September 10 - 12, 2007
Pilsen, Czech Republic

ELECTROMAGNETIC TORQUE ANALYTICAL APPROACH FOR SPHERICAL INDUCTION MOTOR

D. SPAŁEK, K. WALECZEK.

Abstract: *The paper has dealt with approach to electromagnetic field analysis for spherical electromechanical converters taking into account its magnetic anisotropy analytically. The electromagnetic field is evaluated analytically using the separation method proposed for the magnetic vector potential. Subsequently, electromagnetic torque and power losses are calculated analytically for an exemplary spherical induction motor.*

Key words: *Analytical magnetic field solution, spherical induction motor, anisotropic rotor*

INTRODUCTION

The paper deals with the problem of electromagnetic field analysis for spherical electromechanical converter taking into account magnetic anisotropy in analytical way. The electromagnetic field is evaluated analytically with the help of proposed separation method for magnetic vector potential. Power balance and electromagnetic torque for electromagnetic field is presented and checked up. The results obtained can be used as test task for electromagnetic field analysis and can support design for electromechanical converter.

1 MAIN EQUATIONS

The analytical approach to electromagnetic torque constitutes the basis for further calculations either analytical or numerical. Moreover, analytical approach gives wide-range insight into the influence of electromagnetic circuit parameters on the electromechanical converters work. On the other hand, the analytical solution (that is given by one or more closed formulas) requires simplifications of the real electromechanical converter geometry, usually. The fewer the number of the simplifying assumptions the more general solution is obtained. One of the most commonly used analytical methods is the separation of variables [1], [2], [3]. The separation method proposed in this paper leads to the analytical solution for the spherically symmetric field problem considering magnetic anisotropy features. The aim of this contribution is to present an analytical solution for a spherical induction motor with magnetically anisotropic conductive rotor that could be treated as a benchmark task for numerical analyses.

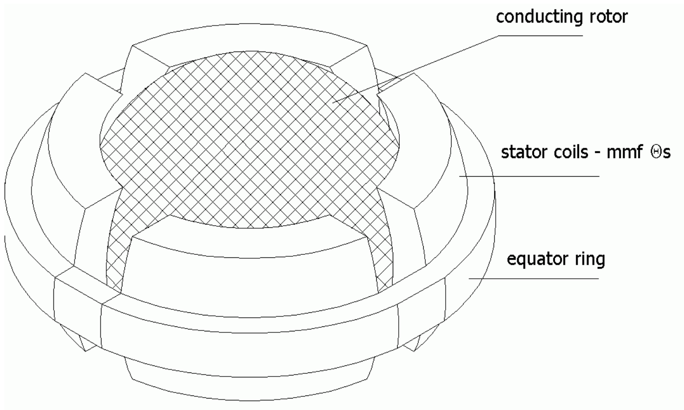

Fig.1: Spherical rotor - view

The electromagnetic field could be evaluated analytically with the help of separation method for spherical co-ordinate system [5, 11, 1 2]. The separation method proposed in the paper leads to the analytical solution for spherical symmetry problem considering magnetic anisotropy. For electromechanical converter with magnetic field dominance the magnetic flux density can be calculation by means of magnetic vector potential as follows

$$B = \mathrm{curl}(\vec{A}) \tag{1}$$

Let us consider an induction motor with spherical rotor (see Fig.1). The stator currents constitute the magnetomotive force $\Theta_s(t,\varphi,\theta)$, that can be expressed in form of complex form as follows

$$\Theta_s(t,\varphi,\theta) = \sum_h \left(\Theta_{sh}(\theta)\exp(i\omega_h t \mp iph\varphi + \mathrm{const}_h)\right), \tag{2}$$

where Θ_{sh} denotes the magnitude of the h^{th} harmonic of stator mmf, ω_h is angular speed for h^{th} rotating field harmonic, p is number of pair pole, φ is the longitude of given point. The mmf magnitudes could depend on colatitude θ due to the fact that the colatitudinal currents flow across decreasing surfaces out of equator (see Fig.2). The stator mmf is exerted by the stator currents placed on the inner surface of the stator at r=R+g (R is rotor outer surface radius, g denotes air-gap width, see Fig.2).

