Frequency-based Operators for Freeform Feature Shape Reuse

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ABSTRACT

Freeform feature shape reuse is an important issue in industrial design supporting. In this paper, the frequency-based operators for freeform feature shape reuse are discussed. Unlike the shape modeling approaches in the spatial domain that manipulate shapes by controlling the low-level geometric elements, the frequency-based operators work in the 2D frequency domain, and control the shape by manipulating its frequency components. This directly reduces the computational complexity of shape manipulating from 3D to 2D problem. Working with the surface distribution of the region of interest, our approach for feature reuse is screening the designer off the low-level geometric entities while performing a shape editing during the feature reuse process. By discretization within specified tolerance, our approach can apply to solid models, surface models, mesh models, or point cloud models.

Keywords

Feature reuse, frequency-based operator, discrete model, Fourier transform, sampling, surface signal.

1. INTRODUCTION

Freeform feature as a complex high-level entity enables the fast creation and modification of the geometric model. In both mechanical and industrial design, freeform feature has been adopted as the pivot entity for associating specific functional meaning to groups of geometric elements (such as faces, edges, vertices and so on), thus offering the advantage of treating sets of elements as a single entity [Fontana99]. It is widely believed that freeform feature reuse plays an important role in facilitating fast shape design.

Conventionally, feature reuse is implemented in the spatial domain by hard copying of the feature geometry and then recreating it on a specified region of a target surface [Biermann02] [Wallace03] [Wang02]. Those methods may work with all digital models, including solid model, surface model, mesh model, or point cloud model. Feature fitting is always served as a complimentary means to identify the feature geometry when working the discrete model [Vergeest02] [Vergeest03]. They provide a convenient way to reuse regional geometry, for instance the feature geometry, in product shape modeling process. However, their main drawbacks are that the distortion of the feature itself has not yet been taken into consideration in the reuse process. For instance, in case a feature was located on a curved surface, the existing methods may not be able to separate the feature from the domain shape. To obtain the "pure" feature shape, the underlying domain shape should be "filtered off". Because of the complexity of the shape, retrieval of the "pure" feature geometry is hardly to achieve by geometrical means in most cases.

Alternatively, digital signal processing provides the possibility to decompose various signals in the frequency domain. By treating a shape as surface signals, the shape descriptors are widely employed to bridge shape transitions between the spatial and the frequency domain, such as Fourier, Hartley or wavelet transforms [Bracewell86] [Bracewell90] [Stollnitz96]. Shape descriptors set up the correspondence between the spatial shape and the frequency spectrum, which might be applied to facilitate feature reuse. In addition, there are a

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number of advantages to working in the frequency domain. First, it provides an intuitive way for shape editing from cognition point of view. By screening off the sophisticated low-level geometric elements, such as control points, edges, faces or topology, the designer can focus on shape editing on the bases of his general cognitive understanding of shape, for instance, the base surface signal (generally composed of low frequency components), or the feature signal (high frequency components). Secondly, spatial filtering operations are also simple and computationally inexpensive to implement in the frequency domain. And more, the cross-correlations, which are useful for feature detection in image and 3D scalar data sets processing, can be computed more efficiently in the frequency domain [Anuta70] [Malzbender93] [Taubin95].

Our approach suggests a series frequency-based shape-editing operators. The main idea is that, by applying Fourier transforms (FTs) or Hartley transforms (HTs), shapes can be decomposed into a series of frequencies which contribute to specific geometric aspects [Ghosh93] [Gonzaalez02] [Hughes92]; and by means of computational operations in the frequency domain, it is feasible to retrieve the "pure" feature and to reuse it in shape modeling.

2. PREVIOUS WORK

In [Ghosh93], the algebra of geometric shapes is introduced, in which a series of FT-based shape operators are defined on the frequency domain of the planar shape, including shape summation, scaling, and multiplication. Attempts to use 2D FT to construct 3D shapes can be found in [Tello95] [WangPC98]. Those approaches describe the 3D shape as a series of 2D planner sections, and create the 3D shape by interpolating its section contours.

