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Multibody simulations of trolleybus vertical dynamics and influences of spring-damper structural elements

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Abstract

Vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on an artificial test track when driving with a real vehicle and when simulating driving with a multibody model along a virtual test track. Driving along the artificial test track was aimed to determine vertical dynamic properties of the real trolleybus and on the basis of them to verify computer trolleybus models. Time histories and extreme values of the air springs relative deflections are the monitored quantities. Due to differences of the experiments and the computer simulations results the influences of the characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires used in the trolleybus multibody model on the extreme values of the monitored quantities are evaluated.

Keywords: vehicle dynamics, multibody model, spring-damper elements, sensitivity analysis, trolleybus

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1. Introduction

Optimum dynamic properties of the vehicle intended for the public transport can usually be achieved in dependence on its structural design by the proper choice of axles suspension elements (in some cases in combination with the proper choice of seats suspension elements). The design must be the compromise of the requirements for the vehicle behaviour during driving manoeuvres, for the riding comfort and for the body and the chassis parts lifetime when driving along an uneven road surface, and for the passenger safety (e.g. [26]).

Driving along the uneven road surface can reveal a lot about the vehicle vertical dynamic properties and about the suitability of the applied axles suspension elements. Especially time histories of relative deflections of springs, relative velocities in the shock absorbers, stress acting in the axles radius rods or radius arms and acceleration in various points in the vehicle interior are the monitored quantities [8]. On the basis of those quantities it is possible to determine the forces acting in the suspension elements of axles, which can be utilized for the stress analysis of structures, for the prediction of the fatigue life of the body and the chassis parts of the tested vehicle. The frequency domain responses of the acceleration in the vehicle interior can be used for the assessment of a riding comfort. In order to evaluate the vertical dynamic properties of the vehicle when driving along the uneven road surface it is necessary to know the surface characteristics, i.e. statistical properties of unevennesses of the surface or just its geometry (e.g. [26]). The geometry of the uneven surface profile of the run through the section is known in test polygons. Test tracks, which are created by distributing artificial vertical unevennesses (obstacles) on the smooth road surface, also are often used (e.g. [13]).

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Fig. 1. The ŠKODA 21 Tr low-floor trolleybus

Vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus (see fig. 1; its design concept is described in [22]) were investigated on the artificially created test track when driving the real vehicle and when simulating driving with the computer models along the virtual test track. Driving along the artificial test track was aimed to determine the vertical dynamic properties of the real trolleybus and on the basis of them to verify computer models. The verified computer models will be further utilized for the simulations of driving along the virtual uneven road surfaces, which will be generated on the basis of the statistical evaluation of the measured quantities in the course of driving along the real city road with the real trolleybus [11, 12].

This article continues the work presented in [10, 17, 21, 22, 23, 24, 25]. Those papers deal with the influences of characteristics of various spring-damper structural elements and of the multibody models complexity on the air springs relative deflections determined by the simulations with the selected multibody models of the empty trolleybus. The extreme values of time histories of the air springs relative deflections are compared. It follows from those papers that the results of the simulations and the experimental measurement are not completely identical, especially in the rebound stage of the rear axle suspension. Therefore the sensitivity analysis of the influence of various model parameters has to be performed.

Usually the sensitivity analysis is connected with the problems of the parameter selection for design optimization and with the problems of gradient calculations in gradient-based optimization procedures [6]. The sensitivity analysis is also a tool used in many applications in order to qualitatively analyse the behaviour of a chosen system. The general recursive approach to the calculation of sensitivities of multibody systems by means of direct differentiation is shown in [1]. The extension of this analytical approach for rigid-flexible systems is presented in [5]. However, in most practical cases of real multibody systems, the numerical approaches are the most suitable and efficient methods. Handling properties of road vehicles were investigated using the sensitivity analysis in [4]. The sensitivity analysis for the tyre wear evaluation was employed in [3]. Another application in rail vehicles and pantograph interaction can be found in [14].

The results of the sensitivity analyses of the multibody model of the ŠKODA 21 Tr low-floor trolleybus [20] created in the *alaska 2.3* simulation tool [18] at simulating driving along the virtual test track are given in [23] and [25]. The parameters of the sensitivity analysis presented in [23] are the loading characteristics of the decisive spring-damper structural elements of the axles suspension and the influence of changes of those characteristics on the extreme values of relative deflections of the air springs is monitored. The results of the sensitivity analysis of the influence of the different tire inflation are given in [25]. Sensitivity analyses in both cases were performed during the simulations of the trolleybus drive along the real test track at the trolleybus speed 44.13 km/h. The possibility of bounce of the tire from the road surface, which really occurs in the course of the vehicle relative speeding along the relatively demanding test track, is considered in the trolleybus multibody model. The influence of the change in the radial characteristics of the tires is not fully deterministic in the course of the simulation of drive along this test track and the results of the performed sensitivity analyses would be biased.

