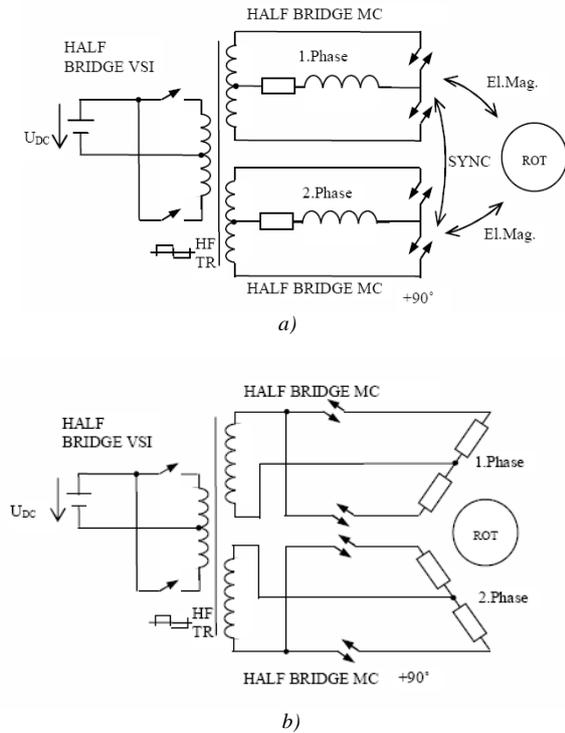


Fig. 2. Circuit diagram of half-bridge converters system with central points of AC source (a) and with central points of motor loads (b)

There is chosen matrix- or cyclo-converter connection no.2 - two half bridge converter systems with central points of source [4]. The advantage is then less number of semiconductor devices of the converters (four instead six). In this case is both hard and ZVS commutations of half-bridge connection of matrix converters. So, they should be operated using bipolar PWM modulation.

## SIMULATION OF HF HALF-BRIDGE MATRIX CONVERTER SYSTEM WITH CENTRAL POINTS OF SOURCE

### Voltage and Current Analysis of AC/AC Half-Bridge Matrix Converter System

Equivalent circuit diagram of half-bridge single phase converter (one of two-phase orthogonal systems) is depicted in Fig. 2a, and bipolar pulse-width modulation in Fig. 3b.

Switching-pulse-width can be determined based on equivalence of average values of reference waveform and resulting average value of positive and negative switching pulses during switching period (Fig. 3b). There are defined both amplitude- and frequency modulation ratios  $m_a$  and  $m_f$  as

$$m_a = \frac{U_{1m}}{U} \quad m_f = \frac{f_s}{f_1} \quad (1a,b)$$

where  $U_{1m}$  is reference amplitude of fundamental harmonic,

$U$  magnitude of supply voltage,

$f_s$  switching frequency,

$f_1$  fundamental frequency.

So, the peak amplitude of the fundamental harmonic component (equal to reference voltage) is  $m_a$  times  $U$ , and varies linearly with  $m_a$  (provided  $m_a \leq 1$ ).

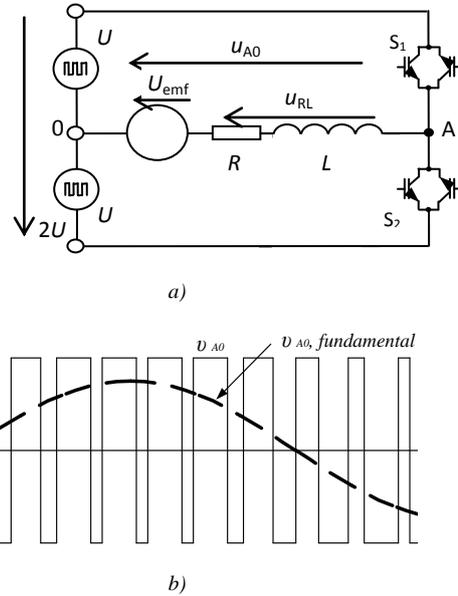


Fig. 3. Single-phase half-bridge matrix converter (a) with bipolar PWM (b)

