A Stable and Invariant Three-polar Surface Representation: Application to 3D Face Description

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ABSTRACT

In this paper, we intend to introduce a new curved surface representation that we qualify by three-polar. It is constructed by the superposition of the three geodesic potentials generated from three reference points of the surface. By considering a pre-selected levels set of this superposition, invariant points are obtained. A comparative study between this representation and the unipolar one based on the level curves around one reference point is established in the sense of the stability under errors on the reference points positions. The three-polar representation is applied, finally, for 3D human faces description. Its accuracy is performed in the mean of the Hausdorff shape distance.

Keywords

Three-polar, geodesic, 3D, potential, superposition, level set, face, curve, Hausdorff, stability, shape, surface, invariant.

1 INTRODUCTION

For few years, there have been several advances in 3D scanning technologies and tools enabling accelerated 3D graphics. Thus, 3D shape analysis and description have become more and more popular and useful for many varieties of visual tasks. Actually, R^3 surfaces description plays an important role for pattern recognition, computer vision and 3D movement analysis. In practice, the data obtained from 3D sensors is generally not organized or partially organized like the 3D triangular mesh known as the conventional 3D discrete surface representation. Therefore, one of the major challenges faced today in the three dimensional imaging field is the construction of a surface representation that ensures several properties such as the invariance under some transformations and different parametrisations, the independence from the point of view and the stability under some local variations in shape. Several past works have been performed in order to construct 3D surface representations. In the literature, the three dimensional surface description methods can be classified into four major categories: the graph based approaches, the 2D views, the transform ones and those based on statistical features.

The graph based approaches have the potential to code geometrical and topological shape properties in an intuitive manner. In this approaches category, the problem of comparing between shapes is transformed onto a comparison between graphes. The usually used descriptors are Reeb graphs [Tun05] and the skeletal ones [Sun03].

In the two dimensional view based methods, a collection of 2D projections of the 3D object from canonical viewpoints is realized. Planar image descriptors are then computed as Zernike moments [Che03] and Fourier descriptors [Vra04].

For the transform based approaches, the first step consists on the conversion of the surface onto 3D voxels or a spherical grid. Specific transformations are then applied. The most known ones are 3D Fourier [Bur92], the 3D Radon [Dar04], , the angular radial transform [Ric05] and the uniformization [Khe08].

In the fourth description category, numerical attributes of the 3D object (local or global) are collected. Several past works adopted this approach for invariant features extraction like the works of high curvature area determination [Fau86], the extend Gaussian image [Kan93] and the generalized shape distribution [Liu06]. Bannour et al. [Ban00] proposed a new surface pseudo-reparametrisation by the extraction of a curves network determined by iso-curvature features computation. Other methods used the local coordinates system by the exponential map around a point belonging to the two dimensional manifold (unipolar representation) obtained by constructing a set of geodesic circles relatively to a given reference point [Sam06, Sri08, Gad12]. The stability of these last methods remains dependent on the robustness of the reference point extraction. In recent works, Ghorbel et al. [Gho13] and Jribi et al. [Jri12] proposed a new representation that they called a bipolar one. It consists on the superposition of the two geodesic potentials generated from two reference points instead of one reference point. The goal was to provide a more stability to the representations based on only one reference point [Sam06, Sri08, Gad12] in the case of errors on the reference point positions.

We intend in this paper to study a novel curved surface representation, that we qualify by three-polar, introduced in [Jri13]. It is an attempt to generalize what is known by local coordinates around one reference point. It is constructed from the superposition of the three geodesic potentials generated from three reference points of the surface. The proposed representation is obtained by sampling the sum of these three geodesic potentials. The stability of this novel representation under errors on the positions of the reference points is established. Its accuracy for 3D human face description is performed in the mean of the Hausdorff shape distance.

Thus, this paper will be structured as follows: We present in the second section the mathematical formulation of the three-polar representation. The used similarity metric to compare between shapes is illustrated in the third section. We establish in the fourth section a comparative study between the three-polar representation and the unipolar one in the sense of the stability under errors on the positions of the reference points. We apply finally the novel three-polar representation for human face description in the fifth section. The cases of correct and wrong positions of the reference points are considered.