Direct representation of a 3D shape using Fourier descriptor has been discussed in [Wu98], in which the shape distribution is represented as a function of ω and θ, i.e., $S: s(\omega,\theta) = (x(\omega,\theta), y(\omega,\theta), z(\omega,\theta))$, where ω and θ are the measures of phase angle in E^3, and $\omega \in [0,\pi)$, $\theta \in [0,2\pi)$. By means of FT, the proposed approach attempts to reduce the size of the data set representing the shape surface by cutting off the frequency harmonics within a certain precision.

In [Taubin95], the freeform volume is treated as an n-dimensional discrete surface signal $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$. By generalizing classical Fourier analysis to 2D surface signals, the proposed approach reduces the problem of surface smoothing or faring to low-pass filtering. The transform of the surface signal is implemented by $\mathbf{x}' = f(\mathbf{K})\mathbf{x} = \sum_{i=1}^{N} \xi_i f(k_i) u_i$, where $\mathbf{K}$ is the circulant matrix of the Laplacian transformation of $\mathbf{x}$, k_i the real eigenvalue of $\mathbf{K}$, and $\mathbf{K}u_i = k_i u_i$. The $f(k_i)$ constitutes the transfer function of the filter and ξ_i the sinusoidal components of the Laplacian transformation. Shape smoothing is achieved by altering the high frequency coefficients. A scheduled Fourier volume morphing is presented by Hughes [Hughes92], in which the volumetric shape morphing from one to another is formulated as a frequency-based linear interpolation.

Recently, Pauly et al [Pauly01] introduced a windowed FT framework for processing point-sampled objects using spectral-based approaches, in which the object model is split into a number of overlapping patches. A patch is defined as a collection of sampling points that represent a connected region of the underlying surface. The patch surface is re-sampled on a regular grid using a fast scattered data approximation (SDA) algorithm, and then transferred from spatial domain into frequency domain by applying discrete Fourier transform (DFT). Hence the direct shape processing is implemented by operations on the Fourier spectrum to achieve a variety of effects, such as de-noising, smoothing and feature enhancing.

3. ASSUMPTIONS

In this paper, we focus on the issues of freeform feature reuse via operators in the frequency domain. To simplify the problem under discussion, we perfer to restrict our topic under some assumptions. we assume that models in editing, both the feature shape and the target shape, have a regular parameterization or could be parameterized without greater distortion. For irregular parameterization or extremely sophisticated models that cannot be simply parameterized, techniques introduced in [Pauly01] for patch dividing and merging can be employed. In addition, we limit our approach to the region of interest (ROI) of the model, which can be sampled under the previous assumption. Obviously, by setting up topology relationship among surface patches, the proposed approach can still be extended to cover global shape modeling, as well.

In addition, by the word "feature", we mean that a shape is distinct contrasting the domain surface where it located, for instance, the dragon shapes in this paper. we refer that a shape without drametic contrast with its domain surface to a ROI, from which the feature can not be easily separated out. And more, throughout this paper, we assume that the low frequency components in the FT is

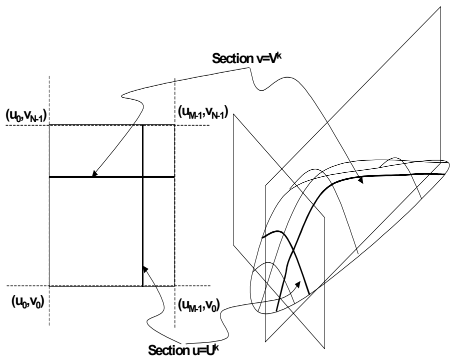

Figure 1. Sampling scheme of the freeform shape feature

corresponding to the domain surface signal, and the high frequency components the feature.

4. THE FOURIER MODEL

Under the assumption in Section 3, a 3D freeform feature or the ROI on an existing model can be represented by a series of discrete sections. The sampling of these section contours represents a spatial shape distribution (a vector field), with which the feature shape can be rebuilt by interpolating. They constitute the overall shape geometry in the form of a matrix of the sampling points.