Harmonic contents of converter output voltage is equivalent to those of DC/AC inverter bipolar modulation [5]. Consequently, the harmonics in the converter output voltage waveform appears as sidebands, centered on the switching frequency  $f_s$  and its multiples, that is, around harmonics  $m_f$ ,  $2 \cdot m_f$ ,  $3 \cdot m_f$ , and so on. This general pattern holds true for all  $m_a$  smaller (or equal) 1. For a frequency modulation ratio  $m_f \geq 9$  (which is our case), the harmonic amplitudes are almost independent on  $m_f$ , though  $m_f$  defines the frequencies at which they occur. Theoretically, the frequencies at which voltage harmonics occur can be defined as

$$f_v = (i \cdot m_f \pm k) \cdot f_1 \quad (2)$$

that is, the harmonic order  $v$  corresponds to the  $k$ -th sideband of the  $i$ -times the frequency modulation ratio  $m_f$

$$v = (i \cdot m_f \pm k) \cdot f_1 \quad (3)$$

where the fundamental harmonic frequency corresponds to  $v = 1$ . For odd values of  $i$ , the harmonics exist only for even value of  $k$ , and opposite, for even values of  $i$ , the harmonics exist only for odd value of  $k$ .

The discrete Fourier transformation has been used for calculation of individual harmonics coefficients [3]:

$$X(v) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nv}{N}} \quad (4)$$

resp.

$$\operatorname{Re}\{X(v)\} = \frac{2}{N} \sum_{n=0}^{N-1} x(n) \cos\left(2\pi \frac{nv}{N}\right) \quad (5)$$

$$\operatorname{Im}\{X(v)\} = \frac{-2}{N} \sum_{n=0}^{N-1} x(n) \sin\left(2\pi \frac{nv}{N}\right) \quad (6)$$

The carried-out results are identical ones with those of given in [5] for DC/AC inverter. The harmonic spectrum is plotted in Fig. 4, which is plotted for  $m_f = 39$ .

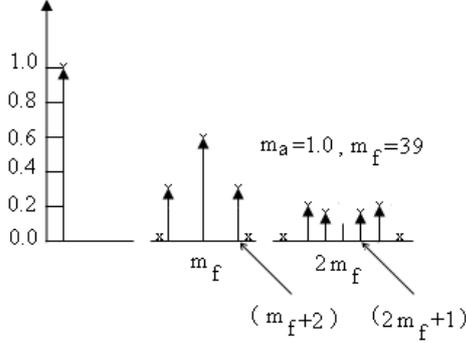


Fig. 4: Amplitude harmonic spectrum of the bipolar PWM voltage

The frequency modulation ratio  $m_f$  should be an odd integer. Choosing  $m_f$  as odd integer results in an odd symmetry [ $f(-t) = -f(t)$ ] as well as half-wave symmetry [ $f(-t) = -f(t+T_s/2)$ ] with the time origin shown in Fig. 3b. Therefore, only odd harmonics are present and the even harmonics disappear from the wave form of  $u_a$ . Moreover, only the coefficients if the sine series in Fourier analysis are finite; those for the cosine series are zero.

Harmonic components can be compute using above methodology and work [12]. For modulation indexes  $m_a = 0.2; 0.4; 0.6; 0.8; 1$  and  $m_f = 39$  resulting Fourier components and the amplitudes for  $U = 150V$  are given in Tab. 1.

Tab. 1: Calculated voltage Fourier coefficients  $C_v(v)=A_v/A_1$  and voltage amplitude  $A_v$  for  $m_a = 0.2; 0.4; 0.6; 0.8; 1$  and  $m_f = 39$  and  $U = 150V$

$v$	$m_a = 0.2$	$m_a = 0.4$	$m_a = 0.6$	$m_a = 0.8$	$m_a = 1.0$	$A_v [V]$ $m_a = 1.0$
<b>1</b>	0.2	0.4	0.6	0.8	1.0	150
$m_f$	1.242	1.15	1.006	0.818	0.601	90.15
$m_f \pm 2$	0.016	0.061	0.131	0.220	0.318	47.7
$m_f \pm 4$					0.018	2.7
$2m_f \pm 1$	0.190	0.326	0.370	0.314	0.181	27.15
$2m_f \pm 3$		0.024	0.071	0.139	0.212	31.8
$2m_f \pm 5$				0.013	0.033	4.95
$3m_f$	0.335	0.123	0.083	0.171	0.113	16.95
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062	9.3
$3m_f \pm 4$		0.012	0.047	0.104	0.157	23.55
$3m_f \pm 6$				0.016	0.044	6.6
$4m_f \pm 1$	0.163	0.157	0.008	0.105	0.068	10.2
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009	1.35
$4m_f \pm 5$			0.034	0.064	0.119	17.85
$4m_f \pm 7$				0.017	0.050	7.5

### Current harmonics investigation under resistive-inductive load with *emf*

Current time-waveforms for 1-harmonic components in steady-state  $i_{S1}(t)$  are given [12]

$$i_{S1}(t) = \frac{A_1 - U_{emf}}{Z_1} \cdot \sin(\omega t - \phi_1) = \frac{U}{Z_1} C_1(1) \cdot \sin(\omega t - \phi_1), \quad (7)$$

where:  $A_v = A_1 \cdot C_v(v)$  - amplitude of  $v$ -harmonic voltage component,