As it is possible to compare the influence of the loading characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics (force-deformation characteristics and damping coefficient) of the tires on the results of the simulations, the speed of 10 km/h, at which the tire bounce from the uneven road surface does not occur yet, is chosen at simulating the trolleybus drive along the test track. The contribution of this paper is mainly in the complex analysis of the influences of various characteristics on the trolleybus vertical dynamics investigated by means of the comprehensive multibody model.

2. Experimental measurements with the real trolleybus

The experimental measurements on the empty ŠKODA 21 Tr low-floor trolleybus were carried out in the depot of Hradec Králové Public City Transit Co. Inc. (Dopravní podnik města Hradce Králové, a. s.) in October 2004.

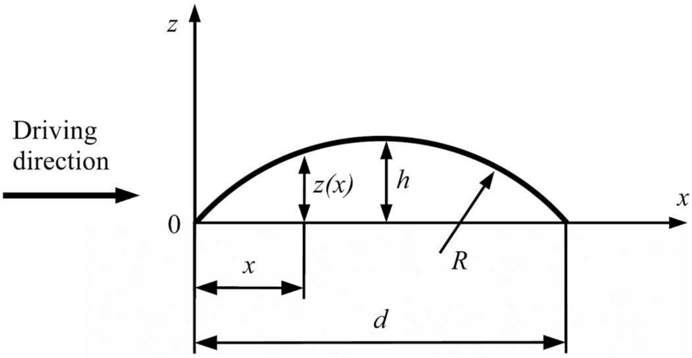

Fig. 2. The standardized artificial obstacle

The test track consisted of three standardized artificial obstacles (in compliance with the Czech Standard ČSN 30 0560 Obstacle II: $h = 60\,\mathrm{mm}$, $R = 551\,\mathrm{mm}$, $d = 500\,\mathrm{mm}$ — see fig. 2) spaced out on the smooth road surface 20 meters one after another. The first obstacle was run over only with right wheels, the second one with both and the third one only with left wheels (see fig. 3).

In the course of the test driving the already mentioned time histories of the relative displacements between the axles and the chassis frame were recorded (altogether four displacement transducers, which were placed in the lateral direction approximately on the level of the air springs: on the left front half-axle, on the right front half-axle, on the rear axle to the left and on the rear axle to the right, were used). Further time histories of stress on twelve places of

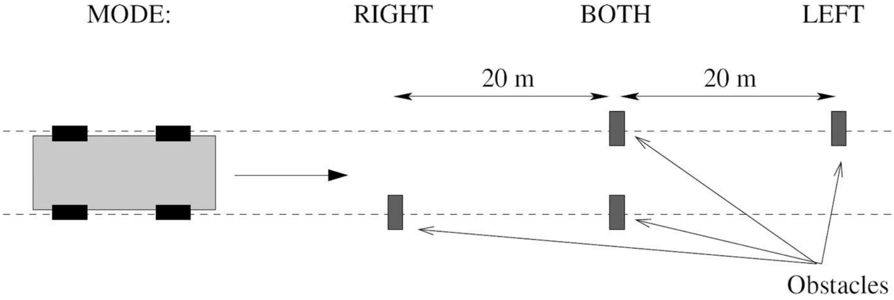

Fig. 3. A track scheme

the trolleybus structure and time histories of the vertical acceleration on seven places of the trolleybus structure were recorded during the test drives. The records of the time histories of the measured quantities were made during three test drives. Trolleybus speed moved within the range from 43 km/h to 47 km/h at that drives.

3. Trolleybus multibody model

In order to simulate drives along the virtual test track, which corresponded to the artificially created test track in the depot of Hradec Králové Public City Transit Co. Inc., the most complex multibody model [20] (see fig. 4) created in the *alaska 2.3* simulation tool [18] is used to investigate the influences of the loading characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires.

