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Electro-osmosis in a cortical bone porous structure: Parametric study

Jana Turjanicová[1], Eduard Rohan[2]

1 Introduction

The electro-osmosis is responsible for important physiological processes in the cortical bone tissue. Cortical bone is seen as a highly hierarchical structure with multiple porosities on different scale levels of the osteon; usually the three main levels are distinguished (Moyne and Murad (2002)). These are (from the largest to smallest) the vascular porosity level, lacuno-canalicular porosity and porosity associated with collagen-apatite.

The present work is focused on electro-osmosis phenomena at the lacuno-canalicular porosity level (characteristic scale $l \approx 10nm$), further referred to as the microscopic level. It can be modeled as a porous medium with fluid filled pores in the solid matrix. The fluid is a solution with two types of monovalent ions of opposite polarizations (cations Na+ and anions Cl-). Further, we consider the solid phase and the solid-fluid interface, both featured by negative electric charges.

2 Model of electro-osmosis

We consider a porous medium occupying domain Ω, which is decomposed into solid matrix Ω_s and fluid filled channels Ω_f, with boundaries $\partial\Omega_s$ and $\partial\Omega_f$.

The material characteristics of the solid and fluid part play important role in the modeling. The material at the solid part Ω_s is considered to have a small negative charge π_s. The fluid part is modeled as incompressible electrolyte solution with dissolved positive and negative particles with Q^+ and Q^- concentrations and z^+ and z^- valences. Diffusivity of $+/-$ particles is represented by diffusion coefficients D^+ and D^-. Both, solid and fluid parts, can be characterized by permitivity $\varepsilon_s, \varepsilon_f$, respectively.

2.1 Mathematical model

A mathematical model of electro osmosis is introduced by the following set of 4 equations for potential Φ and concentrations Q^+, Q^-, as follows

Electrostatics

$$-\nabla \cdot \epsilon_s \nabla\Phi = \pi_s \quad \text{in } \Omega_s, \tag{1}$$

$$-\nabla \cdot \epsilon_f \nabla\Phi = \mathrm{F}(z^+Q^+ - z^-Q^-) \quad \text{in } \Omega_f, \tag{2}$$

Convection-diffusion of ionized fluid

$$\partial_t Q^\pm + \boldsymbol{w}_f \cdot \nabla Q^\pm - \nabla \cdot \mathbb{D}^\pm \cdot \left(\nabla Q^\pm \pm \frac{z^\pm \mathrm{F}}{\mathrm{R}T} Q^\pm \nabla\Phi\right) = 0 \quad \text{in } \Omega_f, \tag{3}$$

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[1] Ing., Dept. of Mechanics, Faculty of Applied Sciences, University of West Bohemia, mail: turjani@kme.zcu.cz

[2] Prof. Dr. Ing. DSc., Dept. of Mechanics, New Technologies for Information Society – the Center of Excellence,Univerzitní 8, Plzeň, mail: rohan@kme.zcu.cz

with Faraday F and ideal gas R constants. Symbol $\boldsymbol{w}_f$ stands for convection velocity and T for absolute temperature of the fluid part. This system of equations is completed by proper boundary conditions; the electrokinetics equation are completed by condition of insulation on boundary and jump condition on the solid-fluid intersection, equations of convection-diffusion then by no-mass-transfer condition and electro neutrality condition, see Rohan et al. (2010).

While assuming material with Y-periodicity, we follow unfolding homogenization approach, Rohan et al. (2010), to upscale problem (1) - (3) from micro- to macroscopic scale in order to obtain effective coefficients appearing in the macroscopic problem.

3 Numerical results

On the upscaled model of electro-osmosis we can perform a parametric study of microstructure influence on effective coefficients and macroscopic solutions. We represent microstructure by simple 3D representative periodic cell Y with three mutually connected channels in direction of main axes. By changing one of channel diameter while preserving the other two, we are able to change the volume ration of fluid part in the solid matrix, known as porosity ϕ. The influence of this study on the distribution of solid and fluid potential and $+/-$ ions concentration can be seen on the Fig. 1.

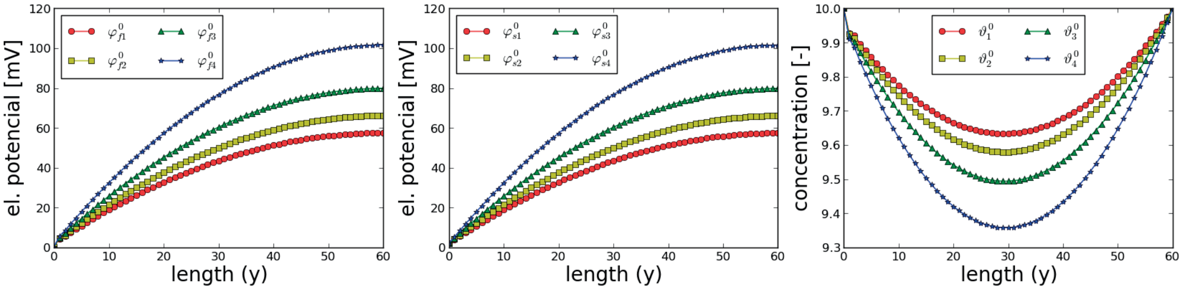

Figure 1: Parametric study of distribution of macroscopic solution alongside y-length of macroscopic body; Left - fluid potential, middle - solid potential, right - $+/-$ ionic concentration

4 Conclusion

The model of the electro-diffusion in cortical bone porous structure was represented. Using unfolding homogenization method, we obtained the effective coefficients relevant to the behavior of macroscopic body. Homogenization and macroscopic problem were implemented in in-house developed software *SfePy*. Finally, the parametric study of porosity change influence on the effective coefficients and macroscopic problem solutions was performed.

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