2 CONSTRUCTION OF THE THREE-POLAR REPRESENTATION

Let consider here a two dimensional differential manifold S, and let denote by U_r the geodesic potential generated from a reference point r of S. U_r is the function that computes for each point p of S the length of the geodesic curve joining p to r. For a given real value λ, the points of S with geodesic potential values from r equal to λ form a curve that we denote by C_r^λ. It is called the geodesic of level λ and it belongs to the surface.

We describe in this section the construction process of the three-polar representation. It is based on the superposition of the three geodesic potentials generated from three reference points of S. Thus, let consider $\{r_i, i = 1..3\}$ three points of the 2-differential manifold S. We denote by $\{U_{r_i}, i = 1..3\}$ their corresponding potentials functions. $U_3 = \sum_{i=1}^{3} U_{r_i}$ is the geodesic potential constructed by the sum of the three geodesic potentials generated from the three reference points $\{r_i, i = 1..3\}$.

Let p^* be a point of S. Thus, there exist three real values $\{\lambda_i^*, i = 1..3\}$ such that p^* belongs to the three level curves $\{C_r^{\lambda_i^*}, i = 1..3\}$. Let $U_3^* = min\{U_3\}$. The points of the surface with the same geodesic sum are invariant under the rotations' group $SO(3)$. By selecting a levels set of this sum, we construct a system of invariant points under the same transformations group . The representation that we propose is obtained by varying these levels from 0 to the integer K. This integer represents the maximum value determined by the interest region extend on the surface. Therefore, the three-polar representation can be formulated as following:

$$M_3^k(S) = \{p^* \in S; U_3(p^*) = U_3^* + \frac{k}{K}(\alpha_K - U_3^*), k = 0..K\} \quad (1)$$

Where α_K is the maximum of the geodesic sum.

3 SIMLARITY METRIC

We present here the used similarity metric to compare between different shapes. We choose the well known Hausdorff shape distance introduced by Ghorbel in [Gho98, Gho12]. By following the same process, we denote by G the group representing all possible normalized parametrisations of surfaces. It can be the real plane R^2 or the unit sphere S^2. we consider the space of all surface pieces as the set of all 3D objects assumed diffeomorphic to G. It can be assimilated to a subspace of $L^2_{R^3}(G)$ formed by all square integrated maps from G to R^3. The direct product of the Euler rotations group $SO(3)$ by the group G , acts on such space in the following sense:

$$SO(3) \times G \times L^2_{R^3}(G) \to L^2_{R^3}(G) \quad (2)$$

$$\{A, (u_0, v_0), S(u, v)\} \to AS(u + u_0, v + v_0)$$

The 3D Hausdorff shape distance Δ can be written for every S_1 and S_2 belonging to $L^2_{R^3}(G)$ and g_1 and g_2 to $SO(3)$ as follows:

$$\Delta(S_1, S_2) = max(\rho(S_1, S_2), \rho(S_2, S_1)) \quad (3)$$

Where:

$$\rho(S_1, S_2) = \sup_{g_1 \in SO(3)} \inf_{g_2 \in SO(3)} \| g_1 S_1 - g_2 S_2 \|_{L^2} \quad (4)$$

$\| S \|_{L^2}$ denotes the norm of the functional banach space $L^2_{R^3}(G)$.
Since the euclidean rotations preserve this norm, it is easy to show that this distance is reduced to the following quantity:

$$\Delta(S_1, S_2) = \inf_{h \in SO(3)} \| S_1 - hS_2 \|_{L^2} \quad (5)$$

We consider after that, a normalized version of Δ so that its variations are confined to the interval [0,1]. In practice, this distance is obtained by using the well known Itertive Closest Point (ICP) algorithm [Bes92].

4 STABILITY OF THE THREE-POLAR REPRESENTATION

The stability is one of the most important properties of a given 3D surface representation. This property is of a paramount importance since we wish that a small local deformation on the surface don't lead to a big change of the corresponding representation.
In our work, we are interested in studying a special stability under errors of the reference points positions. We propose to make here a comparison between the three-polar representation and the unipolar one in the sense of the proposed stability. We used the object "Stanford Bunny" [Tur94] which is known as a standard model for testing graphical algorithms. The figure 1(a) illustrates this object and the figure 1(b) shows the used three reference points chosen randomly on this object.

