Visualization of the nonlinear dynamical systems using WWW

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Abstract

Various 3D-view and 2D-projections of the abstract mathematical objects (strange attractors) have been discussed. We are studying the nonlinear dynamical systems using WWW technologies VRML, HTML and HTTP. Such visualization technique gives an essential improvement in the scientific investigation of the nonlinear dynamical systems. The search of the homoclinic points for dissipative dynamical systems is shown as application of such technique. Current implementation of such software have been presented at WWW.
Keywords: WWW, VRML, visualization, dynamical system, homoclinic point

1 Introduction

During last few years the computer modelling methods have been made more important under investigation of the nonlinear dynamical systems. The chief drawbacks of the most all software programs connected with nonlinear dynamical systems studing are:

1) static picture of system;

2) complication of control panel;

3) practically absence of possibility invoke of the network applications from within the workspace of the browser.

All of that problems have been cancelled using VRML-based graphical output of the computer modelling program.

Article is organized as follows. In Section 2 the brief description of VRML and investigation program of the nonlinear systems have been discussed. Section 3 is devoted the methods for finding of the homoclinic points for dissipative systems.

2 VRML and nonlinear dynamical systems

The Virtual Reality Modelling Language (VRML) [http:97a] is a language for describing multi-participant interactive simulations — virtual words networked via the global Internet and hyper-linked with the World Wide Web. It is the intention of its designers that VRML become the standard language for interactive simulation within the WWW.

VRML is designed to meet the following requirements:

1) platform independence;

2) extensibility;

3) ability to work well over low-bandwith connections.

This features allow VRML to be a natural candidate for output graphical format in the investigations of the nonlinear dynamical systems.

There are three main fields in the investigation of the dissipative dynamical systems:

1) usual compendium of knowing strange attractors;

2) scientific tool for close studying of each knowing strange attractors;

3) scientific tool for detaled investigation any three dimension dissipative systems.

Our VRML–based realization of the first part, which under construction, is placed at http://desert.ihep.su/~smirnova. This workspace contains a various kind of knowing strange attractors as Lorenz, Rikitake, etc.. This part is devoted mainly demonstration purposes, but has a standalone scientific interest as a tool for investigation any dynamical systems by means of user's input parameters, such as number of points of trajectory, step of integration, initial point, color of the output attractor. The query form of that part is presented at Figure 1. Figure 2 contains an output VRML files for Rikitake attractor with 5000 points on trajectory (in the case of Rikitake attractor one can see long transition region from the initial point up to strange attractors).

Main advantage of our software consists in a optimal organization of the FaceSets in VRML environment. The output of the analytical program is an ordered set of points of the needed trajectory. Further, by means of Perl-based converter we transfer all points to the single IndexedFaceSet construction. This long array is organized as one VRML primitive. Any VRML browser consider such IndexedFaceSet instruction as a one long primitive instruction and any operations with all our trajectory is provided in a very fast manner as operation with one primitive. Due to that organization we have a interactive virtual worlds even for a large number of points (up to 10^6 points is under operation in our method) and even on the machine with low virtual memory.

Another parts are devoted more closely investigation of the knowing attractors (part 2) and any three dimensional dissipative systems with you own right side of the system of nonlinear differential equations.

3 Detection of homoclinic points

Most of the natural phenomena exhibit chaotic behavior. Its presence can be defined as the existence of intersections of stable and unstable manifolds of hyperbolic fixed point or as the existence of homoclinic point or a homoclinic trajectories. The existence of the latter in a dynamical system enables us to discuss some of its properties. The existence of hyperbolic set follows from the existence of transversal homoclinic trajectories.

There has not been any rigorous proof, found at the level of a theorem, for the problem of existence of homoclinic trajectories in a common dynamical system. In some cases homoclinic trajectories appear at small periodic disturbances of autonomous Hamiltonian systems with one degree of freedom, having closed separatrix of loop. To determine the presence of homoclinic trajectories, it's possible to use Melnikov's method [Mel'n63a] (or Palmer's method for n-dimensional cases [Palme84a]). But only if the dissipation is wear and the equations for manifolds with zero dissipation are known is this method applicable.