Definition 1. (Model equivalency) If the Hausdorff distance between a point set $\Psi(p)$ and a ROI represented by $S: s(u,v)$ is less than a given tolerance, that is, if $\forall p_i \in \Psi(p)$, $\exists (u_k, v_l)$, or $\forall (u_k, v_l)$, $\exists p_i \in \Psi(p)$ satisfies the following equation:

$$\exists \{\min(\|p_i - s(u_k, v_l)\|)\} \le \varepsilon, \quad (1)$$

then $\Psi(p)$ is called the discrete model of $S: s(u,v)$, i.e., $\Psi(p)$ is equivalent to S, or $\Psi(p) \equiv S$, where ε is the modeling precision, $(u_k, v_l) \subset \Re^2$ and $\|s(u_s, v_t) - s(u_k, v_l)\| \le \varepsilon$, where $s(u_k, v_l)$ and $s(u_s, v_t)$ are the adjancet points on the ROI.

Definition 2. (Spatial shape distribution) For a ROI $S: s(u,v)$ of a model,

$$S^D = \{s(u_m, v_n) \mid m = 0,1,\cdots,M-1; n = 0,1,\cdots,N-1\} \quad (2)$$

is called a spatial distribution of shape S, $(u_m, v_n) \in \Re^2$ the sampling parameter, and $[0, M-1] \times [0, N-1]$ the sampling grid.

Obviously, under definition 1, $S^D \equiv S$. Figure 1 depicts the sampling scheme, in which the correspondence between parametric domain and the spatial domain is shown. Be aware of that the sections on the sampled shape may be curved in some cases. In addition, Figure 1 is only for explanation purpose; the parametric domain is not a must. Strategies on how to regularize an arbitrary point set could found in [Jeff02] [Shashkov01].

Topology. As input data of the FT, the sampled distribution automatically specifies a $[0, M-1] \times [0, N-1]$ grid with a simple topology shown in Figure 2. At the degenerated corner of the sampling grid, the sampling points are assumed to be duplicated several times to maintain the sampling topology.

Periodicity. To apply FT, the sampled data has to satisfy some prerequiste condictions. For instance, the periodicity is one of them. To make the sampled matrix of the ROI periodic, we expend the sampling points on each u section and v section in a reversed order. This makes the sampled data virtually forming a closed path in both u and v direction. The expended sampling matrix may looks like:

$$S^D = \begin{pmatrix} u_0v_0 & u_0v_1 & \cdots & u_0v_{N-1} & u_0v_{N-2} & \cdots & u_0v_1 & u_0v_0 \\ . & & & & & & & \\ . & & & & & & & \\ u_{M-1}v_0 & u_{M-1}v_1 & \cdots & u_{M-1}v_{N-1} & u_{M-1}v_{N-2} & & u_{M-1}v_1 & u_{M-1}v_0 \\ u_{M-2}v_0 & u_{M-2}v_1 & \cdots & u_{M-2}v_{N-1} & u_{M-2}v_{N-2} & \cdots & u_{M-2}v_1 & u_{M-2}v_0 \\ . & & & & & & & \\ . & & & & & & & \\ u_0v_0 & u_0v_1 & \cdots & u_0v_{N-1} & u_0v_{N-2} & \cdots & u_0v_1 & u_0v_0 \end{pmatrix}$$

This results the following FT is purely even, and only the real part of the FT is non-zero. For the sake of consistency, in the following subsections, we still denote the sampling grid as $[0, M-1] \times [0, N-1]$, while the expansion process is implemented implicitly.

Sampling precision. An iterative searching over the ROI is performed to find a minimum curvature within the ROI. Then a step length $\Delta s = 2\sqrt{\frac{\varepsilon(2-\rho\varepsilon)}{\rho}}$ is calculated, where ρ is the minimum curvature

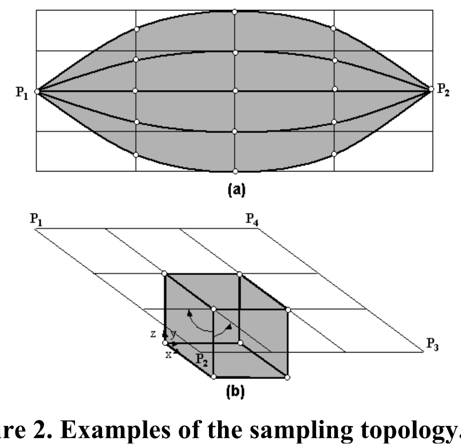

Figure 2. Examples of the sampling topology. (a), A degenerated ROI; (b), A cubic surface.