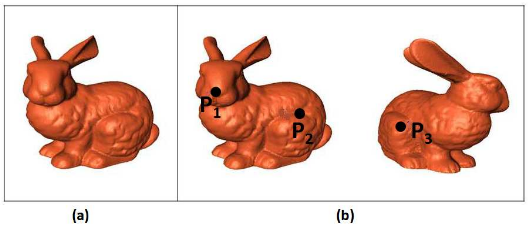

Figure 1: (a) The "Stanford Bunny" object. (b) The used three reference points.

We denote by P_1 the common reference point between the three-polar representation and the unipolar one. P_2 and P_3 are the two other points of the three-polar representation.
In the case of extraction errors of a reference point, we assume that it belongs to a geodesic disc of radius equal to RG_{max} around its correct position. In our work, we qualify by a reference representation the one constructed from reference points that are all extracted without errors. In order to establish this comparative study, the experimentations are performed on two parts. Each part corresponds to a variation of reference points positions. In the first part, only the positions of the point P_1 are extracted with errors for the two representations. In the second part, the positions of the three reference points are incorrect for the three-polar representation. We note that the unipolar representation and the three-polar one undergo the same variation of the point P_1 in the two parts of experimentation. The figure 2 illustrates the variation areas of the reference points.

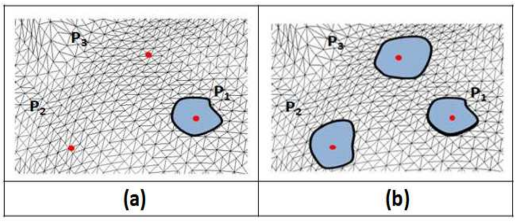

Figure 2: The variation of the reference points. (a) the first part of the experimentation. (b) the second part of the experimentation.

For each part of this study, the Hausdorff shape distance is computed between:

- The three-polar representations with errors on the positions of the reference points and the corresponding reference one.
- The unipolar representation with errors on the positions of the reference point and the corresponding reference one.

All the variations of the Hausdorff shape distance are computed according to the position of the common point P_1 between the three-polar representation and the unipolar one. Here, sixteen positions of the point P_1 are chosen randomly on the disc of radius value equal to RG_{max} around its correct position. The figure 3 shows the variation of this distance for the first part of study. We can note that the three-polar representation is more stable than the unipolar one in the case of one reference point with errors of extraction. In fact, the distance values are smaller in the case of the three-polar representation than the unipolar one.
In the second part of study, for each wrong position of the point P_1, many incorrect positions of the points P_2 and P_3 exist. For each wrong position of the point P_1, we compute the average of the Hausdorff shape distances obtained for incorrect positions of the points P_2 and P_3. The figure 4 illustrates the variation of the Hausdorff shape distance for the second part of study according to the wrong positions of the point P_1. From this figure, we can observe also that the three-polar representation has ensured a more stability than the unipolar one.

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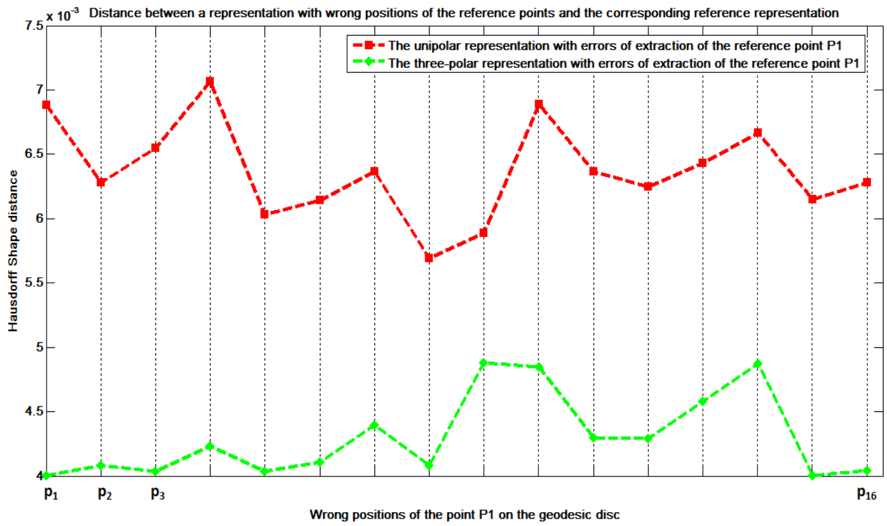