$A_1 = m_a \cdot U$  - amplitude of 1. harmonic voltage component,

$U_{emf}$  - counter-voltage (of electromagnetic force)

$|Z_v| = Z_v = \sqrt{R^2 + (v \cdot \omega \cdot L)^2}$  - module of complex impedance of resistive-inductive load

$\phi_v = \arctan(v \cdot \omega \cdot L / R)$  - argument of complex impedance of resistive-inductive load

$C_1(v)$  -Fourier coefficient of  $v$  -harmonic current component,

$I_v$  - amplitude of  $v$ -harmonic current component.

Harmonic current components can be compute similarly using above methodology and work [12]. The accurate calculation of  $U_{emf}$  can be obtained to use of motor circle diagram.

For modulation indexes  $m_a = 0.2; 0.4; 0.6; 0.8; 1$  and  $m_f = 39$  resulting Fourier components, the impedance of load  $Z_v$  and the current amplitudes  $I_{vm}$  for  $U = 150V$  with counter-voltage  $U_{emf} = 0,9$  of  $A_1$  are given in Tab. 2.

The MatLab programming environment has been used.

Simulation experiments have been done for the parameters:  $R = 10 \text{ Ohm}$ ,  $L = 25 \text{ mH}$ ,  $U = 150 \text{ V}$ ,  $f = 50 \text{ Hz}$  at  $m_a = 1$ ,  $m_f = 39$ , time increment  $\Delta t = 5 \mu s$ .

The total current in steady-state will be summarizing of single harmonics.

Tab. 2: Calculated current Fourier coefficients  $C_f(v)$  and current amplitude  $I_{vm}$  with impedance of load  $Z_v$  for  $m_a = 0.2; 0.4; 0.6; 0.8; 1$  and  $m_f = 39$  with counter-voltage  $U_{emf} = 0,9A_1$

$v$	$m_a = 0.2$	$m_a = 0.4$	$m_a = 0.6$	$m_a = 0.8$	$m_a = 1.0$	$Z_v [k\Omega]$ $m_a = 1.0$	$I_{vm} [A]$ $m_a = 1.0$
<b>1</b>	0.004	0.016	0.036	0.064	0.1	0.013	1.180
$m_f$	0.248	0.46	0.604	0.654	0.601	0.306	0.294
$m_f \pm 2$	0.003	0.024	0.079	0.176	0.318	0.291;	0.294
$m_f \pm 4$					0.018	0.322	0.164;
						0.275;	0.148
						0.338	0.010;0.008
$2m_f \pm 1$						0.621;	0.044;
$2m_f$	0.004	0.130	0.022	0.251	0.181	0.605	0.045
$\pm 3$		0.010	0.043	0.111	0.212	0.636;	0.050;
$2m_f$				0.010	0.033	0.589	0.054
$\pm 5$						0.652;	0.008;
						0.573	0.009

$3m_f$						0.919	0.018
$3m_f \pm 2$	0.067	0.049	0.050	0.134	0.113	0.935;	0.010;
$3m_f$	0.009	0.556	0.122	0.141	0.062	0.903	0.010
$\pm 4$		0.005	0.028	0.083	0.157	0.950;	0.025;
$3m_f$				0.013	0.044	0.888	0.027
$\pm 6$						0.966;	0.007;
						0.872	0.008
$4m_f \pm 1$						1.233;	0.008;
$4m_f \pm 3$	0.033	0.063	0.005	0.084	0.068	1.217	0.008
$4m_f$	0.002	0.028	0.079	0.092	0.009	1.249;	0.001;
$\pm 5$			0.020	0.051	0.119	1.202	0.001
$4m_f$				0.014	0.050	1.265;	0.014;
$\pm 7$						1.186	0.015
						1.280; 1.170	0.006;
							0.006

Simulation results are given in Fig. 5 and Fig. 6.