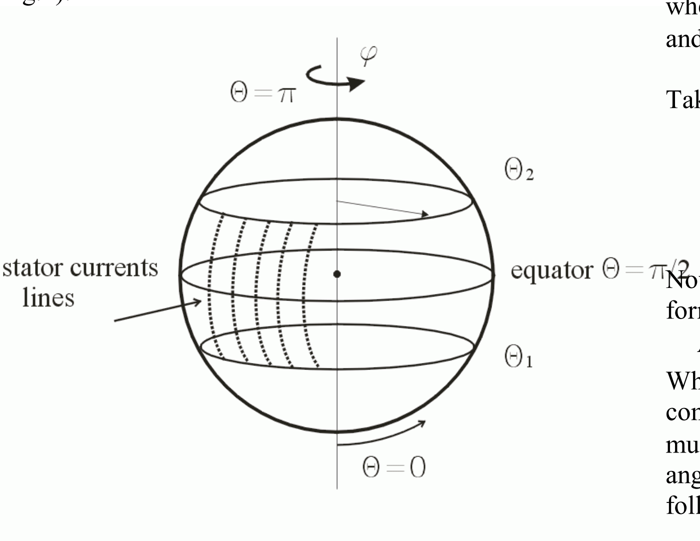

Fig.2: Stator currents lines

2 SOLUTION OF GOVERNING EQUATIONS

The Eqn (2) leads to the following relation in a spherical co-ordinate system

$$\vec{B} = \frac{\vec{1}_r}{r\sin\theta}\left\{\frac{\partial A_\theta}{\partial\varphi} - \frac{\partial(A_\varphi\sin\theta)}{\partial\theta}\right\} - \\ -\frac{\vec{1}_\varphi}{r}\left\{\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right\} + \\ + \frac{\vec{1}_\theta}{r\sin\theta}\left\{\frac{\partial}{\partial r} rA_\varphi\sin\theta - \frac{\partial A_r}{\partial\varphi}\right\} \tag{3}$$

where $\vec{1}_r, \vec{1}_\varphi, \vec{1}_\theta$ denote the unit vectors that satisfy the relation $\vec{1}_r \times \vec{1}_\varphi = \vec{1}_\theta$.

For an induction motor with spherical rotor [4], [5], [6] the magnetic field is directed so as to enable turning around a fixed axis i.e. z-axis. The magnetomotive force (mmf) exerts the magnetic field that has got components in "r" and "φ" only. The magnetic vector potential in a spherical coordinate system can be given in the form of

$$\vec{A} = \vec{A}_\theta = A_\theta\vec{i}_\theta = A\vec{i}_\theta, \tag{4}$$

for describing magnetic flux density.

According to Eqn (4) the magnetic flux density is as follows

$$\vec{B} = \frac{\vec{1}_r}{r\sin\theta}\left\{\frac{\partial A_\theta}{\partial\varphi}\right\} - \frac{\vec{1}_\varphi}{r}\left\{\frac{\partial(rA_\theta)}{\partial r}\right\}, \tag{5}$$

$$B_\theta = 0. \tag{6}$$

Both Amper's law

$$\mathrm{curl}(\vec{H}) = \vec{j}, \tag{7}$$

and the constitutive relation for an anisotropic magnetic medium as follows

$$\begin{bmatrix} H_r \\ H_\varphi \\ H_\theta \end{bmatrix} = \begin{bmatrix} \nu_{rr} & \nu_{r\varphi} & \nu_{r\theta} \\ \nu_{\varphi r} & \nu_{\varphi\varphi} & \nu_{\varphi\theta} \\ \nu_{\theta r} & \nu_{\theta\varphi} & \nu_{\theta\theta} \end{bmatrix} \cdot \begin{bmatrix} B_r \\ B_\varphi \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_{rr}B_r + \nu_{r\varphi}B_\varphi \\ \nu_{\varphi r}B_r + \nu_{\varphi\varphi}B_\varphi \\ \nu_{\theta r}B_r + \nu_{\theta\varphi}B_\varphi \end{bmatrix}, \tag{8}$$

lead to the following equation for colatitudinal direction,

$$\frac{1}{r\sin\theta}\left\{\frac{\partial}{\partial r} rH_\varphi\sin\theta - \frac{\partial H_r}{\partial\varphi}\right\} = j_\theta = \gamma E_\theta = -\gamma\dot{A}_\theta, \tag{9}$$

where γ stands for the electric conductivity of rotor layer, and γ=0 for air-gap.

Taking into account (5) and (9) it can be written

$$\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{\nu_{\varphi r}}{r\sin\theta}\left\{\frac{\partial A_\theta}{\partial\varphi}\right\} - \frac{\nu_{\varphi\varphi}}{r}\left\{\frac{\partial rA_\theta}{\partial r}\right\}\right) - \\ -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{\nu_{rr}}{r\sin\theta}\left\{\frac{\partial A_\theta}{\partial\varphi}\right\} - \frac{\nu_{r\varphi}}{r}\left\{\frac{\partial rA_\theta}{\partial r}\right\}\right) = -\gamma\dot{A}_\theta \tag{10}$$