Figure 3: First part of study: Hausdorff shape distance according to the positions of the point P_1 chosen randomly on the geodesic disc of radius value equal to RG_{max} around the correct position of the point P_1.

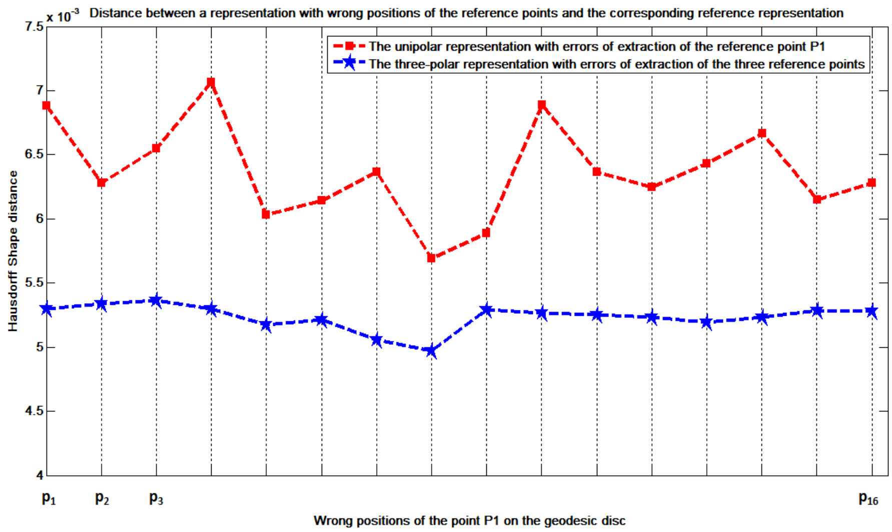

Figure 4: Second part of study: Hausdorff shape distance according to the positions of the point P_1 chosen randomly on the geodesic disc of radius value equal to RG_{max} around the correct position of the point P_1.

5 HUMAN FACE DESCRIPTION WITH THE THREE-POLAR REPRESENTATION

The 3D face description has received a great deal of attention over the last few years because of its various application domains like biometrics which are one of the most important. We test here the performance of the three-polar representation on the 3D meshes of the database Bosphorus [Sav08] in the mean of the Hausdorff shape distance. We use a total of ten faces that can be grouped into two classes. A first class contains five faces of the same person with different expressions and a second one contains five faces of different persons.

5.1 The reference points: choice and automatic extraction

The choice of the reference points consists the first step of the three-polar representation construction. We choose to use the two outer corners of the eyes as two reference points for the proposed three-polar representation. Indeed, there is a general agreement that eyes are the most important facial features [Cam07]. In fact, they are a crucial source of information about the state of the human being and their appearance is less variant to certain face changes. Since the nose tip is commonly used for the unipolar representation [Sam06, Sri08, Gad12], it will be also chosen as a third reference point in the three-polar representation. We refer to the work of Szeptycki et al. [Sze09] for the automatic extraction of these points. This method is based on a curvatures analysis with the use of a generic face model generated from a set of faces of the database Bosphorus.

5.2 3D face description: Good extraction of the reference points

We test in this section the performance of the three-polar representation for human face description in the case of a good extraction of the reference points. The figure 5 shows the three-polar representation with different resolutions which are linked to the number of levels in the representation construction.