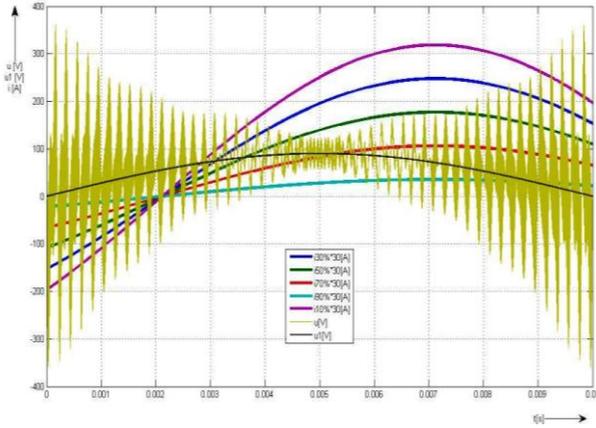


Fig. 5: Time waveform of voltage (1. harmonic component) and load current – with various counter-voltage and modulation index of bipolar PWM  $m_a=1$  and  $m_f=39$

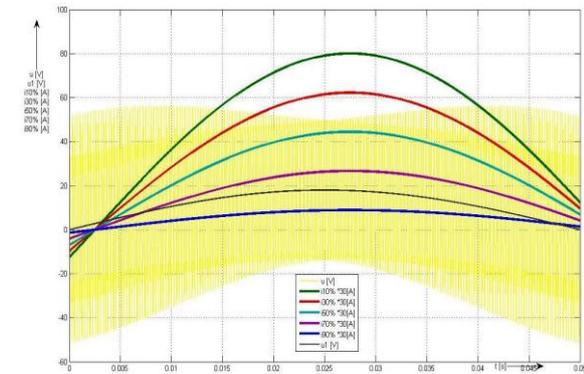


Fig. 6: Time waveform of voltage (1. harmonic component) and load current – with various counter-voltage and modulation index of bipolar PWM  $m_a=0.2$  and  $m_f=39$

Root-mean-square value of the total steady-state current can be calculated as:

$$I = \sqrt{\sum I_v^2} = \sqrt{\sum \left( \frac{I_{vm}}{\sqrt{2}} \right)^2} \quad (8)$$

The total harmonic distortion of the current is given by:

$$\frac{\sqrt{\sum I_v^2}}{I_1} = \sqrt{\frac{I^2 - I_1^2}{I_1^2}} = \sqrt{\left( \frac{I}{I_1} \right)^2 - 1} = 2\% \quad (9)$$

## Transient phenomena investigation using Fourier analysis

The Fourier analysis can be used also for the behaviour of the system in transient state. The total current of  $v$ -harmonic component  $i_v$  will be summarizing of current in steady-state  $i_{sv}$  and current in transient phenomenon  $i_{Tv}$

$$i_v(t) = i_{sv}(t) + i_{Tv}(t) = \frac{A_v}{Z_v} \cdot \sin(v\omega t - \varphi_v) + \frac{A_v}{Z_v} \cdot \sin \varphi_v \cdot e^{-t/\tau} \quad (10)$$

where

$i_v$  - is total current waveform

$i_{sv}$  - steady-state component of total current

$i_{Tv}$  - transient component of total current

$\tau = R/L$  - time constant of resistive-inductance load

Total current as well as both components should be calculated for each harmonics. Simulation experiment results modulation index  $m_a = 1$  and  $m_f = 39$  resulting ratio current with counter-voltage  $U_{emf} = 0,1$  of  $A_1$  are given in Fig. 7.

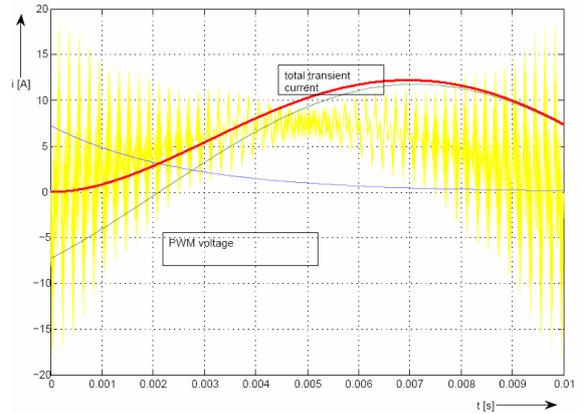


Fig. 7: Time waveform of voltage (1. harmonic component) and load current– with counter-voltage  $U_{emf}=0.1$   $A_1$  and modulation index of bipolar PWM  $m_a=1$  and  $m_f=39$

## CONCLUSION

The proposed system with AC interlink in comparison with currently used conventional systems uses two single phase half bridge matrix converters with bipolar pulse-width modulation. The advantage is then less number of semiconductor devices of the converters.

The Fourier transformation can be considered for two phase orthogonal systems under substitution of the equivalence scheme of the electric motor.

The solution given in the paper makes it possible to analyse more exactly effect of each harmonic component comprised in total waveform on resistive-inductive load or induction motor quantities.

## ACKNOWLEDGEMENT

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