Diffraction of Longitudinal Shear Waves on a Thin Piezoelectric Inclusion of Arbitrary Rigidity

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Abstract **A thin plane piezoelectric inclusion is perfectly bonded to a surrounding elastic matrix (in two-dimensional Euclidean space) and subjected to an incident plane longitudinal shear wave. Using the representation theorem, the problem is reduced to singular integral equation that is solved numerically for arbitrary values of inclusion and matrix parameters.**

Keywords **Piezoelectric, Diffraction, Modelling**

I. Introduction

Nowadays the electromechanical systems with piezoelectric patches are widely used in ultrasonic non-destructive testing of materials and constructions, in ultrasonic welding, cleaning of surfaces, in ultrasonic diagnostics in medicine [1-3]. In such applications the reliable mathematical models adequately describing the wave processes play an important role.

In this paper, the problem of diffraction of longitudinal shear waves on a thin piezoelectric inclusion of arbitrary rigidity is considered. Using a known model of the interaction of inclusion with an elastic medium [4] the corresponding boundary-value problem is reduced to the singular integral equation that is solved numerically.

II. Problem formulation

Let us consider an elastic uniform medium characterized by the shear modulus μ and the mass density ρ, in which there is a piezoelectric inclusion in conditions of the ideal mechanical contact. The longitudinal shear of the elastic system is assumed. The inhomogenity occupies the region $S = \{|x_1| < a, |x_3| < h/2\}, \ |x_2| < \infty$, where h is the thickness, (x_1, x_2, x_3) are the Cartesian coordinates and the quantity $\varepsilon = h/a$ is the small parameter. The material of the heterogeneity with an elastic constant c_{44} falls into the crystallographic class $6mm$, and the axis of symmetry of the sixth order is perpendicular to the plane x_1x_3.

A plane, incident longitudial shear wave of the form

$$u^i(x) = exp\left[ik(l,x)\right], \ x = (x_1, x_3), \ k = \omega/c, \ c^2 = \mu/\rho \quad (1)$$

impinges on the inclusion (the time factor of the form $e^{-i\omega t}$ is omitted throughout the analysis). Here $\mathbf{l} = (\sin\theta_0, -\cos\theta_0)$ is the direction of sounding, k is the wave number and typical wavelength kh satisfies the condition $kh \ll 1$.

The total wave $u = u^i + u^s$ is decomposed into the given incident wave u^i and the unknown scattered wave u^s which is required to satisfy the Sommerfeld radiation condition at infinity, from which it follows that

$$u^s(\mathbf{x}) = \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{x}+\mathrm{i}\pi/4}}{\sqrt{8\pi \mathrm{k}|\mathbf{x}|}}\mathrm{f}(\omega;\mathbf{l},\boldsymbol{\nu}), \ |\mathbf{x}| \to \infty, \quad (2)$$

where $f(\omega, \mathbf{l}, \boldsymbol{\nu})$ is the complex amplitude or far-field pattern of the scattering wave, $\boldsymbol{\nu} = x/|x| = (\sin\theta, \cos\theta)$ is the direction of observation, ω is the circular frequency. Herein $u(x)$ is the displacement in the x_2-direction and for the only non vanishing stress components the notation $\sigma_{\beta 2} = \mu\frac{\partial u}{\partial x_\beta}$ with $\beta = 1, 3$ is introduced. Let $u^0(x), \ x \in S$ and $k_0 = \omega/c_0$ denote the displacement field and wave number in the inclusion, respectively, and $\phi(\mathbf{x})$ is the electric potential for the heterogeneity.

The scattering problem of time harmonic waves is described by the wave equations

$$(\Delta + k^2)u(\mathbf{x}) = 0, \ \mathbf{x} \in R^2/S, \quad (3)$$

$$(\Delta + k_0^2)u^0(\mathbf{x}) = 0, \ \mathbf{x} \in S,$$

$$\Delta u^0(\mathbf{x}) - \Delta\phi_p(\mathbf{x}) = 0, \ \phi_p(\mathbf{x}) = \frac{\epsilon_{11}}{e_{15}}\phi(\mathbf{x}), \ \mathbf{x} \in S,$$

and following conditions along the boundary ∂S of the inclusion:

$$u(\mathbf{x}) = u^0(\mathbf{x}), \ \frac{\partial u(\mathbf{x})}{\partial n} = \gamma_e \frac{\partial}{\partial n}[u^0(\mathbf{x}) + \eta^2\phi_p(\mathbf{x})], \quad (4)$$

$$\phi(\mathbf{x}) = 0, \ \mathbf{x} \in \partial S,$$

where n denotes the outer normal direction, e_{15} and ϵ_{11} are the piezoelectric constant and the permittivity of the material of the heterogeneity, respectively, η is the electromechanical coupling coefficient and ρ_0 is the mass density for the inclusion,

$$c_0^2 = \frac{c_{44}}{\rho_0}(1 + \eta^2), \ \gamma_e = \frac{c_{44}}{\mu}, \ \eta = \frac{e_{15}}{\sqrt{c_{44}\epsilon_{11}}}.$$

Our objective is to find the numerical representations of the solution to the problem (1)-(4) for arbitrary electrical and elastic parameters of inclusion and matrix for small but non vanishing ε.

References

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