Ultrasonic horn design for ultrasonic machining technologies

M. Nad$^{a, *}$

$^a$Faculty of Materials Science and Technology in Trnava, STU in Bratislava, Paulínska 16, 917 24 Trnava, Slovak Republic

Received 7 September 2009; received in revised form 4 June 2010

Abstract

Many of industrial applications and production technologies are based on the application of ultrasound. In many cases, the phenomenon of ultrasound is also applied in technological processes of the machining of materials. The main element of equipments that use the effects of ultrasound for machining technology is the ultrasonic horn – so called sonotrode. The performance of ultrasonic equipment, respectively ultrasonic machining technologies depends on properly designed of sonotrode shape. The dynamical properties of different geometrical shapes of ultrasonic horns are presented in this paper. Dependence of fundamental modal properties (natural frequencies, mode shapes) of various sonotrode shapes for various geometrical parameters is analyzed. Modal analyses of the models are determined by the numerical simulation using finite element method (FEM) design procedures. The mutual comparisons of the comparable parameters of the various sonotrode shapes are presented.

© 2010 University of West Bohemia. All rights reserved.

Keywords: ultrasound, ultrasonic machining technologies, modal properties, ultrasonic horn, longitudinal vibration, finite element method

1. Introduction

The use of ultrasound phenomenon is becoming increasingly used feature in many industrial applications. Ultrasonic vibrations have been harnessed with considerable benefits for a variety of production applications, for example, ultrasonic cleaning, plastic welding, etc. and has proved to offer advantages in a number of other applications. These applications include the automotive, food preparation, medical, textile and material joining and mainly applications in manufacturing industries. Significant increasing in performance and qualitative improvements are achieved by using ultrasonic vibrations in machining technological processes.

Applications of ultrasonic vibration energy in machining technologies are realized by two different approaches. The first approach, called as an ultrasonic machining, is based on abrasive principle of material removal. The tool which is shaped in the exact configuration to be ground in workpiece and it is attached to a vibrating horn. The second approach is based on the conventional machining technologies – ultrasonic assisted machining.

The ultrasonic vibrations are transmitted directly on cutting tools, respectively directly to a cutting process. These techniques are used for high precision machining application and for non-brittle materials and difficult-to-cut materials machining such as hardened steels, nickel-based alloys, titanium and aluminium-SiC metal matrix composites. The repetitive high-frequency vibro-impact mode brings some unique properties and improvements into metal cutting process [2, 5, 9, 10], where the interaction between the workpiece and the cutting tool is transformed into a micro-vibro-impact process.

*Corresponding author. Tel.: +421 335 511 601 ext. 32, e-mail: milan.nad@stuba.sk.
The application of ultrasonic vibration energy in the machining process provides many benefits and improvements in the process of cutting. In recent published work is reported that the high-frequency ultrasonic vibration of the cutting tool has reportedly allowed a significant reduction of cutting forces and tool wear, a surface finish improvement up to 25–40%, as well as roundness improvements up to 40–50%. When cutting low alloy steels, the ultrasonic vibration means a reduction in cutting forces up to 50% approximately, and produces smaller chips and a better surface finish in comparison to conventional turning.

Generally, in all manufacturing systems using ultrasonic vibrations, the electromechanical transducer acts as the source of mechanical oscillations, transforming the electrical power received from the generator into mechanical vibrations. The electromechanical transducers are based on the principle utilizing magnetostrictive or piezoelectric effects. The electromechanical ultrasonic transducers generate the vibration with resonant frequency \( f_{\text{res}} \approx 20 \) kHz and more. The amplitude of the resulting ultrasonic vibrations is inadequate for realization of the cutting process. To overcome this problem, the amplifying wave guided elements of the ultrasonic machining equipments are connected to the electromechanical transducer enabling to achieve the necessary size of amplitude. The wave-guide focusing device known as ultrasonic horn (also known as concentrator, sonotrode or tool holder) is fitted onto the end of the transducer. Ultrasonic horn transfers the longitudinal ultrasonic waves from the transducer end to the toe end with attached the cutting tool and it amplifies the input amplitude of vibrations so that at the output end the amplitude is sufficiently large to perform of required machining process.

