Phase resonance in series fractional order $RL_\beta C_\alpha$ circuit

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Abstract The paper describes the results of studies on the phase resonance phenomenon in a series RLC circuit with fractional order reactive elements. Formulas for frequency characteristics and phase resonance conditions have been derived. Simulations of concerned fractional order system have been conducted too.

Keywords phase resonance, series RLC circuit, fractional order inductance and capacitance.

I. INTRODUCTION

Fractional order elements $L_\beta, C_\alpha$ represent a generalization of classic reactive elements $LC$ [2]. Their mathematical models in frequency domain are frequently described by relations [1], [2]:

$$Z_L(j\omega) = R_L + (j\omega)^\beta L, \quad \beta \in \mathbb{R}^+,$$
$$Z_C(j\omega) = R_C + (j\omega)^{-\alpha} C^{-1}, \quad \alpha \in \mathbb{R}^+. \tag{1}$$

Many practical realizations of these elements are known [2], and supercapacitors are one of the best known implementation of the fractional order elements. Fractional order elements find various applications in electrical engineering, electronics and control theory [2]. Properties of systems containing fractional order elements differ from those of systems with classic $RLC$ elements. Research on fractional order systems is conducted in various directions [2]. One of them concerns the analysis of fractional order system features in frequency domain [2], [3]. Studies of the resonance phenomena in a series $RLC$ circuit were presented in the article [3]. This article is its continuation and concerns the analysis of resonance in a series $RL_\beta C_\alpha$ circuit.

II. FREQUENCY MODEL OF THE SYSTEM

The considered $RL_\beta C_\alpha$ model is shown in Fig. 1. It consists of a fractional coil (inductor) $L_\beta$ and a supercapacitor $C_\alpha$, which impedances are described by relations (1) and (2) respectively.

![Fig. 1. Series $RL_\beta C_\alpha$ circuit.](image)

The impedance of the circuit (Fig. 1) seen from the terminals 1 - 1’ is represented by:

$$Z(j\omega) = \left[ R + (j\omega)^\beta L + (j\omega)^{-\alpha} C^{-1} \right] =$$
$$= \left[ R + \omega^\beta L \cos\left(\frac{\pi}{2} \beta \right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2} \alpha \right) \right] +$$
$$+ j \left[ \omega^\beta L \sin\left(\frac{\pi}{2} \beta \right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2} \alpha \right) \right] =$$
$$= |Z(j\omega)| \text{exp}(j\varphi(\omega)). \tag{3}$$

where:

$$R = R_s + R_L + R_C, \quad A = C^{-1}, \tag{4}$$

$$|Z(j\omega)| = \left[ R + \omega^\beta L \cos\left(\frac{\pi}{2} \beta \right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2} \alpha \right) \right]^2 +$$

$$+ \left( \omega^\beta L \sin\left(\frac{\pi}{2} \beta \right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2} \alpha \right) \right)^2. \tag{5}$$

$$\varphi(\omega) = \arctg \frac{\omega^\beta L \sin\left(\frac{\pi}{2} \beta \right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2} \alpha \right)}{R + \omega^\beta L \cos\left(\frac{\pi}{2} \beta \right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2} \alpha \right)} \tag{6}$$

III. PHASE RESONANCE CONDITIONS

The formula (3) suggests, that the phase resonance conditions:

$$\text{Im}\{Z(j\omega)\} = 0, \quad \text{Im}\{Y(j\omega)\} = 0,$$

are the same. Hence, based on the formulas (3) and (7) there can be derived a relationship for the resonance frequency $f_r$ in the system from Fig.1:

$$f_r = \frac{1}{2\pi} \frac{1}{\sin\left(\frac{\pi}{2} \alpha \right)} \frac{1}{\sin\left(\frac{\pi}{2} \beta \right)} \tag{8}$$

It can be noticed, that in specific cases:

1. $\alpha = \beta$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \tag{9}$$

2. $\alpha = \beta = 1$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \tag{10}$$

The case defined by formula (10) describes the classic resonance in a series RLC circuit.

IV. FUTURE RESULTS

More detailed as well as complex results from the conducted analysis for the considered fractional order $RL_\beta C_\alpha$ series circuit will be presented during the conference, including the results from model simulations.

V. REFERENCES

