Phase resonance in series fractional order $RL_\beta C_\alpha$ circuit

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Abstract **The paper describes the results of studies on the phase resonance phenomenon in a series *RLC* circuit with fractional order reactive elements. Formulas for frequency characteristics and phase resonance conditions have been derived. Simulations of concerned fractional order system have been conducted too.**

Keywords **phase resonance, series RLC circuit, fractional order inductance and capacitance.**

I. INTRODUCTION

Fractional order elements L_β, C_α represent a generalization of classic reactive elements LC [2]. Their mathematical models in frequency domain are frequently described by relations [1], [2]:

$$Z_L(j\omega) = R_L + (j\omega)^{\beta} L, \qquad \beta \in R^+, \tag{1}$$

$$Z_C(j\omega) = R_C + (j\omega)^{-\alpha} C^{-1}, \qquad \alpha \in R^+. \tag{2}$$

Many practical realizations of these elements are known [2], and supercapacitors are one of the best known implementation of the fractional order elements. Fractional order elements find various applications in electrical engineering, electronics and control theory [2]. Properties of systems containing fractional order elements differ from those of systems with classic *RLC* elements. Research on fractional order systems is conducted in various directions [2]. One of them concerns the analysis of fractional order system features in frequency domain [2], [3]. Studies of the resonance phenomena in a series RLC_α circuit were presented in the article [3]. This article is its continuation and concerns the analysis of resonance in a series $RL_\beta C_\alpha$ circuit.

II. FREQUENCY MODEL OF THE SYSTEM

The considered $RL_\beta C_\alpha$ model is shown in Fig. 1. It consists of a fractional coil (inductor) L_β and a supercapacitor C_α, which impedances are described by relations (1) and (2) respectively.

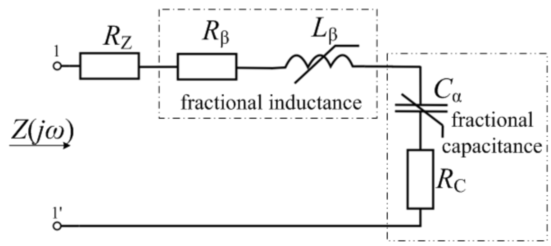

Fig. 1. Series $RL_\beta C_\alpha$ circuit.

The impedance of the circuit (Fig. 1) seen from the terminals 1 -1' is represented by:

$$\begin{aligned} Z(j\omega) &= \left(R + (j\omega)^{\beta} L + (j\omega)^{-\alpha} C^{-1}\right) = \\ &= \left(R + \omega^{\beta} L \cos\left(\frac{\pi}{2}\beta\right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2}\alpha\right)\right) + \\ &+ j\left(\omega^{\beta} L \sin\left(\frac{\pi}{2}\beta\right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2}\alpha\right)\right) = \\ &= |Z(j\omega)| \cdot \exp\left(j\varphi(\omega)\right), \end{aligned} \tag{3}$$

where:

$$R = R_Z + R_L + R_C, \qquad A = C^{-1}, \tag{4}$$

$$|Z(j\omega)| = \left[\left(R + (\omega)^{\beta} L \cos\left(\frac{\pi}{2}\beta\right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2}\alpha\right)\right)^2 + \left(\omega^{\beta} L \sin\left(\frac{\pi}{2}\beta\right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2}\alpha\right)\right)^2\right], \tag{5}$$

$$\varphi(\omega) = arctg \frac{\omega^{\beta} L \sin\left(\frac{\pi}{2}\beta\right) - \omega^{-\alpha} A \sin\left(\frac{\pi}{2}\alpha\right)}{R + \omega^{\beta} L \cos\left(\frac{\pi}{2}\beta\right) + \omega^{-\alpha} A \cos\left(\frac{\pi}{2}\alpha\right)}. \tag{6}$$

III. PHASE RESONANCE CONDITIONS

The formula (3) suggests, that the phase resonance conditions:

$$\mathrm{Im}\{Z(j\omega)\} = 0, \qquad \mathrm{Im}\{Y(j\omega)\} = 0, \tag{7}$$

are the same. Hence, based on the formulas (3) and (7) there can be derived a relationship for the resonance frequency f_r in the system from Fig.1:

$$f_r = \frac{1}{2\pi} \sqrt[\alpha+\beta]{\frac{1}{LC} \cdot \frac{\sin\left(\frac{\pi}{2}\alpha\right)}{\sin\left(\frac{\pi}{2}\beta\right)}}. \tag{8}$$

It can be noticed, that in specific cases:

1. $\alpha = \beta$

$$f_r = \frac{1}{2\pi} \sqrt[2\alpha]{\frac{1}{LC}}, \tag{9}$$

2. $\alpha = \beta = 1$

$$f_r = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}. \tag{10}$$

The case defined by formula (10) describes the classic resonance in a series *RLC* circuit.

IV. FUTURE RESULTS

More detailed as well as complex results from the conducted analysis for the considered fractional order $RL_\beta C_\alpha$ series circuit will be presented during the conference, including the results from model simulations.

V. REFERENCES

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