Now, a non-standard separation variable is defined in the form given below

$$A = A_\theta(r,\varphi,\theta) = R(r,\theta)F(\varphi) = R\cdot F. \tag{11}$$

When using complex notation for field vector components the time derivative can be replaced by its multiplication by iω (i is the imaginary unit, ω is the angular pulsation of field in rotor). Thus (10) takes the following form

$$\frac{\nu_{\varphi r}}{rRF\sin\theta}\frac{\partial^2 RF}{\partial r\partial\varphi} - \frac{\nu_{\varphi\varphi}}{rR}\frac{\partial^2 rR}{\partial r^2} - \frac{\nu_{rr}}{r^2F\sin^2\theta}\frac{\partial^2 F}{\partial\varphi^2} + \frac{\nu_{r\varphi}}{r^2RF\sin\theta}\frac{\partial^2 rRF}{\partial\varphi\partial r} = \frac{i\omega\gamma}{RF} \tag{12}$$

for $\theta\in(0,\pi)$ and subsequently

$$\frac{\nu_{\varphi\varphi}}{rR}\frac{\partial^2 rR}{\partial r^2} + \frac{\nu_{rr}}{r^2F\sin^2\theta}\frac{\partial^2 F}{\partial\varphi^2} - \\ -\left(\frac{\nu_{\varphi r}}{rR}\frac{\partial R}{\partial r} + \frac{\nu_{r\varphi}}{r^2R}\frac{\partial rR}{\partial r}\right)\frac{1}{F\sin\theta}\frac{\partial F}{\partial\varphi} = i\omega\gamma \tag{13}$$

For the function F(φ) it is assumed that the separation constant equals to p^2 for the first mmf space harmonic h = 1 i.e.

$$\frac{1}{F}\frac{\partial^2 F}{\partial\varphi^2} = -p^2, \tag{14}$$

and for mmf higher space harmonics p is replaced by ph.

The angular field frequency ω in Eqn (13) is determined for each mmf harmonic at the synchronous rotatory speed $\pm 2\pi f_1/ph$.

The Eqn (14) has got the general solution in the form of

$$F = F(\varphi) = C\exp(ip\varphi) + D\exp(-ip\varphi). \tag{15}$$

The Eqns (14) and (15) are adequate for rotating magnetic field generated by the stator mmf. The solution (15) for a unidirectional rotating field (constant C = 0, D = 1) leads to equation in the form of

$$\frac{\partial^2 R}{\partial r^2}+2(1-h(\theta))\frac{\partial R}{\partial r}+R\left(-\beta^2-\frac{p^2\nu_{rr}-ip\sin\theta\nu_{r\varphi}}{r^2\nu_{\varphi\varphi}\sin^2\theta}\right)=0\,, \qquad (16)$$

with the following analytical solution for an anisotropic region [7] (p. 363 Eqn B110(3), or it could be checked by putting in):

$$R(r,\theta)=(\beta r)^{\lambda(\theta)-\frac{1}{2}}\left(C_1 I_{\lambda(\theta)}(\beta r)+C_2 K_{\lambda(\theta)}(\beta r)\right) \qquad (17)$$

where

$$\lambda(\theta)=\pm\sqrt{(h(\theta)-\tfrac{1}{2})^2+\frac{p^2\nu_{rr}-ip\sin\theta\nu_{r\varphi}}{r^2\nu_{\varphi\varphi}\sin^2\theta}}\,,$$

$$h(\theta)=-\frac{\nu_{r\varphi}+\nu_{\varphi r}}{2\nu_{\varphi\varphi}\sin\theta}ip\,,\quad \beta^2=\frac{i\gamma\omega}{\nu_{\varphi\varphi}}\,. \qquad (18a,b,c)$$

The solution in the form of (17) confirms that the proposed non-standard separation (11) is correct. The Eqn (13) for the non-conductive region (e.g. the air-gap γ=0, $\nu_{r\varphi}=\nu_{\varphi r}=0$) takes the simple form of

$$\frac{\partial^2 rR}{\partial r^2}-(\kappa(\theta)+1)\kappa(\theta)\frac{R}{r}=0\,, \qquad (19)$$

where

$$\kappa(\kappa+1)=\frac{\nu_{r\delta}p^2}{\nu_{\varphi\delta}}\,, \qquad (20)$$

and $\nu_{r\delta}$, $\nu_{\varphi\delta}$ the mean radial and latitudinal reluctivities for the air-gap (they can be different form vacuum reluctivity [4]). The solution of (19) is as follows

$$R=R(r,\theta)=a_\delta r^{\kappa_1(\theta)}+b_\delta r^{\kappa_2(\theta)}\,. \qquad (21)$$

The analytical solution for the spherical motor can be presented in terms of separated functions R(r,θ) and F(φ) occurred due to separation defined by Eqn (11) - see Table 1. The general solutions presented should be combined with the boundary conditions for particular geometry conditions.

