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ALTERNATIVE METHODS OF FIELDS SOLUTIONS

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Abstract: *The paper describes principles of two non-standard numerical iterative methods of investigating the spatial arrangement of stationary or quasi-stationary fields, inspired by the anticipated real behaviour of the physical reality. This wave method simulates a field in which quantities propagate in accordance with generalized Huygens' Principle in a discrete mesh. The basis of the method of the uniform energy distribution follows directly from the method's name. The submitted illustrative examples show the possible applications of both the methods.*

Key words: *Methods of fields solutions, wave method, stationary field, Huygens' Principle, uniform distribution of energy*

INTRODUCTION

Although nowadays there are available a lot of more or less efficient methods of solutions to electromagnetic and other physical fields, it is evident that not all possibilities have been exploited, or rather, processed. With respect to concrete practical needs the authors have developed a wave method particularly suitable for solutions to fields in a discrete medium. At this stage, the other submitted method has rather a character of a study. The approach is based on geometrical algorithms which lead to finding the uniform energy distribution to all the field elements.

1 THE WAVE METHOD

For the wave method of the field analysis the generalized Huygens' Principle is applied [1], [2]. This principle states that each element, at which the wave has already arrived, further distributes the wave symmetrically to all directions. The total wave front is then given by the interference of the vector flow to all the elements.

Huygens' Principle has been formulated for an acoustic field and thus the homogeneous isotropic medium. The above-mentioned model of energy wave propagation appears to be also applicable to other types of physical fields, including the electrical and magnetic ones. However, for these purposes it was necessary to generalize Huygens' Principle for the environment with the variable conductivity with respect to the considered quantity. At the same time a discrete model of the spatial area was assumed, consisting of orthogonal equidistant mesh of branches and nodes. The distribution of the quantity, which had arrived at the given node from all the directions at the n-th time step, should then be directly proportional to the standardized conductivity of the individual branches which correspond to the node. The standard is the sum of the conductivities of all the branches corresponding to the node.

The solution converges to the equilibrium state in the entire area by means of the transient phenomenon when the quantity spreads in waves from sources (of a flow character in arbitrary nodes). The method on principle ensures the constant validity of Kirhoff´s laws, possibly of their analogies (of zero nodal and loop balances) in the entire mesh. The calculation of the main block terminates at the moment that all the relative changes of all flows in all branches drop below the set limit.

The algorithm of the method is shown in the following description of the 2D version of the program: There is given the following: the D_x and D_y flows matrices, the matrix of the residual currents R, the matrix of the nodal current sources S, the matrices of the C_x and C_y conductivities and the matrix of the φ potentials. The residual currents r_{ij} represent the differential flow that is in the given moment inconsistent with the First Kirchoff´s Law validity for the considered node. Consequently, the zero balance for the node is restored by the distribution of the residual flow to corresponding branches proportionally to the conductivity of these branches.

After setting the initial field values, i. e. zero values of all currents and potentials, and after entering and generating the conductivity matrix of the field, we look for such a path between the electrodes whose total conductivity should be near the maximum possible one. After its finding the path is recorded in the memory in a form of the matrix of the turning nodes coordinates. This path is used to check the total voltage between the electrodes during the calculation.

The solution to the field goes ahead gradually in the particular time levels, in which the flows propagation is simulated analogically to Huygens´ Principle. At the first stage, the standardized values of the residual currents r_{ij} are always calculated in the entire field.

$$r_{i,j} = \frac{\sum d_{i,j} + s_{i,j}}{\sum c_{i,j}} \tag{1}$$

where s_{ij} is the element of the S matrix and represents the flow source (possibly the charge density) of the corresponding element.

At the other stage there follows the calculation of the flows d_{ij} in both directions including aliquot parts of both nodes´ residues, with which the branch corresponds, with respect to the branch orientation.

$$\begin{aligned} dx_{i,j} &= dx_{i,j} + (r_{i,j} - r_{i,j+1}) \cdot cx_{i,j} \\ dy_{i,j} &= dy_{i,j} + (r_{i,j} - r_{i+1,j}) \cdot cy_{i,j} \end{aligned} \tag{2}$$

Fig. 1 shows the result of the illustrative example of the method. It displays the 2D current field on the square discrete area fed with the current at the central node. (The potential 10 V is assigned to this node). The other electrode is represented by the outer boundary of the area functioning as an equipotent line with an unknown potential at the beginning of the calculation.

It is worth noticing that 3D tasks naturally converge faster than comparable 2D problems whose partial waves are absorbed more slowly. Non-linear behaviour of the medium can also be admitted.

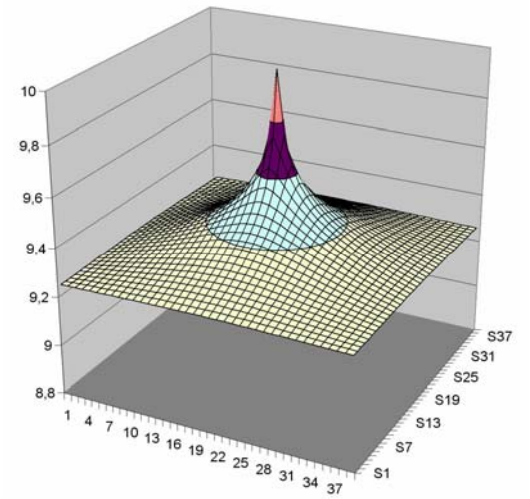

Fig. 1: Illustrative example, current field potential

2 THE METHOD OF THE UNIFORM DISTRIBUTION OF ENERGY

As with all other methods, the analysis lies in seeking the orthogonal mesh of equipotent lines and the lines limiting the constant flow of the corresponding quantity. The lines border the elements whose shape (at sufficient density) approximates a rectangle or a block. In the non-dissipative non-active environment each of the elements possesses the same amount of field energy and at the same time the same conductivity.