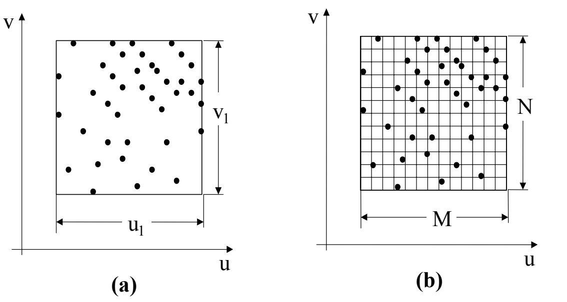

Figure 3. Regular sampling from irregular point set. (a) Irregular sampled point set and its smallest enclosure box; (b) The regular sampling grid.

within the ROI and ε the given modeling tolerance. Δs is used as the uniform sample step to obtained the input samples for FT.

Irregular ROI. For a ROI with highly irregular boundary, methods introduced in [Pauly01] can be applied to group the adjacent sample points into clusters, and then the adjacent clusters are merged into patches. Generally, these patches are a set of irregular sampled points without any additional knowledge about the spatial relations between them. To create regular sampled data from the irregular ROI, we use the scattered data approximation method presented by Gortler et al. [Gortle96] for image based rendering, and please refer to there for the details of how to project the irregular data set onto a regular sampled grid. Figure 3 illustrates this conversion in the parameter plane.

Definition 3. (The general shape distribution) Distributions derived from definition 2 are called general shape distribution.

A general shape distribution could be a discrete pointset of a surface, a surface normal distribution, a curvature distribution, or others. General shape distributions can be used to construct the Fourier model to depict a specific shape aspect.

Definition 4. (3D Fourier model) For a $[0, M-1]\times[0, N-1]$ regular sampling grid on a freeform feature or a target surface, the sampled distribution given by Equation (2) can be equivalently represented by means of the discrete Fourier transform (DFT) as:

$$S = \Im^{-1}\{f(\tau,\zeta) | \tau \in 0,1,\cdots,M-1, \zeta \in 0,1,\cdots,N-1\} \qquad (3)$$

where $\Im^{-1}(\cdot)$ denotes the reverse FT, and $f(\tau,\zeta)$ its elements given by

$$f(\tau,\zeta) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} s(u,v) e^{-j2\pi\left(u\tau/M + v\zeta/N\right)} \qquad (4)$$

We call Equation (3) the 3D Fourier model of a shape S.

5. FREQUENCY-BASED OPERATORS FOR FEATURE SHAPE REUSE

Given a 3D Fourier model of the feature (or ROI) defined by Equation (3), and another shape distribution $G = \{g(u_m, v_n)\}$ with 3D Fourier model

$$G = \Im^{-1}\{g(\tau,\zeta) | \tau \in 0,1,\cdots,M-1, \zeta \in 0,1,\cdots,N-1\},$$

the generalized shape reuse formula could be defined as:

$$C = \Im^{-1}\{h_1(\tau,\zeta) f(\tau,\zeta) + h_2(\tau,\zeta) g(\tau,\zeta) \mid \tau \in [0, M-1], \zeta \in [0, N-1]\} \qquad (5)$$

where C is the synthesized shape; $\Im^{-1}[\cdot]$ denotes the 2-D reverse DFT; $h_1(\tau,\zeta)$, $h_2(\tau,\zeta)$ are called control functions, which have the following characteristics:

(1) If $h_1(\tau,\zeta)$ or $h_2(\tau,\zeta)$ is constant on (τ,ζ), it will scale the global shape of S or G accordingly;