Region	anisotropic layer (index a)	gap (index δ)
A = R·F Solutions for R(r, θ) F(φ)	$R(r,\theta)=(\beta r)^{\lambda(\theta)-\frac{1}{2}}\left(C_1 I_{\lambda(\theta)}(\beta r)+C_2 K_{\lambda(\theta)}(\beta r)\right)$ $\lambda(\theta)=\pm\sqrt{(h(\theta)-\frac{1}{2})^2+\frac{p^2\nu_{rr}-ip\sin\theta\nu_{r\varphi}}{r^2\nu_{\varphi\varphi}\sin^2\theta}}$ $h(\theta)=-\frac{\nu_{r\varphi}+\nu_{\varphi r}}{2\nu_{\varphi\varphi}\sin\theta}ip$ $F=\exp(-ip\varphi)$	$R(r,\theta)=a_\delta r^{\kappa_1(\theta)}+b_\delta r^{\kappa_2(\theta)}$ $\kappa(\kappa+1)=\frac{\nu_{r\delta}p^2}{\nu_{\varphi\delta}}$ $F=\exp(-ip\varphi)$
Magnetic flux density radial components B_r( )	$B_r=\frac{1}{r\sin\theta}\frac{\partial A}{\partial\varphi}=\frac{-ip}{r\sin\theta}R(r,\theta)$	
Magnetic flux density tangential components B_φ( )	$B_\varphi=-\frac{1}{r}\frac{\partial(rA)}{\partial r}$	
	$B_\varphi=-\frac{\partial}{r\partial w}\left(w^{\lambda-\frac{1}{2}}\left(a_a I_\lambda(w)+b_a K_\lambda(w)\right)\right)$ $w=\beta r$	$B_\varphi=-\frac{1}{r}\left((\kappa_1+1)a_\delta r^{\kappa_1}+(\kappa_2+1)b_\delta r^{\kappa_2}\right)$
constans	a_a b_a	a_δ b_δ

Fig.1: Solutions for differential equations for vector magnetic potential and magnetic flux density

3 BOUNDARY CONDITIONS

There are four conditions defined for the electromagnetic field vectors. They enable to calculate the four unknown constants a_a, b_a, a_δ, b_δ shown in Table 1. The boundary conditions result from physical principles [8], [9].

a) The magnetic field strength disappears at the inner layer surface (r=R-a)

$$H_\varphi=\nu_{\varphi r}B_r+\nu_{\varphi\varphi}B_\varphi=0\,, \qquad (22)$$

as a consequence of the fact that magnetic reluctivity of the rotor core is assumed to be zero.

b) The continuity of the normal (radial) magnetic flux density (r=R)

$$B_{r\delta}=B_{ra}\,, \qquad (23)$$

and

c) the longitudinal (tangential) component of the magnetic field strength (r=R)

$$\nu_{\varphi\delta}B_{\varphi\delta}=\nu_{\varphi\varphi}B_{\varphi a}+\nu_{\varphi r}B_{ra}\,. \qquad (24)$$

d) The magnetomotive force of the electromechanical converter stator currents leads to the following condition for the longitudinal (tangential) component of magnetic field strength at the stator surface (r=R+g)

$$B_{\varphi\delta} = \frac{1}{\nu_{\varphi\delta} r \sin\theta} \frac{\partial \Theta_s}{\partial \varphi}, \qquad (25)$$

derived under the assumption that the magnetic field strength vanishes on the outer side of the winding surface (stator frame iron is infinitely permeable) [10], [11], [12], [13]. The Table 2 presents constant values of a_a, b_a, a_δ, b_δ for the first space harmonic of the stator mmf.