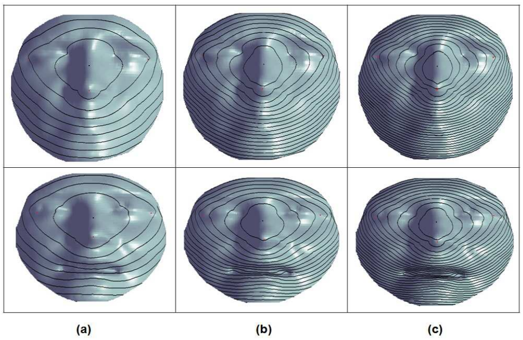

Figure 5: Row 1: A neutral face. Row 2: A face with a surprise expression. (a) The three-polar representation with 10 levels. (b) The three-polar representation with 20 levels. (c) The three-polar representation with 30 levels.

In order to illustrate the effectiveness of the three-polar representation, the matrix representing the pairwise normalized Hausdorff shape distance is computed between the ten faces. The first five faces correspond to the first class. The rest belongs to the second class. The figure 6 illustrates this matrix.

This matrix shows that the distances between the faces of the same person are smaller compared with the ones computed between faces of different individuals.

5.3 3D face description: Extraction errors of the reference points

A comparative study between the three-polar representation and the unipolar one is established in the sense of the 3D face description when extraction errors of the reference points positions exist. Two study cases are considered. For the first case, only the common point between these two representations is extracted with errors. It corresponds to the nose tip which is chosen randomly in this case in a geodesic disc of radius value equal to RG_{max} around its correct position. The figure 7 illustrates the variations area of the reference points in

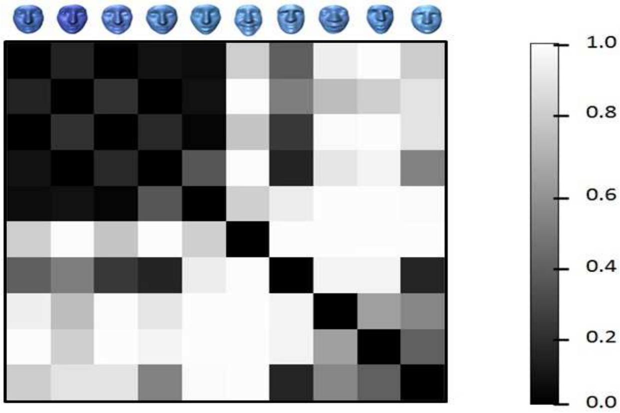

Figure 6: Matrix of pairwise normalized Hausdorff distances between the ten facial surfaces. The first five faces correspond to the same person while others belong to different individuals

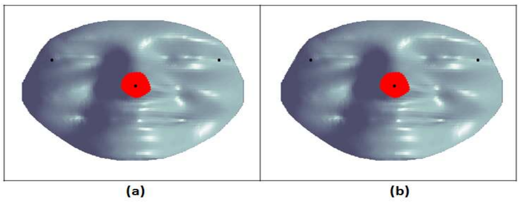

Figure 7: Variation area of the reference points for the first case of study: (a) the unipolar representation. (b) the three-polar representation

the first case of study for the two representations.
In the second case of study, the three reference points are chosen randomly in the three geodesic discs of radius values equals to RG_{max} around their correct positions for the three-polar representation. We note that the nose tip has the same variations for the both representations. The figure 8 illustrates the variation area of the reference points in the second study case.

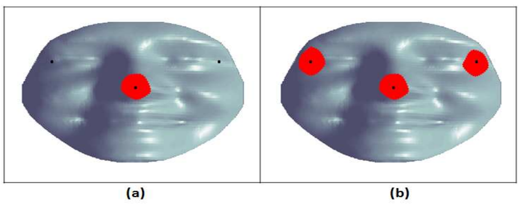

Figure 8: Variation area of the reference points for the second case of study: (a) the unipolar representation. (b) the three-polar representation

For each study case, the matrices of pairwise normalized Hausdorff shape distance are computed for the two representations. The results of the first study case are illustrated in the figure 9.