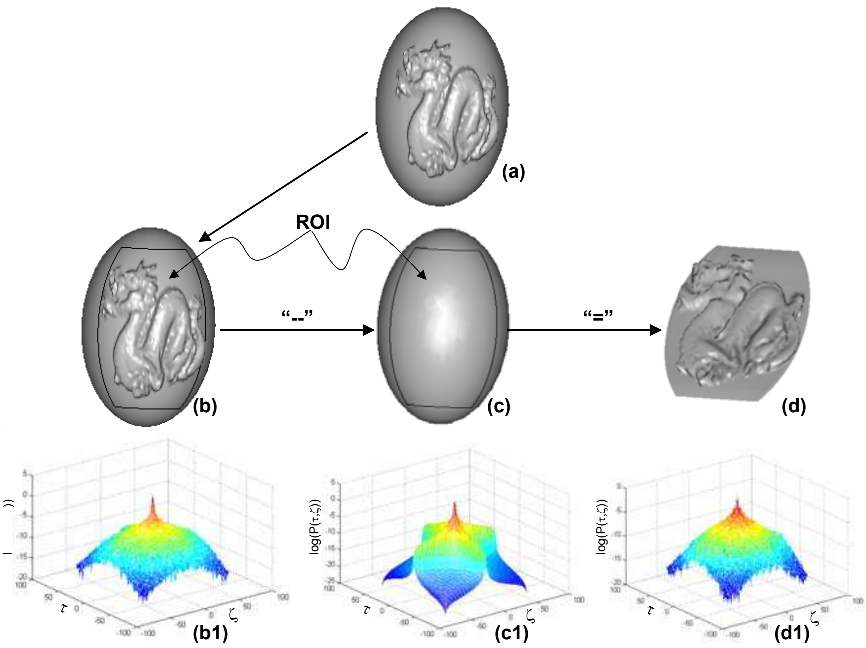

Figure 4. Feature retrieval. (a), The existing model; (b), The ROI; (c), The base surface; (d), The extracted feature shape; (b1), (c1)and (d1) are the corresponding Fourier power spectrum.

(2) If $h_1(\tau,\zeta)$ or $h_2(\tau,\zeta)$ varies on (τ,ζ), it will affect the coefficients of FT of S or G in such a way that it can be regarded as a filter [Ghosh93] [Malzbender93].

(3) If both $h_1(\tau,\zeta)$ and $h_2(\tau,\zeta)$ are functions of t, τ and ζ, where t is the time trajectory, a shape morphing process along t is then defined. For instance, let

$$h_1(t,\tau,\zeta) = 1 - h(t,\tau,\zeta) \text{ and } h_2(t,\tau,\zeta) = h(t,\tau,\zeta),$$

a complete shape morphing from S to G is defined. It is possible to control the introduction of frequency components during the morphing process by means of the control function to achieve diverse shape morphing effects [Hughes92].

Frequency-based operators for feature reuse could be derived from Equation (5) as follows:

Feature retrieval. To retrieve the "pure" feature shape, in Equation (5), let $h_1(\tau,\zeta)=1$ and $h_2(\tau,\zeta)=-1$, then:

$$C = \Im^{-1}\{f(\tau,\zeta) - g(\tau,\zeta)\} = \Im^{-1}\{S - G\} \quad (6)$$

where S is the Fourier model of the ROI, and G the domain surface (base signal). C is the "pure" feature shape decomposed, as shown in Figure 4. Since the domain surface is "filtered off", the "pure" feature shape is always situated on the zero-plane. For instance, if we work with the height field model, then the height of the points outside the feature boundary will be zero after "filter off" the base surface signal.

Alternatively, feature shape could be retrievaled by filtering applied directly on the FT of the ROI under the assumption in Section 3.

Feature smoothing or enhancing. In 2D DFT, the compound frequency at (τ,ζ) is $\omega_{\tau\zeta} = {}^{u\tau}/_{M} + {}^{v\zeta}/_{N}$. The higher the $\omega_{\tau\zeta}$ value, the more detail the shape element corresponds to [Hughes92]. The feature smoothing or enhancing effects can be achieved by scaling down or boosting up the high frequency coefficients [Pauly01] [Taubin95]. The spatial shape filtering operator can then be derived as

$$C = \Im^{-1}\{h(\tau,\zeta) f(\tau,\zeta)\} \quad (7)$$

where $h(\tau,\zeta)$ is the specified filter.