Boundary condition	**Field excited by stator currents**	**Constants for solutions (the constants depend on θ, that is not denoted explicit)**
Stator current mmf $r=R+g=R_g$	$B_{\varphi\delta} = \frac{1}{\nu_{\varphi\delta} r \sin\theta} \frac{\partial \Theta_s}{\partial \varphi}$	$a_a = \frac{p_\theta \Theta_s}{\nu_{\varphi\delta}} \{U(1+\kappa_1)R_g^{\kappa 1} + W(1+\kappa_2)R_g^{\kappa 2})\}^{-1}$ $b_a = -a_a S$, $a_\delta = a_a U$, $b_\delta = a_a W$
Rotor outer surface $r=R$	$B_{r\delta} = B_{ra}$, $\nu_{\varphi\delta} B_{\varphi\delta} = \nu_{\varphi\varphi} B_{\varphi a} + \nu_{\varphi r} B_{ra}$	$S = \frac{\nu_{\varphi\varphi}[n w_a^f I_\lambda(w_a) + w_a^{f+1} I'_\lambda(w_a)] + i p_\theta \nu_{\varphi r} I_\lambda(w_a) w_a^f}{\nu_{\varphi\varphi}[n w_a^f K_\lambda(w_a) + w_a^{f+1} K_\lambda(w_a)] + i p_\theta \nu_{\varphi r} K_\lambda(w_a) w_a^f}$, $f = h - \frac{1}{2}$, $n = f + 1$, $w_a = \beta R_a$, $w = \beta R$, $\beta = \sqrt{i\omega\gamma / \nu_{\varphi\varphi}}$, $p_\theta = p / \sin(\theta)$,
Inner layer surface $r=R-a=R_a$	$\nu_{\varphi\varphi} B_\varphi + \nu_{\varphi r} B_r = 0$	$P = \frac{\nu_{\varphi\varphi}}{\nu_o}[n w^f I_\lambda(w) + w^n I'_\lambda(w) - S[n w^f K_\lambda(w) + w^n K_\lambda(w)]] + Q \frac{i p_\theta \nu_{\varphi r}}{\nu_{\varphi\delta}}$ $Q = w^f [I_\lambda(w) - S K_\lambda(w)]$ $U = \frac{Q(\kappa_2+1) - P}{(\kappa_2 - \kappa_1)R^{\kappa 1}}$, $W = \frac{P - Q(\kappa_1+1)}{(\kappa_2 - \kappa_1)R^{\kappa 2}}$

Tab.2: The boundary conditions for magnetic field

4 EXAMPLE – SPHERICAL INDUCTION MOTOR

Exemplary, analytical calculations are provided for the spherical induction motor which parameters are presented in Table 3. The mmf space harmonics can be chosen arbitrary so as they can describe both one-phase and three-phase induction motors.

Data for	Value
stator mmf harmonics Θ_1 Θ_5 Θ_7 Θ_{11}	p = 1 250 A (h=1) 50 A (h=5) 35 A (h=7) 23 A (h=11)
rotor radius R	0.05 m
solid rotor (conductive) layer width a	0.005 m
radial reluctivity $\nu_{rr} = \nu_r$	$\nu_0/20$
tangential (longitudinal) reluctivity $\nu_{\varphi\varphi} = \nu_\varphi$	$\nu_0/10$, $\nu_0/20$, $\nu_0/30$
solid rotor conductivity γ	$5 \cdot 10^6$ S/m
gap width g	0.001 m
air-gap reluctivity $\nu_{\varphi\delta} = \nu_{\varphi r} = \nu_0$	ν_0
angles interval of stator mmf longitudinal width $\theta_1 \div \theta_2$	$\pi/4 \div 3\pi/4$

Tab.3: Spherical three-phase induction motor parameters

The mmf harmonics of stator windings for three phase motor are presented in Fig.3.

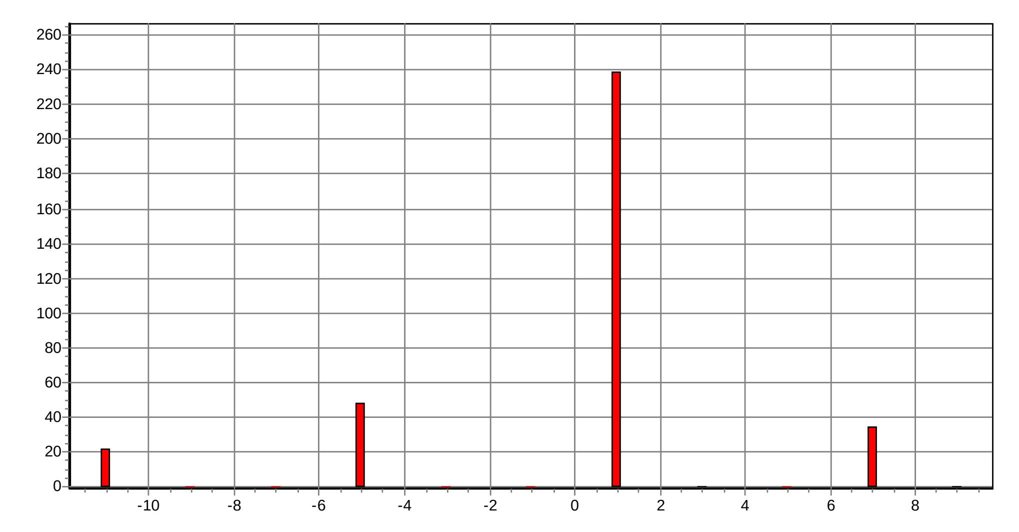

Fig.3: MMF harmonics for one separated phase of stator windings (p=1)

