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DESIGN OF TWO-MODE FILTERS

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Abstract: The design of lowpass and highpass filters operating in two modes is described. The main idea of the design procedure is based on the reciprocity theorem.

Keywords: reciprocal two-ports, frequency filters, current conveyors, transconductors

1 Introduction

Let us consider a two-port network the circuit diagram of which is shown in Fig. 1.

Fig. 1. General two-port network

Suppose that this network is characterized by a regular admittance matrix. We say that such a circuit is *reciprocal* when the following condition is fulfilled [1], [2], [3]:

$$\Delta_{a:c} = \Delta_{c:a} \ . \tag{1}$$

Here the symbol $\Delta_{a:c}$ denotes the cofactor of the admittance matrix considered [4].

Condition (1) satifies every network that contains only two-port elements, because its admittance matrix is symmetrical with respect to the main diagonal. It is known that a reciprocal two-port network possesses the following feature:

The open-circuited voltage transfer function from the input terminal pair to the output terminal pair equals the short-circuited current transfer function from the output port to the input one.

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Therefore any two-port network containing only one-port elements can operate in two modes. It is necessary to say that the one-ports mentioned above can contain arbitrary active (non-reciprocal) elements (devices).

To simplify the design of current processing circuits, the *adjoint network theorem*, originally defined to facilitate expressing the network sensitivities [5], can be used. The original network and its adjoint counterpart are said to be *interreciprocal* to each other. Two mutually adjoint (interreciprocal) two-ports behave as one reciprocal two-port network.

2 Frequency filters operating in two modes

The simplest reciprocal two-port network is a voltage/current divider drawn in Fig. 2 [6]. Its transfer functions are described as follows:

$$H(s)=\frac{V_{out}}{V_{in}}=\frac{I_{out}}{I_{in}}=\frac{Y_1(s)}{Y_1(s)+Y_2(s)}=\frac{Z_2(s)}{Z_1(s)+Z_2(s)}\ , \tag{2}$$

where $Y_1(s)$ and $Y_2(s)$ are admittance functions, whereas $Z_1(s)$ and $Z_2(s)$ are impedance functions of the one-port elements in Fig. 2a,b.

It is evident from Fig. 2 that the one-port element $Y_1(s)$ is floating, whereas $Y_2(s)$ can be grounded. In the following, we will consider that the floating element is either a resistor or a capacitor. The grounded one-port element can contain arbitrary active elements, thus assuring the realization of a suitable immittance function.

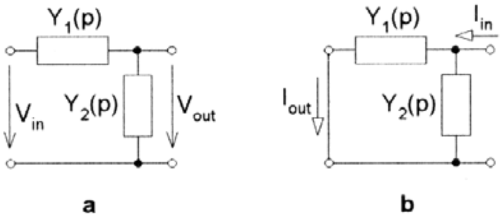

Fig. 2. Reciprocal two-port network

2.1 N-th order lowpass filter

If we consider the floating resistor in Fig. 2, i.e. if

$$Y_1(s)=G_1=\frac{1}{R_1}\ , \tag{3}$$

then we can write the transfer function of an nth-order lowpass filter in the following form acc. to eqn (2):

$$H(s)=\frac{\alpha_0}{\beta_0+\beta_1 s+\beta_2 s^2+\ldots+\beta_n s^n}=\frac{G_1}{G_1+b_1 s+b_2 s^2+\ldots+b_n s^n}\quad . \tag{4}$$

To fulfil eqns (2) and (4), we must design a one-port element having the following admittance function:

$$Y_2(s) = \sum_{i=1}^{n} b_i s^i \tag{5}$$

As an example, we present a third-order lowpass filter shown in Fig. 3. The admittance function of the grounded one-port element is as follows:

$$Y_2(p) = \frac{p^3 C_1 C_2 C_3 + p^2 C_1 C_2 (G_2 + G_3) + p G_2 G_3 (C_1 + C_2)}{G_2 G_3} \ . \tag{6}$$