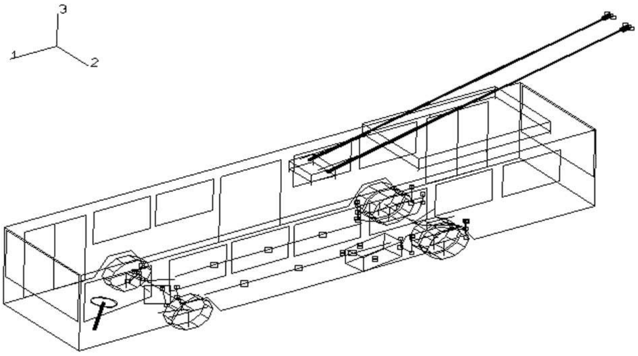

Fig. 4. Visualization of the multibody model of the ŠKODA 21 Tr low-floor trolleybus in the *alaska 2.3* simulation tool

3.1. Structure of multibody model

The multibody model of the ŠKODA 21 Tr low-floor trolleybus is formed by 35 rigid bodies and two superelements (2×4 bodies) mutually coupled by 52 kinematic joints. The rigid bodies correspond generally to the vehicles individual structural parts. The superelements correspond to the flexible parts of the chassis frame. The number of degrees of freedom in kinematic joints is 136. Rigid bodies are defined by inertia properties (mass, centre of mass co-ordinates and moments of inertia). Air springs and hydraulic shock absorbers in axles suspension and bushings in the places of mounting some trolleybus structural parts are modelled by connecting the corresponding bodies by nonlinear spring-damper elements [18]. When simulating driving along the uneven road surface the contact point model of tires is used in the multibody model; radial stiffness and radial damping properties of tires are modelled by nonlinear spring-damper elements considering the possibility of bounce of the tire from the road surface [16].

3.2. Characteristics of spring-damper structural elements

Dynamic properties of road vehicles are most influenced by the suspension springs, shock absorbers, bushings and tires (e.g. [2]). In order that vehicle virtual computer model should reliably approximate kinematic and dynamic properties of the real vehicle, knowledge of the characteristics of those decisive spring-damper structural elements is the important presumption (besides the proper approach to the model creating and knowledge of all the substantial vehicle parameters).

The characteristics of the air springs (force in dependence on deflection) of the ŠKODA 21 Tr trolleybus were determined on the basis of the test reports of ŠKODA OSTROV s. r. o. and of the Hydrodynamic Laboratory of the Technical University of Liberec [20].

In the multibody model of the ŠKODA 21 Tr trolleybus the damping force dependence on the relative velocity of compression and rebound of the shock absorbers is used as the shock absorbers characteristics. The characteristics were measured by BRANO a. s. (the shock absorbers producer) on the Schenck testing device [21].

In the shock absorbers structure rubber bushings are used in the places of mounting to the chassis frame and to the axles of the trolleybus [21]. In the multibody model the bushings are modelled by means of spring elements, the nonlinear force-deformation characteristics of which were determined under the laboratory conditions (they are taken over from [15]) and which are coupled in series to the damping elements representing the hydraulic shock absorbers themselves.

The review of the tire models used in the field of vehicle multibody dynamics can be found in the monograph [19]. The most important tire characteristics needed for solving the vehicle vertical dynamics tasks are their radial properties [2]. The used tire model for the vertical dynamics is the already mentioned contact point model based on the tire substitution by a single parallel spring and a damper. Radial stiffness and radial damping properties of the tires were experimentally measured in the Dynamic Testing Laboratory ŠKODA VÝZKUM s. r. o. The evaluation of the measured quantities for the purpose of generation of multibody models is given in [9].

4. Results of the simulations

As it has been already stated the results of the simulations at speed 10 km/h (at which the tire bounce from the uneven road surface does not occur yet) during simulating the trolleybus drive along the test track are given in this article.

When simulating movement with the multibody models, nonlinear equations of motion, which are solved by means of numerical time integration, are generated. Results of the simulations were obtained using the Shampine-Gordon integration algorithm [18]. Fig. 5 shows the time histories of the air springs relative deflections at simulating the test drive with the trolleybus multibody model (with the consideration of the MICHELIN tire radial characteristics model at 100 % inflation — [25]). The extreme values of the air springs relative deflections read from the time histories are in tab. 1.