The cutting performance of ultrasonic machining equipment primarily depends on the well-taken design of the sonotrode [6]. The sonotrode is the only part of the ultrasonic machining system which is unique to each process. They are used in various shapes and sizes, according to the application, but like other components should be resonant at the operating frequency. The sonotrode material used is a compromise between the needs of the ultrasonics and the application – titanium alloys, steel, stainless steel. As was mentioned earlier, the shape of ultrasonic horn depends on technological process for which will be used.

The most frequently used shapes of ultrasonic horns are: cylindrical, tapered, exponential and stepped. To achieve optimal performance of ultrasonic machining system is necessary to take into account all relevant effects and parameters that affect the dynamics of the system [4]. One of the most important elements of the ultrasonic system – sonotrode, must have the required dynamic properties, which must be determined already in design phase.

In the recent works, the selection of a suitable shape and corresponding dimensions of sonotrode are usually determined by numerical simulations using finite element method [1, 2, 7, 8, 11]. To the best author’s knowledge, no mutual comparison of the modal properties (natural frequencies, amplification factors) various sonotrode shapes, presented in this paper, is available in the literature.

In this paper, the dynamical analysis of various shapes of sonotrodes is presented. The effect of relevant sonotrode dimensions on natural frequencies and mode shapes is analyzed by finite element method (FEM). The mutual comparisons of the comparable parameters of the various sonotrode shapes are presented. The main aim of this paper is to present generally valid results leading to the selection of suitable shape and corresponding geometrical dimensions of sonotrode with required dynamical properties.

2. The sonotrode design

The principal function of the sonotrode is to amplify the amplitude of ultrasonic vibrations of the tool to the level required to the effective machining. The sonotrode serves also as an
element transmitting the vibration energy from the transducer towards to the tool interacting with workpiece. It does so by being in resonance state with the transducer. The design and manufacture of the sonotrode require special attention. Incorrectly manufactured sonotrode will impair machining performance and can lead to the destruction of the vibration system and cause considerable damage to the generator.

Generally, the sonotrodes are made of metals that have high fatigue strengths and low acoustic losses. The most important aspect of sonotrode design is a sonotrode resonant frequency and the determination of the correct sonotrode resonant wavelength. The wavelength should be usually integer multiple of the half wavelength of the sonotrode. The resonant frequency of sonotrode, which has simple geometrical shape can be determined analytically (cylindrical shape). For complicated geometrical shape, the resonant frequency is usually determined numerically using finite element method.

The required performance of sonotrode is assessed by an amplification factor

\[ \vartheta = \left| \frac{A_1}{A_0} \right|, \]  

(1)

where \( A_0 \) – amplitude of input end of sonotrode,

\( A_1 \) – amplitude of output end of sonotrode.

The basic requirement for the size of the amplification factor is

\[ \vartheta > 1. \]  

(2)

### 2.1. The analytical solution of the free sonotrode vibrations

The governing equation of longitudinally vibrating sonotrode with variable circular cross-section \( S(x) \), which is valid for 1D continuum (thin elastic bar), is expressed in the form

\[ \frac{\partial^2 u(x,t)}{\partial t^2} = c_p^2 \left[ \frac{1}{S(x)} \frac{\partial S(x)}{\partial x} \frac{\partial u(x,t)}{\partial x} + \frac{\partial^2 u(x,t)}{\partial x^2} \right], \]  