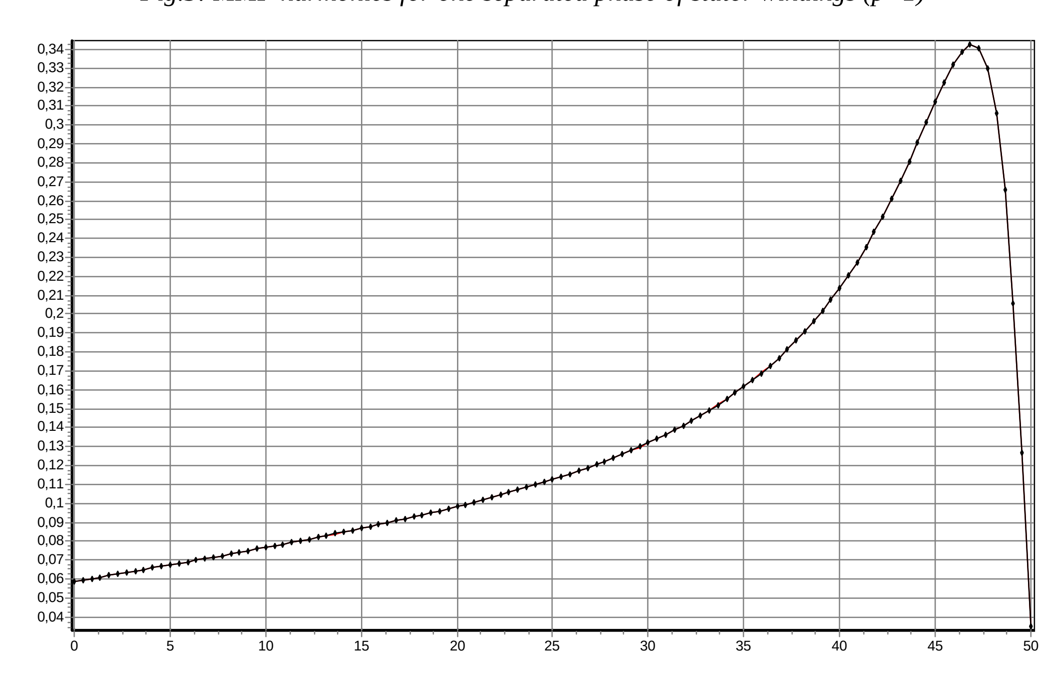

Fig.6: Torque calculated by means of both Maxwell and co-energy methods

5 ELECTROMAGNETIC TORQUE CALCULATIONS – MAXWELL, COENERGY AND LORENTZ METHODS EFERENCES

Based on the magnetic field vector potential distribution, the magnetic flux density components and the electromagnetic torque can be evaluated analytically, subsequently. The Maxwell stress tensor leads to the total electromagnetic torque by means of the well-known surface integral

$$T_e = \int_0^{2\pi}\int_{\theta_1}^{\theta_2} H_\varphi B_r r^3 \sin^2\theta d\theta d\varphi, \qquad (27a)$$

where r is the radius of the surface (over it follows the integration) placed in the air gap $r \in [R, R+g]$. Based on the magnetic field vector components presented in Table 1 for the air-gap region, the electromagnetic torque can be given as follows

$$T_e = \pi p \nu_{\varphi\delta} \int_{\theta_1}^{\theta_2} \mathrm{Im}\{r \frac{\partial(rR(r,\theta))}{\partial r} R^*(r,\theta)\} \sin\theta d\theta . \quad (27b)$$

The total torque can be also by means of the coenergy method ([14], [15]) as follows

$$T_e = \left.\frac{\partial W_C}{\partial \varphi}\right|_{j=const} = \int_V (\vec{j}\frac{\partial \vec{A}}{\partial \varphi} + \vec{B}\frac{\partial \vec{H}}{\partial \varphi}) dV, \qquad (28a)$$

where W_C stands for magnetic coenergy of anisotropic rotor. For the rotor region the volume integral can be presented as follows

$$T_e = \int_0^{2\pi}\int_{R_a}^{R}\int_{\theta_1}^{\theta_2} (j_\theta \frac{\partial A_\theta}{\partial \varphi} + B_r \frac{\partial H_r}{\partial \varphi} + B_\varphi \frac{\partial H_\varphi}{\partial \varphi}) r^2 \sin\theta d\theta dr d\varphi \cdot \qquad (28b)$$

The both values given by Eqn (27a,b) and Eqn (28a,b) should be the same and describe the total torque value (no specific indexes).

Exemplary, calculations for both methods: Maxwell and co-energy are derived for the exemplary data (Table 3) and are presented in Fig.4.

