Influence of BDF order on FEM computation

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Abstract The influence of the time derivative formula and time adaptive predictor-corrector scheme on the solution accuracy and computation time is studied. An exemplary electromagnetic field problem is solved by the finite element method. Backward differentiation (BDF) formulae of higher order are applied.

Keywords backward differentiation formula, finite element method, predictor-corrector.

I. INTRODUCTION

The paper presents an initial study of certain aspects that are applied when solving electromagnetic field problems with the finite element method. The influence of the time derivative approximation and the time-step adaptation on the computation time and the accuracy of the result is studied.

II. PROBLEM FORMULATION

An exemplary problem of the electromagnetic field is considered. The field distribution is described by the magnetic vector potential component differential equation:

\[ \frac{\partial^2 A}{\partial t^2} - \mu \frac{\partial A}{\partial t} = 0. \]  

(1)

Neumann boundary conditions on \( x = x_1 \) and \( x = x_2 \) are imposed in the form of time-periodic functions:

\[ \frac{\partial A}{\partial x} = a \sin(2\pi f_0 t) + 0.1a \sin(2\pi 9f_1 t). \]  

(2)

III. TIME DERIVATIVE FORMULAE

For the approximation of the time derivative the BDF formulae [1] are used (Backward Differentiation Formula):

\[ \left. \frac{\partial A}{\partial t} \right|_{t=t_n} = \left( \sum_{i=0}^{n} \alpha_i \frac{\partial A}{\partial t} A_m - (t_{m-i}) \right) \bigg|_{t=t_n} \]  

(3)

where \( n \) denotes the order of the BDF (\( n = 1 \) means the backward Euler scheme), \( t_m \) is the time value for the current step, \( A_m \) is the component value for the current time step, \( \ell_j \) is the Lagrange polynomial which is 1 at \( t_j \) and 0 at every other included time step.

An adaptive time-step technique is applied. The error is estimated by means of a predictor-corrector scheme where for a chosen \( n \) the predictor step uses the solution for \( n - 1 \) and the solution for \( n \) is applied in the corrector step. A relative maximum error \( e \) is calculated by means of differences between both solutions. If the error fits a chosen threshold \( e_{tol} \in [e_{\text{min}}, e_{\text{max}}] \) then the next time step is calculated. If \( e < e_{\text{min}} \) then the time step value \( \Delta t \) is doubled. If \( e > e_{\text{max}} \) then \( \Delta t \) is divided by two and the calculations are repeated for the current time step.

IV. ANALYSIS

The main aspect that is studied is the choice of an efficient algorithm in the sense that computation requires less time and the solution accuracy is either improved or at least kept within a relatively close range.

An average relative error \( e \) has been computed for both frequencies (denoted later on by \( f_1 \) and \( f_9 \) respectively for \( f_1 \) and \( f_9 \)):

\[ e = \frac{\left| \Delta_x \right| - \Delta_x}{\Delta_x} \]  

(4)

where \( \Delta_x \) are the complex values obtained by applying an FFT algorithm on the numerical solution. \( \Delta_x \) are the values of the analytical solution. The error values and computation times are presented in Tables 1 and 2 respectively.

V. CONCLUSION

The study shows that satisfactory accuracies can be obtained when using predictor-corrector schemes that apply backward differentiation formulae. An increase in the BDF order has allowed to compute the result in a shorter period of time while keeping a relatively close error value.

VI. REFERENCES