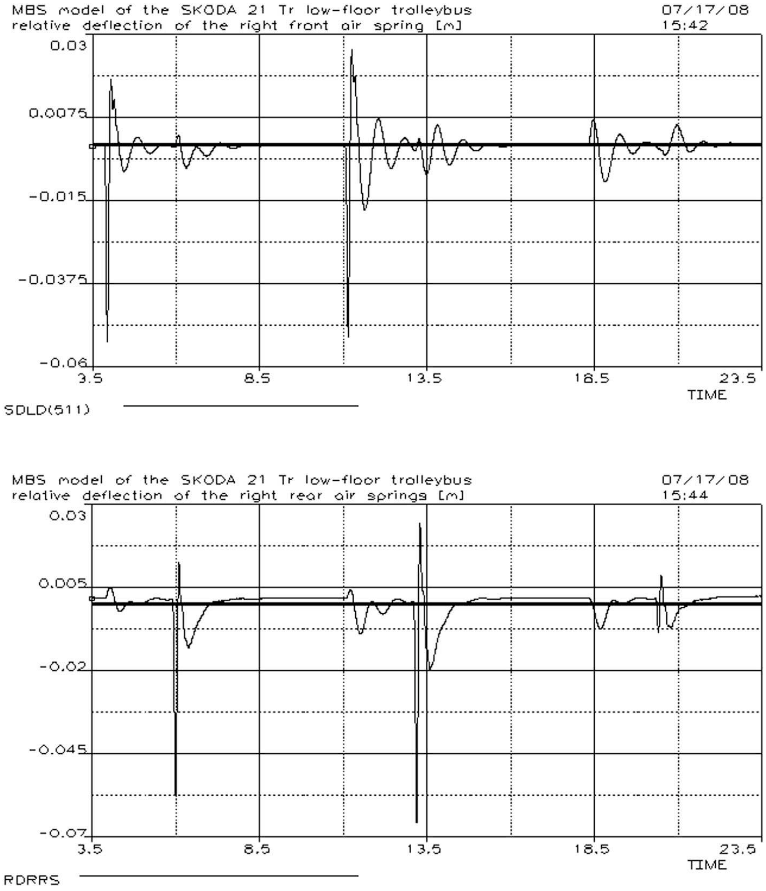

Fig. 5. Time histories of the front right and the rear right air springs relative deflection when simulating the test drive with the trolleybus multibody model at speed 10 km/h

Table 1. Extreme values of the relative deflections of air springs

Obstacle	Value	Extreme values of relative deflection of air springs [mm]			
		Right front	Left front	Right rear	Left rear
1st	min.	-53	-9	-57	-9
	max.	18	6	12	8
2nd	min.	-52	-51	-66	-68
	max.	26	25	24	23
3rd	min.	-10	-53	-9	-58
	max.	7	17	8	12

5. Sensitivity analysis of the multibody model

The sensitivity analysis of the influence of the change of the characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires in the trolleybus multibody model is performed.

The sensitivity analysis of the influences of the selected parameters characterizing the system behaviour for the change in various system parameters is applied especially in the field of optimization, identification and correction of the mathematical models of the investigated systems. By means of that it is possible to determine which parameters influence the change of the selected quantities most significantly and subsequently to select the parameters as the optimizing ones and to try to define them more precisely or to correct them.

As it was already mentioned the results of the computer simulations and the experimental measurements with the ŠKODA 21 Tr low-floor trolleybus, compared on the basis of the evaluation of the accordance of extreme values of the time histories of the air springs relative deflections with the measured extreme values of the relative displacements during the run along the test track, are not identical especially in the course of the rebound stage of the rear axle suspension (e.g. [23]). It is obvious that this fact is influenced by the course of the characteristics of the spring-damper structural elements. That is why the sensitivity analysis of the influences of the force-velocity characteristics of the hydraulic shock absorbers, the force-deflection characteristics of the air springs and the force-deformation characteristics of the shock absorbers bushings on the extreme values of the time histories of the air springs relative deflections was performed in [23] and the results of the sensitivity analysis of the influence of the different tire inflation were given in [25]. As it has been already stated it is possible to compare the influence of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires on the results of the simulations, the speed of 10 km/h, at which the tire bounce from the uneven road surface does not occur yet, is chosen at simulating the trolleybus drive along the test track.

The influence of the changes in the parameters of the characteristics of the spring-damper structural elements and the radial characteristics of the tires on the extreme values of the relative deflections of all the air springs when running over each obstacle of the artificial test track is monitored.

5.1. The sensitivity analysis of the dynamic response of the trolleybus multibody model

Like in most cases of the complicated multibody systems it is not possible to derive analytical relations to express the dynamic response of the given multibody model to the general excitation. Neither it is possible to derive analytical formulas to calculate the sensitivity of the dynamic response to the change in the system parameters. In order to express the partial derivative of the certain monitored quantity $y = y(\boldsymbol{p})$ regarding the vector of the S selected parameters of the system $\boldsymbol{p} = [p_1, p_2, \ldots, p_S]^T$ it is necessary to use relations for the numerical calculations of sensitivity, so called difference formulas [7].