(3)

where \( x \) – coordinate in the longitudinal direction,

\( u(x,t) \) – longitudinal displacement of cross-section,

\( S(x) = \pi (r(x))^2 \) – cross-section area,

\( r(x) \) – radius of circular cross-section,

\( c_p = \sqrt{E/\rho} \) – velocity of the longitudinal waves in 1D continuum,

\( E \) – Young’s modulus of sonotrode material,

\( \rho \) – density of sonotrode material

The free sonotrode vibration of cylindrical shape \( r(x) = r \) is described by wave equation

\[ \frac{\partial^2 u(x,t)}{\partial t^2} = c_p^2 \frac{\partial^2 u(x,t)}{\partial x^2}. \]  

(4)

The solution of equation (4) is supposed in the form \( u(x,t) = U(x)T(t) \). Then partial differential equation (4) is divided into following two ordinary differential equations

\[ \frac{d^2 U(x)}{dx^2} + \frac{\omega_0^2}{c_p^2} U(x) = 0, \]  

(5)

\[ \frac{d^2 T(t)}{dt^2} + \omega_0^2 T(t) = 0, \]  

(6)

where \( \omega_0 \) – natural angular frequency.
Introducing the following non-dimensional quantities

- non-dimensional coordinate in the longitudinal direction: \( \xi = \frac{x}{l_0}; \xi \in (0; 1) \),
- non-dimensional longitudinal displacement of cross-section: \( \Psi(\xi) = \frac{U(x)}{l_0} \),

into the first of equations (5), we obtain non-dimensional equation

\[
\frac{d^2 \Psi(\xi)}{d\xi^2} + \beta^2 \Psi(\xi) = 0 \quad \text{and its solution} \quad \Psi(\xi) = A \cos(\beta \xi) + B \sin(\beta \xi), \tag{7}
\]

where \( \beta = \frac{c_p \omega}{l_0} \) – frequency parameter,
\( l_0 \) – sonotrode length.

Both sides of sonotrode have the possibility of motion in the axial direction. To the input side is attached electromechanical transducer which generates ultrasonic axial vibrations and to the output end is attached vibrating tool. Then the boundary conditions for free vibration of sonotrode are supposed as a free-free edge on both sides [7] in the form

\[
\frac{d\Psi(\xi)}{d\xi} \bigg|_{\xi=0} = 0, \quad \frac{d\Psi(\xi)}{d\xi} \bigg|_{\xi=1} = 0. \tag{8}
\]

Then, after application of boundary conditions (8) into solution (7), the following modal parameters of sonotrode are obtained

- natural frequency (in [Hz]) of the \( k^{th} \) mode shape
  \[
  f_{0k} = \frac{k}{2l_0} \sqrt{\frac{E}{\rho}}, \tag{9}
  \]
- non-dimensional wave length of the \( k^{th} \) mode shape
  \[
  \lambda_k = \frac{2\pi}{\beta_k} = \frac{2}{k}, \tag{10}
  \]
  where \( \beta_k \) is \( k^{th} \) root of characteristic equation and \( k = 1, 2, \ldots \)

In order to achieve the desired effect on ultrasonic machining, only the first two mode shapes of sonotrode are used, i.e. for \( k = 1 \) so-called “half wave” shape and \( k = 2 \) “wave” shape (Fig. 1).

As it is seen, the analytical determination of mode shapes and the natural frequencies of cylindrical shape of sonotrode is relatively simple. Analytical determination of these parameters for non-cylindrical shapes is more complicated. Therefore, to the determination of modal properties for more complicated geometrical sonotrode shapes, the numerical method (FEM) is better to use.
2.2. Finite element analysis of free sonotrode vibrations

The determination of modal properties for various shapes of sonotrode and assessment of the effect of relevant geometrical parameters on the modal properties, the finite element method is used. The FEM modelling and calculation of modal properties was done using the software package ANSYS. The element SOLID45 was used to the sonotrode FE model creation.

The equation of motion to description of free vibration of sonotrode FE model, by which the modal properties are determined, is expressed in following form

\[ M\ddot{u} + B\dot{u} + Ku = 0, \quad (11) \]

where \( M \) (\( B \), resp. \( K \)) is mass (damping, resp. stiffness) matrix,
\( \ddot{u} \) (\( \dot{u} \), resp. \( u \)) is vector of nodes acceleration (velocity, resp. displacement).