This criterion does not imply anything about the appearance of a strange attractor in dissipative dynamical system showing a stable chaotic behavior in a large area of the phase space. It should be noted that an attractor is not a manifold. Let us assume that a dissipative dynamical systems are given in which the presence of homoclinic trajectories or homoclinic points should be tested, but the equations for the manifolds of hyperbolic fixed point are unknown.

We consider three-dimension dissipative dynamical systems with strange attractor.

Let us assume that for the given nonlinear differential equations there exists intersection stable and unstable manifolds for hyperbolic fixed point and a Poincare map. In this case the unstable manifold returns to the crossing plane. In this case the Poincare map is not determined in the intersection of the stable manifold and the crossing plane, and is not continuous in its neighbourhood.

There are several ways of searching for homoclinic points. We consider three of them [Klime96a].
Method 1. One constrict the stable and unstable manifolds for hyperbolic fixed point and finds the intersection of these manifolds.

While building the stable manifold for a hyperbolic fixed point x_0 with real eigenvalues, the latter manifold can be approximated with a plane stretched over eigenvectors belonging to negative eigenvalues. Approximation of curved manifolds W^s and W^u with planes in some neighborhood of the hyperbolic fixed point induces error in the computation of invariant manifolds. The error can be estimated using quadratic asymptotics of the manifolds mentioned [Hassa80a]. Some other difficulties in constructing the stable manifolds are discussed in paper [Parke87a]. For example, let us consider the construction of the intersection of the stable manifold with the crossing plane, and of the Poincare map, for Lorenz attractor (Figure 3). Because of the exponential instability of trajectories on strange attractors, the probability of getting intersection of the stable manifold with the crossing plane with Poincare map is a small value [Sparr82a].
Method 2. For building the stable and unstable manifolds of hyperbolic fixed point the λ-lemma [Palme84a] can be used. The lemma can be applied to a local diffeomorphism and even to a C^1-mapping in a Banach space in some neighborhood of the hyperbolic fixed point. It demands that the partial derivatives are uniformly continuous, and that

both the stable and unstable manifolds are of finite dimensions. So the local stable (unstable) manifold of the hyperbolic fixed point of the diffeomorphism should always be considered in a neighborhood of the fixed point in the stable (unstable) subspace of linear part of mapping f.

As an example we show the construction of the stable and unstable manifolds for the Lorenz attractor (Figure 4) by means of the λ-lemma. It was made clear that Cantor structure exists in neighborhoods of homoclinic points of these attractors [Klime91b].

Method 3. Now, we find the homoclinic points in the strange attractor wihtout requiring the calculation of the manifolds of the fixed point and the construction of the intersection of the stable manifold and crossing plane W of the Poincare map.

Let $f(x)$ be the first point where the unstable manifold originating at $x \in W$ intersects the plane W. This defines the map $f : W \backslash S \rightarrow W$. Here S is a line of discontinuty, it divides W into two parts, W_1 and W_2. Then there is a unique limit p_1 of the images of the points from W_1, approaching S from one side, and there is a unique limit p_2 of the images of the points from W_2, approaching S from the other side.

Let us consider the process of searching for homoclinic points. To find a homoclinic point we use the fact that the Poincare map is not determined in the intersection of manifolds and is not continous in its neighborhood. First, we test the Poincare map for discontinuity in the neighborhood of some point. As soon as the neighborhood is found, we search for a point from the crossing plane where the Poincare map is not defined. Let us denode this point by x on the plane. Then $f^k(x)$ — the images of point x obtained by letting the mapping f act k times — are situated along some curves (or curve) on the plane. Let these curves be called the right (RB) and the left (LB) branches respectively. Let P_{12} be a segment of curve LB. We select points $p_1, p_2 \in P_{12} \subset LB$ such that $p_1 \in LB \cap W_1$, $p_2 \in LB \cap W_2$, and $f(p_1) \in LB \cap W_2$, $f(p_2) \in RB \cap (W_1 \cup W_2)$. Then we find sequential points $p_1^j, p_2^j \in P_{12}^j \subset P_{12}^{j-1} \cdots \subset LB$ such that $f(p_1^j) \in LB \cap W_1$ and $f(p_2) \in RB \cap (W_1 \cup W_2)$. It is clear that if as we have homoclinic points, we have $\lim_{j\rightarrow\infty} \bigcap P_{12}^j = p^* \in S$, where point p^* is a point for which $f(p^*)$ is ill-defined. If such a point can not be found, there are no strange attractors in the dynamical system.