Change Δy of the monitored quantity y can be expressed with a small change $\Delta\boldsymbol{p}$ of the initial parameters vector $\boldsymbol{p}_0$, when the specific conditions of the continuity of derivations of the monitored quantity y are fulfilled, using the Taylor formula (approx. by two terms)

$$\Delta y = y\left(\boldsymbol{p}_0 + \Delta\boldsymbol{p}\right) - y(\boldsymbol{p}_0) = \sum_{j=1}^{S} \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \Delta p_j. \tag{1}$$

After the modification of relation (1) it is obtained

$$\frac{\Delta y}{y(\boldsymbol{p}_0)} = \sum_{j=1}^{S} \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y(p_0)} \cdot \frac{\Delta p_j}{p_{j0}}. \tag{2}$$

From relation (2) it is possible to get relative sensitivity $\Delta\bar{y}_j$ of quantity y to the change in parameter p_j

$$\Delta\bar{y}_j = \frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} \cdot \frac{p_{j0}}{y\,(p_0)}. \tag{3}$$

Partial derivative in relation (3) is approximated using the finite difference

$$\frac{\partial y(\boldsymbol{p}_0)}{\partial p_j} = \frac{y(\boldsymbol{p}_0 + \Delta\boldsymbol{p}_j) - y(\boldsymbol{p}_0)}{\Delta p_j}, \tag{4}$$

where vector $\Delta\boldsymbol{p}_j = [0, \ldots, 0, \Delta p_j, 0, \ldots, 0]^T$.

Then differential relation for the calculation of relative sensitivity $\Delta\bar{y}_j$ of quantity y to the change in parameter p_j, using relations (3) and (4), can be written in the final form

$$\Delta\bar{y}_j = \frac{y(\boldsymbol{p}_0 + \Delta\boldsymbol{p}_j) - y(\boldsymbol{p}_0)}{\Delta p_j} \cdot \frac{p_{j0}}{y(\boldsymbol{p}_0)}. \tag{5}$$

Thus in case of the sensitivity analysis of the ŠKODA 21 Tr trolleybus multibody model when driving along the artificial test track the relative deflections of the air springs of axles are successively the monitored quantities y and the relative changes in the characteristics of the decisive spring-damper structural elements are the vectors of parameters $\boldsymbol{p}$.

5.2. Sensitivity analysis results

The loading characteristics of the decisive spring-damper structural elements of the axles suspension and the tire radial characteristics were the parameters of the sensitivity analysis, during which the influence of parameter changes of those characteristics on the extreme values of relative deflections of the air springs was monitored. The results of the sensitivity analysis in the course of the simulations with the multibody model of the ŠKODA 21 Tr low-floor trolleybus show that the influence of the force-velocity characteristics of the hydraulic shock absorbers and the force-deflection characteristics of the air springs have greater influence on the results of driving along the virtual test track than the force-deformation characteristics of the tires. In contradiction to [23] the force-deflection characteristics of the front air springs have greater influence on the extreme values of the time histories of the air springs relative deflections than the force-velocity characteristics of the front hydraulic shock absorbers. It is the other way round (the same as in [23]) with the characteristics of the rear air springs and the rear shock absorbers. In comparison with [25] the tire radial force-deformation characteristics have not a significantly greater influence on the results of driving along the virtual test track than the tire radial damping characteristics and the rear tire radial force-deformation characteristics considerably influence even the extreme values of the time histories of the front air springs relative deflections. Those differences in results are caused by the fact that at speed 10 km/h the tire-road surface contact is kept during the run over the obstacles of the test track. The force-deformation characteristics of the shock absorbers bushings have, as it was found also in [23], a minor influence on the results of the simulations.