Shape filtering is an cheaper way to achieve special shape processing effects compared with performing a more expensive convolution in the spatial domain. In addition, algorithms in digital image processing can be directly applied in shape processing within the proposed framework.

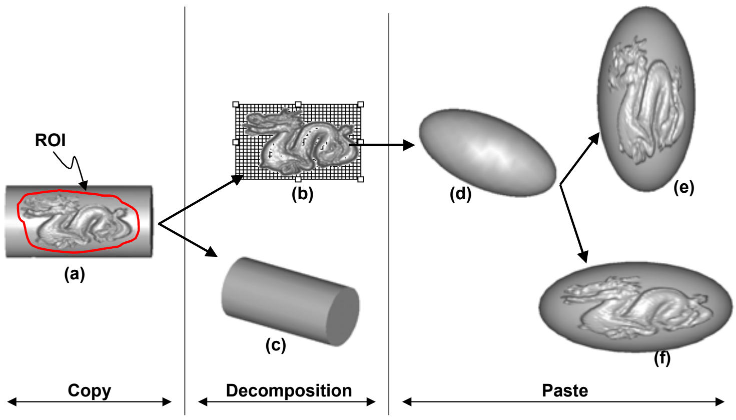

Figure 5. The feature copy-and-paste process

Shape scaling. By assigning control functions $h_1(\tau,\zeta) = \lambda > 0$ and $h_2(\tau,\zeta) = 0$, the shape-scaling operator can be represented as

$$C = \Im^{-1}\{\lambda f(\tau,\zeta)\} \quad (8)$$

where C is the scaled shape of S by $\lambda > 0$. Shape scaling can be achieved in spatial domain as well. However, Equation (8) provides a supplementary control over shape scale in shape composition process.

Feature shape reuse. In Equation (5), let $h_1(\tau,\zeta) = \lambda_1$ and $h_2(\tau,\zeta) = \lambda_2$, we then obtain

$$C = \Im^{-1}\{\lambda_1 f(\tau,\zeta) + \lambda_2 g(\tau,\zeta)\} = \Im^{-1}\{\lambda_1 S + \lambda_2 G\} \quad (9)$$

where λ_1 and λ_2 are the controls that adjust the behavior of S or G on the composed shape C. In case S represents a feature and G the domain shape (or a ROI), Equation (9) formulates the feature reuse

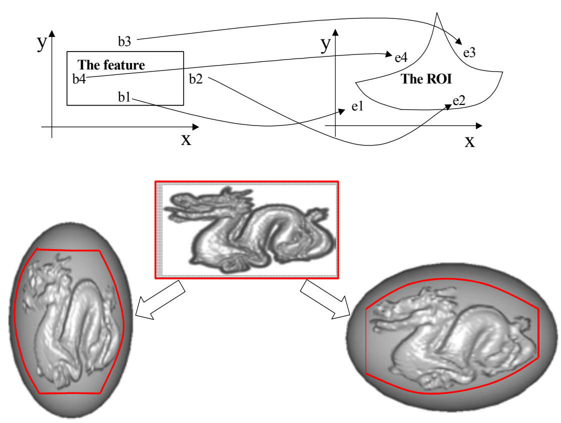

Figure 6. Parameterization of the resulting shape influenced by mapping areas.

process.

Obviously, feature copy-and-paste can be thought as a special case of feature reuse, in which only the height of the feature is mapped onto the normal distribution of the target surface. Issues regarding the behavior control of the pasted shape were discussed in [Wang02]. Figure 5 depicts the process of the feature copy-and-paste operation.

6. ISSUES CONCERNING FEATURE SHAPE REUSE

System Input and application scope: Our system takes resampled data set as input, which may be a sample grid from arbitory volume surfaces, for instance, the surface of a solid volume, a NURBS surface with or without trimming, or trangulated mesh from a mesh model. The modeling presion is depend on the sampling presion.