The electromagnetic torque can be evaluated with the help of the Lorentz force density as well

$$T_{eL} = \int_0^{2\pi}\int_{R_a}^{R}\int_{\theta_1}^{\theta_2} j_\theta \frac{\partial A_\theta}{\partial \varphi} r^2 \sin\theta d\theta dr d\varphi . \qquad (29a)$$

Applying the magnetic field vector components presented in Table 1 for the rotor layer, the Lorentz torque can be rewritten in the form of

$$T_{eL} = \pi\gamma p\omega \int_{\theta_1}^{\theta_2} \sin\theta \int_{R_a}^{R} \left|R(r,\theta)\right|^2 r^2 dr d\theta . \qquad (29b)$$

The torques evaluated with the help of Maxwell, co-energy and Lorentz method should be equal

$$T_e = T_{eL} , \qquad (30)$$

for either magnetically isotropic or normally anisotropic ($\nu_{\varphi r} = \nu_{r\varphi}$) rotor. As an example, torque-speed curves for both torques calculations are presented in Fig.5 (SI units).

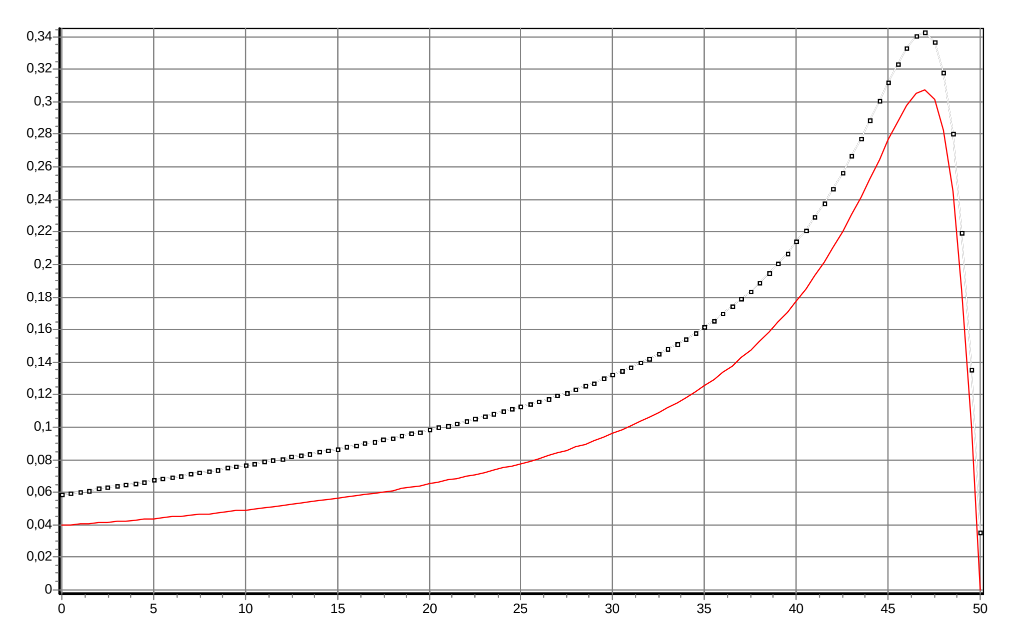

Fig.5: Torque-speed curves for first mmf space harmonic only - solid line, and for all considered mmf harmonics - dash-doted line, (SI units)