Modifications of this method for attractors wich one branch of the Poincare map is obvious. This algorithm was used for finding homoclinic points of the Lorenz and Rikitake attractors, of the Rösler attractor and of a simple attractor.The results obtained for the coordinates of the homoclinic points are shown on Table 1 [Klime96a].

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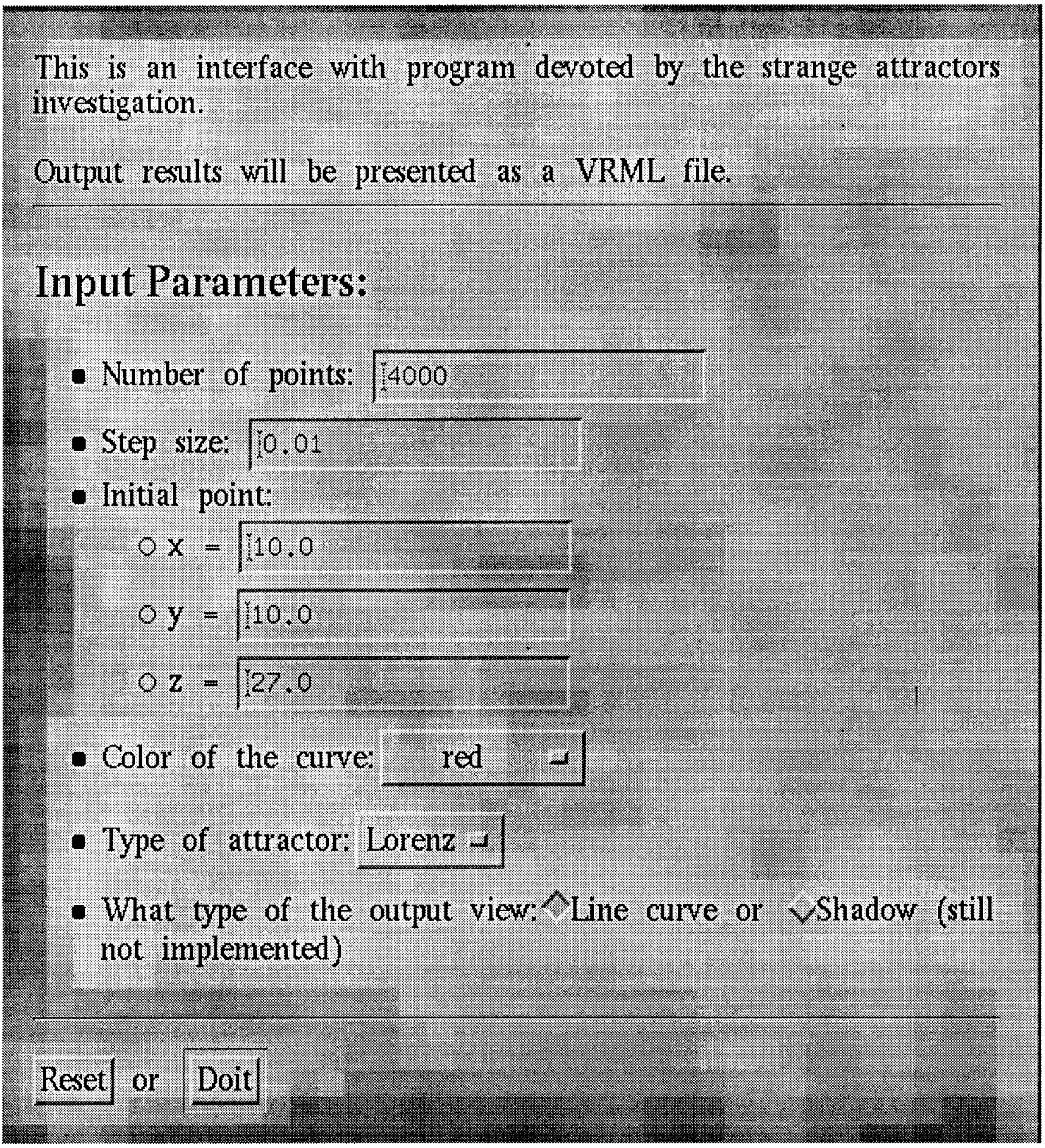