Controls. During feature reuse process, controls over the composed shape can be achieved by dedicated definition of both $h_1(\tau,\zeta)$ and/or $h_2(\tau,\zeta)$. Unlike conventional shape-editing tools which achieve shape control by modifying vertices, edges, or tangent vectors, the frequency-based operators operate on the FT of the shape, thus resulting the modification of the shape in the spatial domain, accordingly [Bracewell86] [Dudgon94].

Resampling considerations. In equation (5) we assume that both S and G have the same sampling grid. In general, the input shapes do not have the same sampling grid. For instance, the sampling grid for the feature is always denser than the target shape in order to reflect more shape details. In this case, a resampling on the sparser grid have to be done. Resampling the in frequency domain is similar to resampling in the spatial domain. Examples please refer to [Pauly01].

Parameterization. By applying Equation (5), the parameterization of the resulting shape is implicitly specified by the mapping areas, i.e., the feature and the target boundaries. A distortion may occur if the area of the feature and the ROI on the target surface are different. However, in most cases we only interested in controlling one aspect of the mapping, for instance the feature height, then the only effective boundary for the resulting shape is that of the target shape. As shown in Figure 6, in the reuse process, feature parameterization is determined by the mapping area.

Incorporating synthesized shape into existing modeling framework. When working with a point cloud or a mesh model, the new point-set generated from the reverse DFT can be merged into the existing model with minor efforts. When the ROI is to be rebuilt as a NURBS surface, continuity issue arises at the boundary of the ROI with adjacent patches. Many different approaches have addressed these issues, for instance, by over-sampling [Pauly01]. In our approach, we apply additional controls obtained by interrogating the adjacent boundaries, such as crossing boundary tangent vectors, or vectors derived from adjacent points. Three continuity conditions are considered:

(1) The G^0 continuity. The new shape can be reconstructed with free boundary constraints;

(2) The G^1 continuity. The new shape will be reconstructed with cross-boundary slope vectors obtained from adjacent patch boundaries in both u and v direction (when blended);

(3) The G^2 continuity. Strategies in [Pauly01] (by overlap sampling adjacent patches, then averaging the overlapped boundary points) can be adopted, or duplicating the knot vector at the patch boundary ($n = k-1$, where n is the repeating times and k the degree of the NURBS basis function). The former yields a smooth transition, and the latter may result boundary effects (e.g., sharp edges).

7. EXAMPLES AND DISCUSSIONS

Figure 7 demonstrates a result of feature reuse in a mesh model, in which the feature height is mapped onto the ROI along it surface normal field. The feature (the eagle head) is created using the height field algorithm and sampled using the given tolerance (a sample grid of 256×256 in this case). Obviously, The sampling grid for the ROI is far simple compared with that of the feature. Therefore a resampling is implemented in the frequency domain of the ROI. When blending the whole model, the boundary curve of the composed shape is re-parameterized and constrained by cross-boundary vectors at the node, so that it can be simply stitched with the adjacent boundaries under G^1 continuity.

Efficiency. For a $M \times N$ sampling grid of the ROI, the computational complexity is $O(M \times N \log(M \times N))$ [Dudgon94]. Figure 7 is implemented in an interactive environment with the PC hardware configuration CPU/800 MB, RAM/256 MB, and a graphic card with 32 MB cache.

Limitations. In spite of the advantages of frequency-based operators in supporting feature reuse, the proposed approach has its limitation as follows:

(1) Due to the fact that frequency-based operators are restricted within the ROI, it is hard to reflect the cross boundary constraints without complementary facilities;