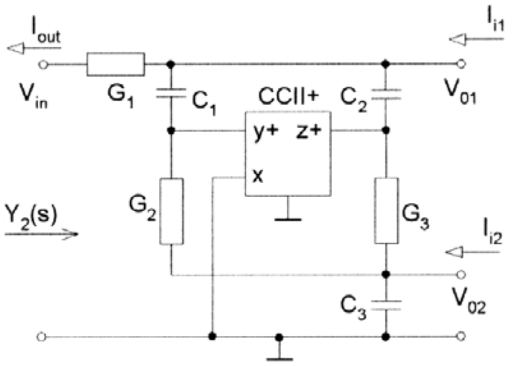

Fig. 3. A three-port network operating as a lowpass and a bandpass filter in two modes

The network in Fig. 3 can operate as a lowpass filter in both modes, with the following transfer function:

$$\frac{V_{o1}}{V_{in}} = \frac{I_{out}}{I_{i1}} = \frac{G_1 G_2 G_3}{D(s)} \ , \tag{7}$$

where

$$D(s) = p^3 C_1 C_2 C_3 + p^2 C_1 C_2 (G_2 + G_3) + p G_2 G_3 (C_1 + C_2) + G_1 G_2 G_3 \ . \tag{8}$$

Simultaneously, this network can operate as a band pass filter in two modes, but with two different transfer functions:

$$\frac{V_{o2}}{V_{in}} = \frac{-pC_1 G_1 G_3}{D(s)} \quad (9) \qquad \text{and} \qquad \frac{I_{out}}{I_{i2}} = \frac{-pC_2 G_1 G_2}{D(s)} \quad (10)$$

2.2 *N*th-order highpass filter

Let us suppose that the floating element is a capacitor. This means that

$$Y_1(s) = pC_1 \ . \tag{11}$$

The transfer function of an *n*th-order highpass filter can be arranged as follows:

$$H(s) = \frac{\alpha_m p^n}{\beta_0 + \beta_1 p + \beta_2 p^2 + \ldots + \beta_n p^n} = \frac{pC_1}{pC_1 + \dfrac{b_0 + b_1 p + b_2 p^2 + \ldots + b_{n-1} p^{n-1}}{p^{n-1}}} . \tag{12}$$

Comparing eqn (12) with eqn (2) we find that a one-port network with the following admittance function must be designed as follows:

$$Y_2(p) = \frac{\sum_{i=0}^{n-1} b_i p^i}{p^{n-1}} . \tag{13}$$

For the realisation of admittance function (13) we can use either operational transadmittance amplifiers (OTAs) or universal current conveyors (UCCs) [6].

3 Final remarks

In the described way we can also design second-order bandpass or notch filters. Filters containing current or voltage conveyors arec easy to transform into the oposite type by mutually interchanging capacitors and resistors.

References

[1] MAYER, D.: Úvod do teorie elektrických obvodů. SNTL-ALFA, Praha, 1978.

[2] VALSA, J., SEDLÁČEK, J.: Teoretická elektrotechnika. VUTIUM Brno, 1997.

[3] TOUMAZOU, C., LIDGEY, F. J., HAIGH, D. G.: Analog IC design: the current-mode approach. Peter Peregrinus, London, 1990.

[4] ČAJKA, J., KVASIL, J.: Úvod do teorie lineárních obvodů. SNTL – ALFA, Praha 1979.

[5] DIRECTOR, S. W., ROHRER, R. A. : The generalized adjoint network and network sensitivities. IEEE Trans. Circuit Theory, CT-16, 1969, pp. 318-323.

[6] ČAJKA, J., VRBA, K., OLŠÁK, M: Dvojbrany, které pracují v napěťovém i proudovém režimu (Two-ports operating in the voltage and also in the current mode). Slaboproudý obzor, 2002, Vol. 59, No. 2-3, pp. 7-11.

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