Figure 1: Installation of parameters for dynamical systems

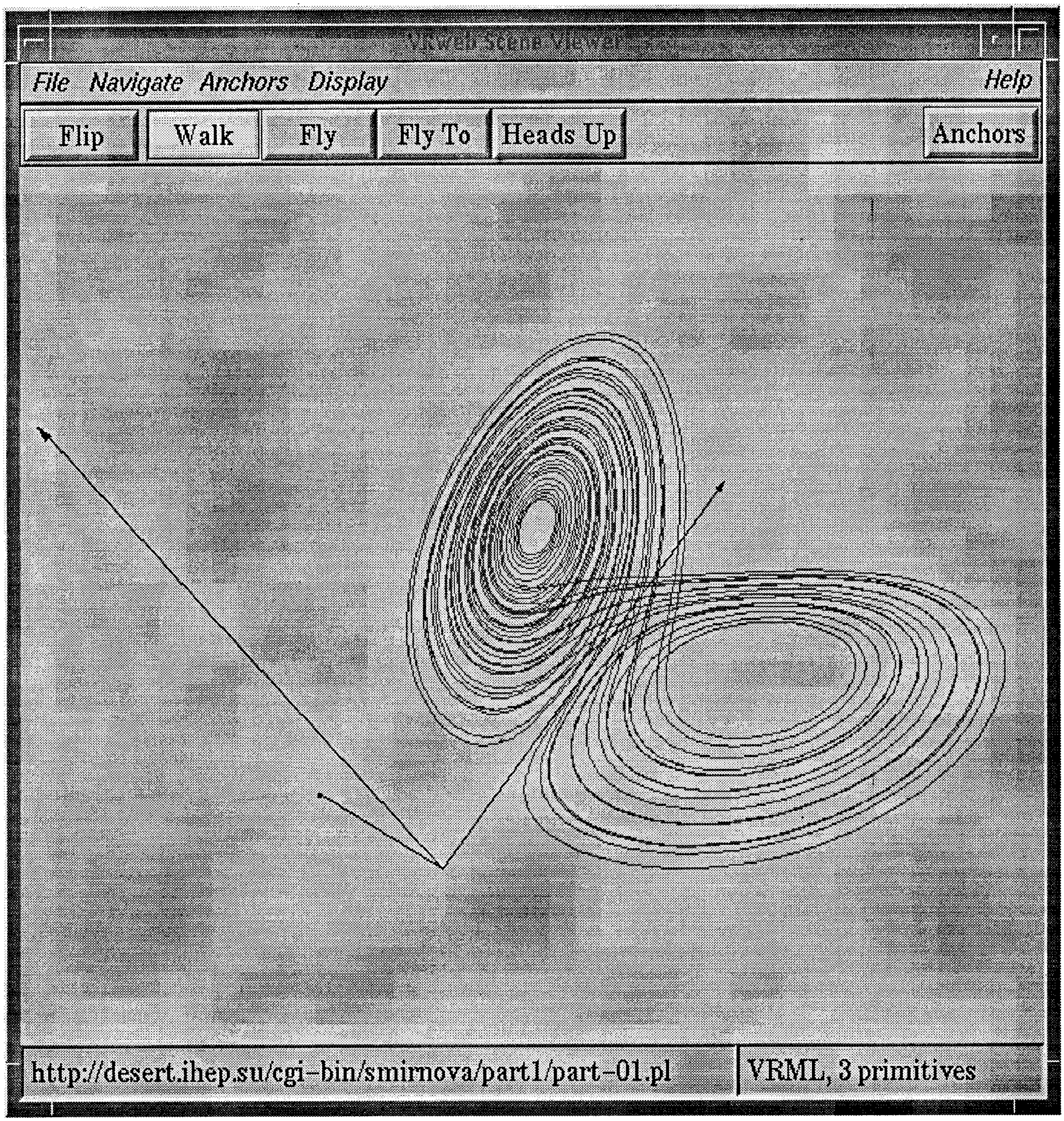

Figure 2: Central part of the Rikitake attractor

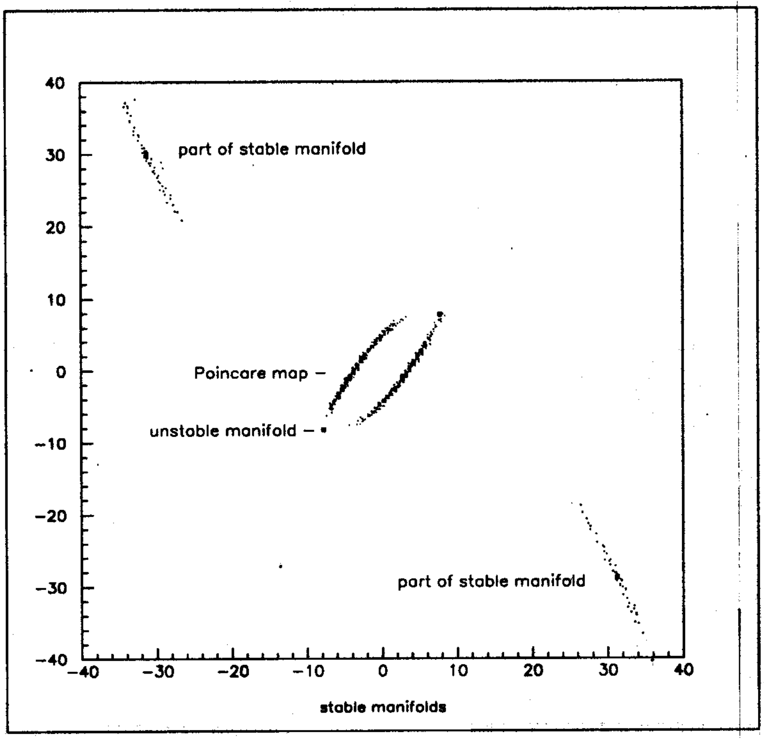