From the observation of these two matrices (figure 9), we can note that the three-polar representation better

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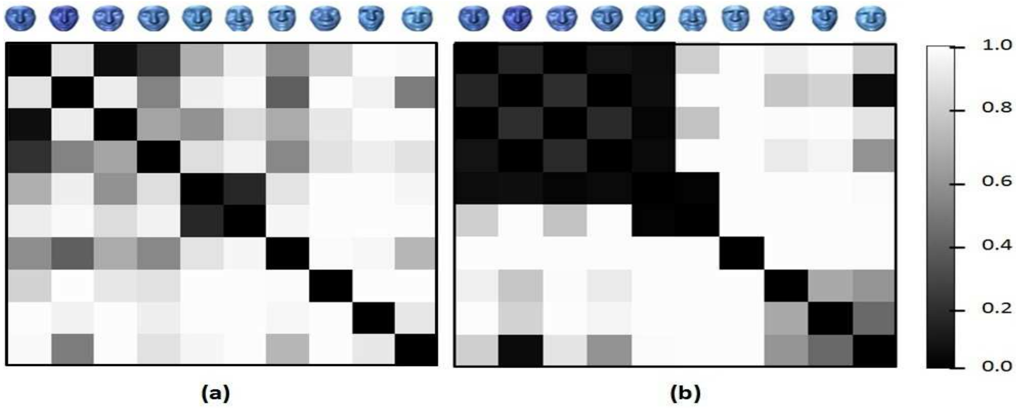

Figure 9: Matrices of pairwise normalized Hausdorff distances between the ten facial surfaces with errors on the positions of the nose tip. (a): The unipolar representation. (b): The three-polar representation.

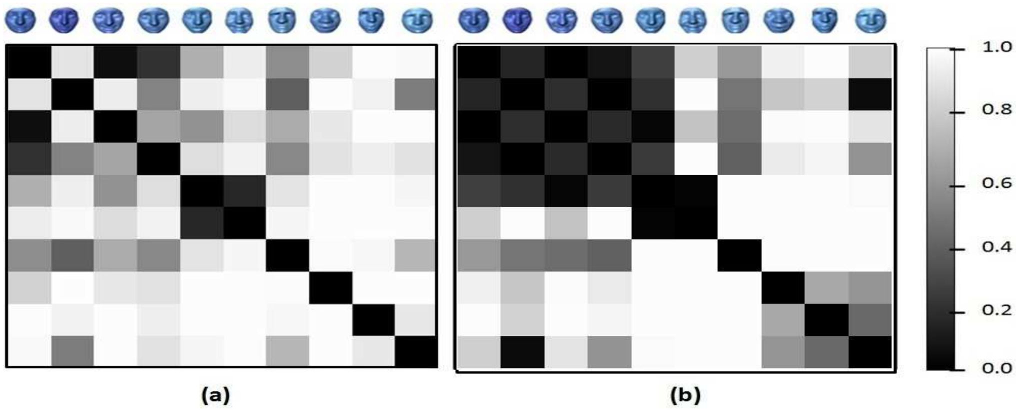

Figure 10: Matrices of pairwise normalized Hausdorff distances between the ten facial surfaces with errors on the positions of the nose tip for the unipolar representation and on the positions of all the reference points for the three-polar representation. (a): The unipolar representation. (b): The three-polar representation.

caracterizes 3D faces than the unipolar one in the case of wrong positions of only one reference point. In fact, the distances between the faces of the same person are smaller for the three-polar representation than the unipolar one.
The figure 10 shows the two matrices corresponding to each representation for the second study case. We can note that the three-polar representation has shown also better performances for the description of 3D faces in the second case of study than the unipolar one.

6 CONCLUSION

In this paper, we have studied a novel 3D invariant curved surface representation. It is qualified by three-polar since it is constructed from the superposition of the three geodesic potentials generated from three reference points of the surface. The goal was to generalize the representation constructed from one reference point and to ensure a more stability in the case of errors on the reference point positions. A comparison study between the three-polar representation and the unipolar one is established in the sense of the stability under errors on the reference points positions. We applied the novel 3D representation for 3D human face description in the mean of the Hausdorff shape distance.
The perspectives of future works involve the application of the three-polar representation on a larger number of 3D face surfaces. We intend also to find the optimal number of the levels of the novel representation. Finally, we propose to generalize the three-polar representation to *n* reference points. The value of *n* must be optimal in the sense that it will not be necessary to add other reference points.

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