Since it can be supposed that the sonotrode materials have a low damping capacity (from dynamical aspect), the damping in equation of motion can be neglected. The equation of motion (11) can be for \( B = 0 \) rewritten into the form

\[ M\ddot{u} + Ku = 0. \quad (12) \]

The modal properties of sonotrode are determined by the solution of eigenvalue problem

\[ (K - \omega_i^2 M)\phi_i = 0, \quad (13) \]

where \( \phi_i \) – \( i^{th} \) eigenvector (mode shape),
\( \omega_i \) – natural angular frequency of \( i^{th} \) mode shape.

As mentioned earlier, the horns are manufactured in various shapes and dimensions. The cross-section of sonotrodes for ultrasonic machining or ultrasonic assisted machining has mostly circular shape. The functions that define the shape of longitudinal shape of sonotrode may be different. The main emphasis is to obtain the required dynamic properties. The geometries of considered sonotrode shapes are based on the dimensions and non-dimensional parameters (Table 1).

### Table 1: Parameters of Sonotrode Shapes

<table>
<thead>
<tr>
<th>Shape Description</th>
<th>Parameters</th>
<th>Mode Shape of Sonotrode Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda/2 ) – “half wave” shape</td>
<td>( k = 1 ), ( \lambda_1 = 2 ), ( f_{01} = \frac{1}{2\pi} \sqrt{\frac{E}{\rho}} ), ( \vartheta_1 = 1 )</td>
<td><img src="image" alt="Half Wave Mode Shape" /></td>
</tr>
<tr>
<td>( \lambda ) – “wave” shape</td>
<td>( k = 2 ), ( \lambda_2 = 1 ), ( f_{02} = \frac{1}{2\pi} \sqrt{\frac{E}{\rho}} ), ( \vartheta_2 = 1 )</td>
<td><img src="image" alt="Wave Mode Shape" /></td>
</tr>
</tbody>
</table>

Fig. 1. Mode shapes of cylindrical sonotrode vibrations

3. Results of numerical simulations

The numerical analyses are performed for various sonotrode shapes and geometrical parameters defined in Table 1. The steel as a sonotrode material is used to numerical simulation (\( E = 210 \) GPa, \( \rho = 7800 \) kg \( \cdot \) m\(^{-3} \), \( \nu = 0.3 \)).
Table 1. Geometrical parameters of sonotrode shapes

<table>
<thead>
<tr>
<th>SONOTRODE SHAPE</th>
<th>SLENDERNESS RATIO</th>
<th>SHAPE PARAMETERS</th>
<th>SHAPE FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylindrical</td>
<td></td>
<td>−</td>
<td>( r(x) = r = \frac{d_0}{2} )</td>
</tr>
<tr>
<td>tapered</td>
<td>( \delta = \frac{d_0}{l_0} )</td>
<td>( \alpha \in (-5^\circ; 5^\circ) )</td>
<td>( r(x) = \frac{d_0}{2} (1 + l_0 \tan(\alpha)) )</td>
</tr>
<tr>
<td>exponential</td>
<td></td>
<td>( a \in (0.3; e) )</td>
<td>( r(x) = \frac{d_0}{2} a^x )</td>
</tr>
<tr>
<td>stepped</td>
<td></td>
<td>( \eta = \frac{l}{l_0} ) ( \eta \in {0.25; 0.5; 0.75} )</td>
<td>( r{x \in (0; l)} = \frac{d_0}{2} ) ( r{x \in (l; l_0)} = \frac{d}{2} )</td>
</tr>
</tbody>
</table>

Note: \( d_0 \) – diameter on input side of sonotrode, \( d \) – step changed diameter, \( l_0 \) – length of sonotrode.