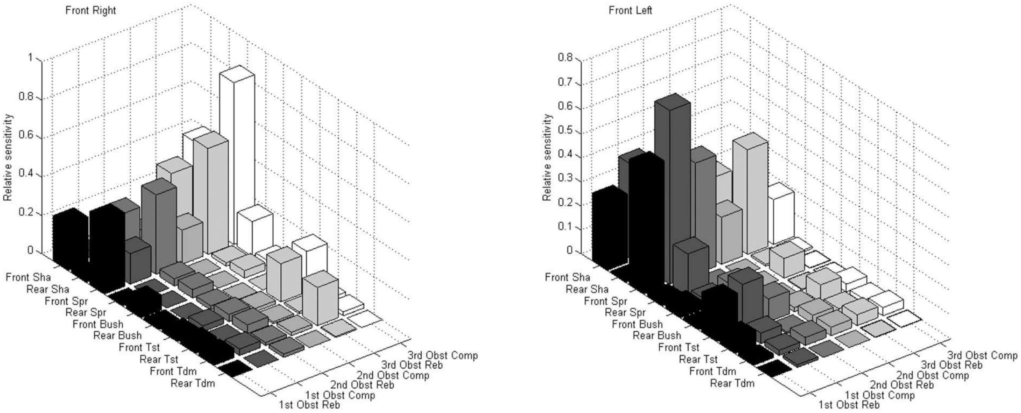

Fig. 6. Relative sensitivity of relative deflection of the front air springs (right and left) on the change in the individual parameters

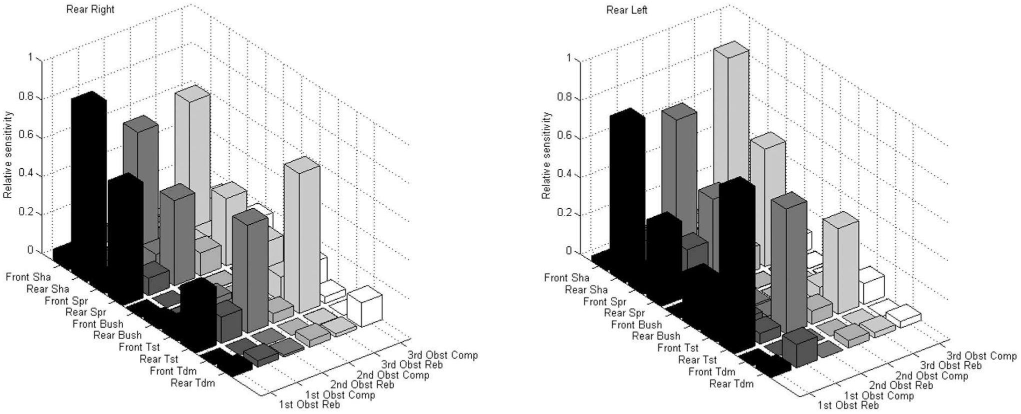

Fig. 7. Relative sensitivity of relative deflection of the rear air springs (right and left) on the change in the individual parameters

The relative sensitivities of relative deflections of the air springs on the change in the individual parameters in the course of the simulations of driving along the virtual test track with the most complex multibody model (e.g. [20]) in the *alaska 2.3* simulation tool is given in fig. 6 and fig. 7 (Sha = influence of the force-velocity characteristics of the shock absorber; Spr = influence of the force-deflection characteristics of the air spring; Bush = influence of the force-deformation characteristics of the shock absorber bushing; Tst = influence of the tire radial force-deformation characteristics; Tdm = influence of the tire radial damping coefficients; 1st to 3rd Obst = obstacle sequence; Comp = compression of air springs; Reb = rebound of air springs).

6. Conclusion

The vertical dynamic properties of the ŠKODA 21 Tr low-floor trolleybus were investigated on the artificially created test track when simulating driving along the virtual test track with the most complex multibody model [20] created in the *alaska 2.3* simulation tool [18].

The sensitivity analysis of the influence of the change of the characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires in the trolleybus multibody model was performed. The results of the sensitivity analysis in the course of the simulations with the multibody model of the ŠKODA 21 Tr low-floor trolleybus showed that the influence of the force-velocity characteristics of the hydraulic shock absorbers and the force-deflection characteristics of the air springs have greater influence on the results of driving along the virtual test track than the force-deformation characteristics of the tires. Results of the simulations at the trolleybus multibody model speed 10 km/h, during which the loss of the tire-road surface contact does not occur at driving over the obstacles of the test track (in contradiction to [23] and [25] at speed 44.13 km/h) partly differ from the results mentioned in [23] and [25]; especially in case of the front axle suspension it was not confirmed, that the results of driving along the virtual test track influence the force-velocity characteristics of the hydraulic shock absorbers more considerably than the force-deflection characteristics of the air springs.

In the nearest future the influence of the same characteristics of the decisive structural spring-damper elements during driving along the test track at the multibody model speed 44.13 km/h, i.e. the same as in [23] and [25], will be investigated. To be able to compare the influence of the characteristics of the spring-damper structural elements of the axles suspension and the radial characteristics of the tires, i.e. in order that the loss of the tire-road surface contact may not occur, the height of the artificial obstacles will be virtually decreased.

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