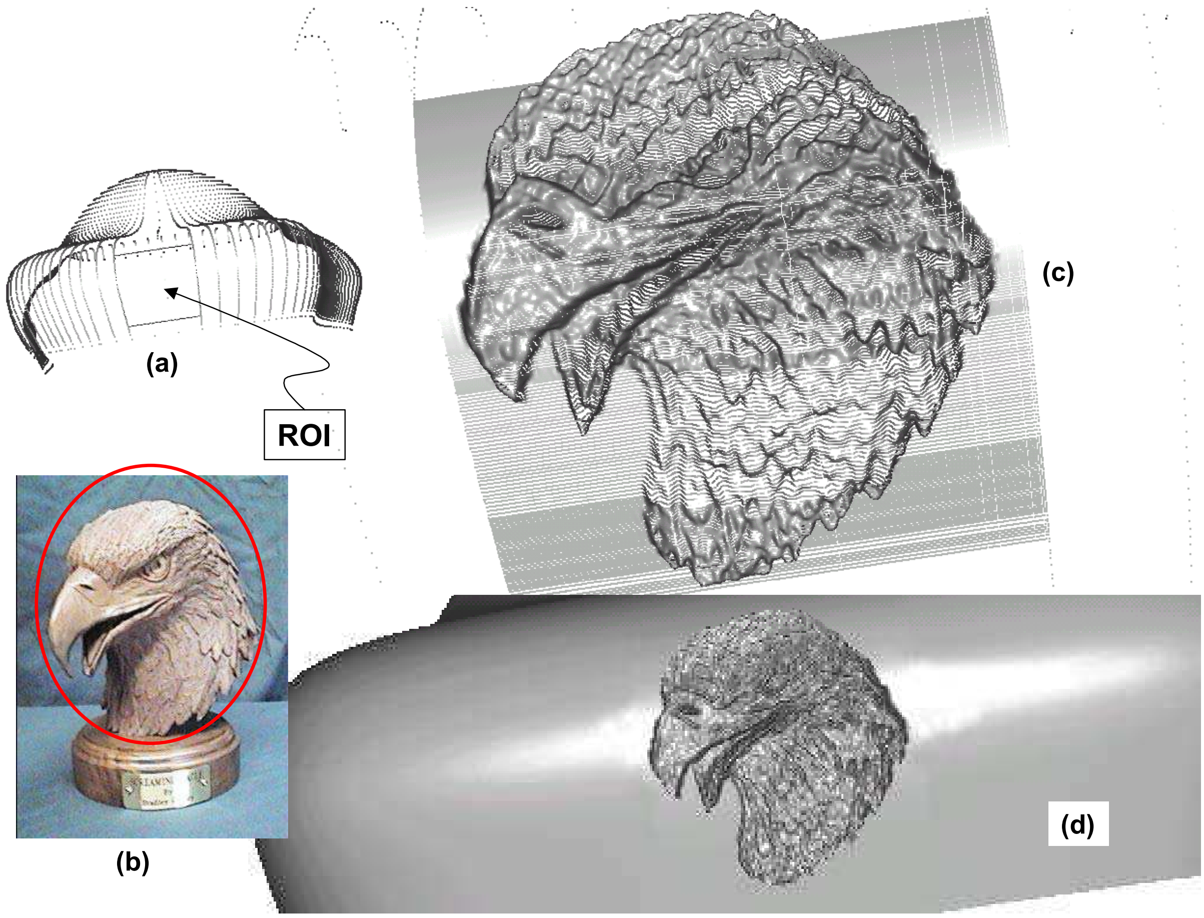

Figure 7. An example of feature reuse. (a), The original point cloud model with a ROI; (b), A picture used to create feature shape using the high-field algorithm; (c), Zoomed view of the ROI after shape mapping (point-cloud model and with low-pass filter applied); (d), The whole model (interpolated).

(2) A unique correspondence between the sampling point-sets and its FT are automatically specified. They cannot represent such cases in which either the sample data set overlapping over itself or non-unique correspondence between the feature points and the target points;

8. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a novel frequency-based approach for supporting feature reuse, and shown that using Fourier transforms in 3D shape modeling is a prospective way for intuitive shape editing. The proposed approach takes the simplest geometry representation scheme, and handling both the feature surface and the domain surface in a unified framework. Examples demonstrate the effectiveness of the approach in supporting the reuse of complex feature geometry, especially for the extremely complicated decorative shape feature on the surfaces of the artifact in industrial design, regardless of the underlying representations. The proposed approach is particularly useful in facilitating the fast conceptulization of product shapes.

Future work includes: to explore further the exact correspondence between shape geometry and its FT; algorithms to approximate higher degree FT from partial or lower degree FT of the shape; to develop adaptive sampling strategies to meets the needs of model simplification; to identify the shape invariance and similarity, and algorithms for precise feature decomposition with arbitrary models.

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