Figure 3: Intersection of stable and unstable manifolds of the origin with the crossing plane and the Poincare map for Lorenz attractor.

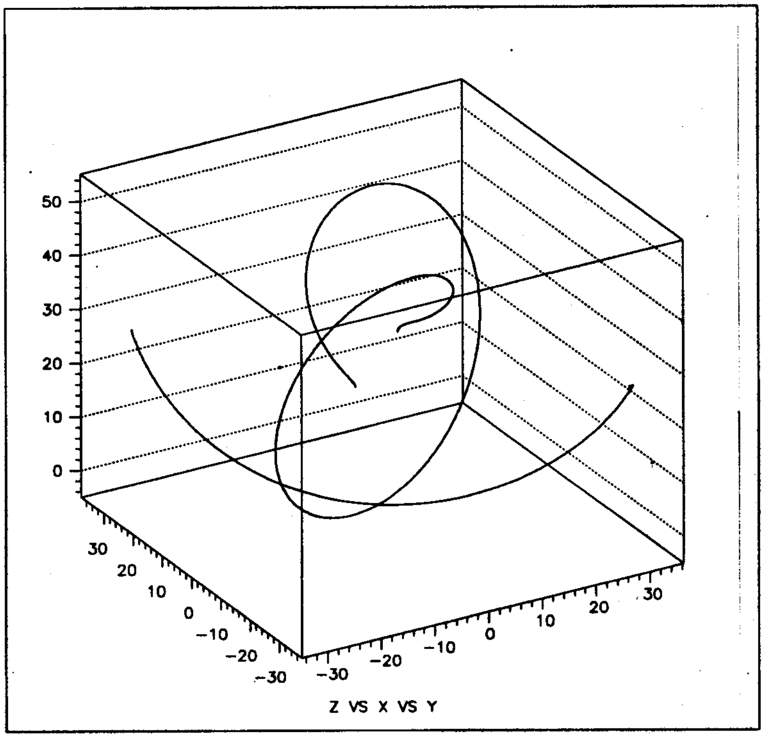

Figure 4: Stable and unstable manifolds of the origin for the Lorenz attractor.