In the following, the non-dimensional resonant frequencies for different geometrical shapes of horn are defined as

\[
\theta_k = \frac{f_k}{f_{0k}},
\]  

(14)

where \( f_k \) – \( k^{th} \) natural frequency of analysed horn, 
\( f_{0k} \) – \( k^{th} \) natural frequency of cylindrical sonotrode shape for corresponding slender-ness ratio.

The results of dynamical analysis of considered sonotrodes are expressed in dependence to the above mentioned dimensionless quantities. This manner of presentation gives an opportunity for generalization of the results and also allows their mutual comparison for the various geometric shapes of sonotrodes. Moreover, this manner of results presentation can be used to the selection of shape and dimensions of sonotrode with the required properties.

The value of resonant frequency of corresponding geometrical horn shape is determined using following equation

\[
f_k = \theta_k f_{0k} = k \frac{\theta_k}{2l_0} \sqrt{\frac{E}{\rho}},
\]  

(15)

where \( k = 1 \) “half wave” shape, \( k = 2 \) “wave” shape.

In the next figures (Fig. 2–5), the dependences of non-dimensional natural frequencies and amplification factors on relevant parameters of sonotrode design are shown.

4. Conclusion

The dynamical analysis of the various geometrical shapes of sonotrodes as one of the most important elements of the ultrasonic machining systems is presented in this paper. The main dynamic characteristics (natural frequencies and amplification factors) of sonotrode in the resonant state were studied according to the geometric shape and dimensions. The efficiency and performance of ultrasonic machining systems depends on specific design and the relatively large number of parameters.
Selection of a geometric shape of the sonotrode depends on technological operation for which the sonotrode will be used. The value of resonance frequency and amplitude amplification factor on the output side of sonotrode are fundamental requirements for the selection.
Exponential sonotrode shape

Fig. 4. Exponential sonotrode shape. Dependence of the non-dimesional frequencies (a) and amplification factors (b) on parameter “a” of exponential function

of appropriate sonotrode shape. The cylindrical sonotrode shape was used as a comparative geometric shape to the expressing of results of other sonotrode shapes. Generally, it can be said that the geometrical shapes and dimensions of the sonotrodes affect the stiffness and mass distribution. With the changing of the cross section towards to the output side of sonotrode, the amplification factor $\vartheta$ is changing in the whole interval of cross-section changing (Fig. 3b–4b) ($\vartheta_i > 1.0$ for increasing cross section, $\vartheta_i < 1.0$ for growing cross section). In addition to changes in cross section, significant effect on the amplification factor has also slenderness ratio. For the increasing cross-section and growing slenderness ratio of sonotrode, the amplification factor decreases. For the interval in which the cross-section increases, the amplification factor increases in dependency on the growing of slenderness ratio (Fig. 3b–4b). Dependency of the natural frequencies on the slenderness ratio changes and cross section changes of sonotrodes are presented in Fig. 2a–4a. The specific case of sonotrode shape is the stepped shape. For this case of sonotrode shape are valid the same conclusions as in the cases of tapered and exponential sonotrode shapes. For this sonotrode shape the dependeces of its natural frequencies and amplification factors on the position of step changed diameter are presented in Fig. 5a and Fig. 5b.

For the design of sonotrode shape the emphasis was given to the fact that these sonotrode shapes are primarily used for the ultrasound technology operations in axial direction sonotrodes. In the event when during the technological operation the loading in the radial direction son-
Stepped sonotrode shape

Fig. 5. Exponential sonotrode shape. Dependence of the non-dimensional frequencies (a) and amplification factors (b) on the non-dimensional location “η” of step diameter change and for ratio \( \frac{d_0}{d} = 2 \)

When a step diameter change in the sonotrode axis arises, it is necessary to ensure the design of geometrical sonotrode shape with sufficient bending stiffness.

The results of the numerical analyses are presented in the form of non-dimensional quantities and parameters. This provides the possibility of selection and comparison of different shapes and to provide an effective tool for selection of suitable sonotrode shape with required properties.

Acknowledgements

The work has been supported by the grant projects VEGA 1/0256/09 and VEGA 1/0090/08.

References