The core our method is a special algorithm of gradual corrections of the nodal points location of the selected mesh so that there could be balanced the conductivity of each element with its neighbours when maximizing the mutual orthogonality of both types of lines. The evaluation of each element is based on comparing the lengths of its diagonals.

The coordinates x, y, z of each node are shifted by a k-multiple of the v vector. This vector is determined in a way that the differences in lengths of body diagonals of corresponding incident elements are minimized.

The corrective algorithm operates with the corrective factor k whose value must not draw near to 1 or exceed this value. In such a case the method becomes instable; diverging oscillations of the solution appear. In order to keep the optimal convergence speed, it is desirable to remain below the aperiodicity level

Fig. 2 shows the convergence of the solution to a simple but – from the method point of view - rather unfavourable case of a field between two shifted parallel electrodes (as all the elements of the initial mesh have had the same conductivity).

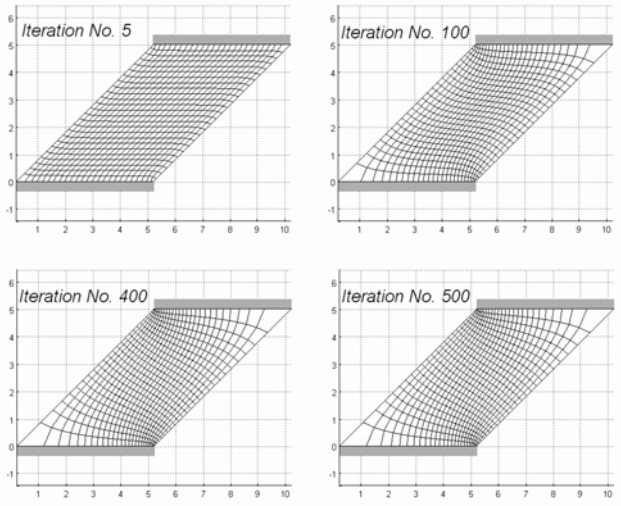

Fig. 2: Procedure of the solution to the field between parallel electrodes

Even the first tests have proven a high sensitivity of the method to the symmetry of the corrective algorithm procedure. It is not possible to permit the preference of any of the directions (or orientations) so that this correction regularly preceded the corrections according to the other coordinates, for instance. The project focus and at the same time the critical stage of the algorithm design was the assurance of a perfect omnidirectional symmetry

of the influence on each of the mesh cell, especially on the boundary and corner elements. Fig. 2 indicates that the problem has been managed well because either the final solution or the continuous ones do not show any asymmetric behaviour.

In order to compare the results, we submit the result of the solution to the same example in the FEMLAB program (fig. 3) and the projection of our mesh to the colour output of the same program (fig. 4).

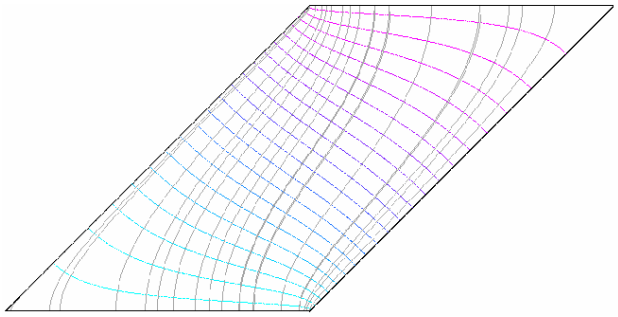

Fig 3: The comparative output of the FEMLAB program

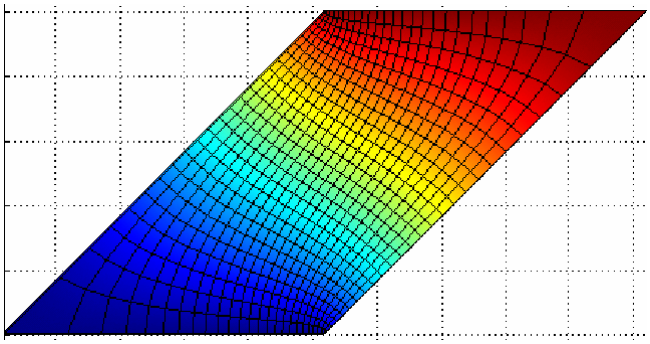

Fig. 4: The analysis comparison with the FEMLAB program (in colour)

3 CONCLUSION

The evaluation of the practical benefits of the submitted methods and the comparison with the memory and timing requirements of conventional computational devices will be published later, after debugging and fine-tuning of the code of both our methods. However, it is possible to state that the computational requirements of both the methods only increase directly proportionally to the number of elements.

It is true that the method of uniform distribution of energy is applicable with the limitation to a conservative field but its advantage is the high accuracy of the analysis even in the boundary areas.

The wave method seems to be capable to became a universal tool for solutions to stationary fields in the linear and non-linear environment with losses and if need be even with the internal sources. The method is particularly effective in the three-dimensional version.

4 ACKNOWLEDGMENT

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