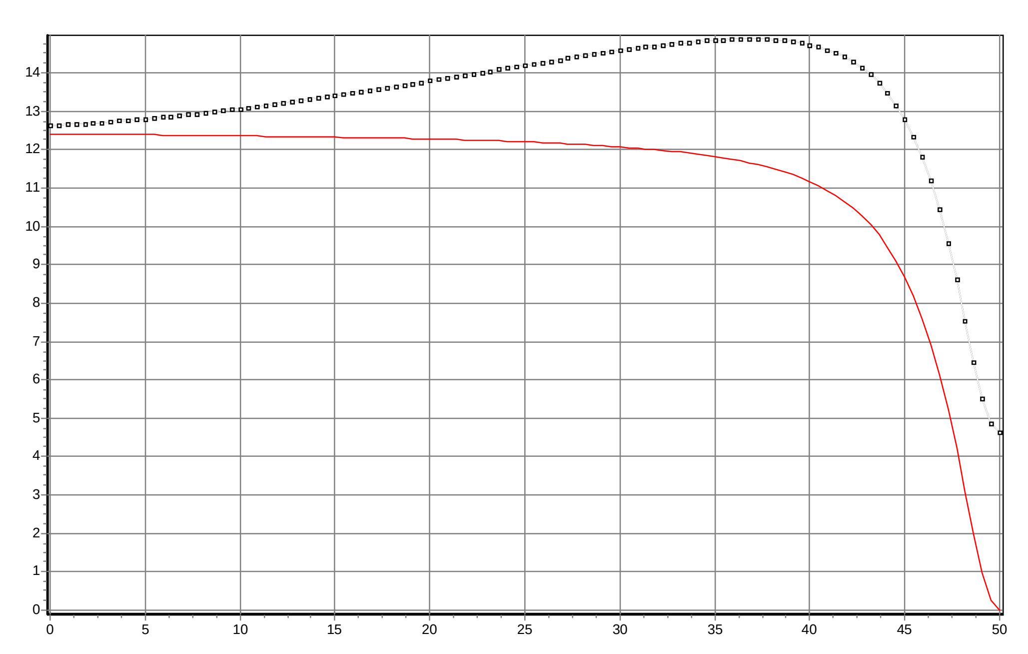

Fig.6: Power losses in conductive layer vs. rotor speed (SI units) for first mmf harmonic (solid line); total losses (dash-dotted line)

6 POWER LOSSES CALCULATIONS

The power losses constitute an important parameter from the thermal point of view [16], [17], [18], [19]. Power losses caused by the induced currents are

$$P = \tfrac{1}{2}\sum_h \int_V \mathrm{Re}\{j_h E_h^*\} dV = \tfrac{1}{2}\sum_h \gamma\omega_h^2 \int_V |A_h|^2 dV \,. \quad (30)$$

They are shown in Fig. 8 (SI units) (h is order number of the h^{th} harmonic of stator mmf). The electromagnetic torque calculated with the help of the Lorentz formula is equal to

$$T_{eL} = \int_V r\sin\theta j_\theta B_r dV =$$
$$= \tfrac{1}{2}\sum_h \int_V \mathrm{Re}\left\{ j_h \frac{\partial A^*}{\partial \varphi} \right\} dV = \quad (31)$$
$$= \tfrac{1}{2}\sum_h \gamma p h \omega_h \int_V |A_h|^2 dV$$

leading to

$$P = \sum_h P_h = \sum_h \omega_h \frac{T_{eL,h}}{ph}, \quad (32)$$

where the electromagnetic torque previously calculated values appear. The power losses for first harmonic and all harmonics of stator mmf are shown in Fig.6.

7 CONCLUSIONS

The electromagnetic field Maxwell equations in a spherical co-ordinate system are solved analytically. The mathematical form of the non-standard separation is given by Eqn (11). The analytical solution has been obtained for a magnetically anisotropic and conductive region (rotor layer).

For electromechanical converter with spherical rotor and polyharmonic stator magnetomotive force the presented model has been applied for presenting the analytical solution provided.

The electromagnetic torque calculations are derived with the help of the Maxwell, co-energy and Lorentz methods.

Moreover, the power losses have been evaluated with the help of the analytical solutions obtained.

The presented solutions for electromagnetic field, electromagnetic torque and power losses can be treated as benchmark task [3], [19].

8 REFERENCES

[1] LANDAU L.D., LIFSZYC E.M.: 'The classical theory of fields'. Pergamon, New York, 1951

[2] JACKSON J.D.: 'Classical electrodynamics'. John Wiley, New York 1999

[3] Workshop TEAM http://ics.ec-lyon.fr/team.html - accessed 2005

[4] BOLTE E., HAHLWEG C.: 'Analysis of steady-state of high speed induction motors with exterior rotor and conductive layer on the slotted stator' *Proceedings of International Conference on Electrical Machine ICEM* , 2002 p.19

[5] DAVEY K., VACHTSEVANOS G., POWERS R.: The analysis of fields and torques in spherical induction motors. IEEE Trans. Magn., Vol. 23, No. 1, pp. 273-282 (1987).

[6] LEE K.-M., ROTH R., ZHOU Z.: Dynamic modeling and control of a ball-joint-like variable reluctance spherical motor. ASME J. Dynam. Syst. Meas. Cont., Vol. 118, No. 1, pp. 29–40 (1996).

[7] GRADSZTAJN I. S., RYZIK I. M.: 'Tables of integrals, sums, series and terms' (Tablicy intjegralow, sum, rjadow i proizwjedjenij in Russian), Moscow, 1962

[8] BINNS K.J, LAWRENSON P.J., TROWBRIDGE C.W.: The analytical and numerical solution of electric and magnetic fields. John Wiley & Sons, 1994.

[9] TAVNER P.J.: 'Cross-magnetisation effects in electrical machines'. *IEE Proc. Electr. Power Appl.*, 2004, **151**, (3), pp. 249-259

[10] NETHE A., SCHOLZ T., STAHLMANN H-D.: An analytical solution method for magnetic fields using the Fourier analysis and its application of ferrofluid driven electric machines. Proc. ISTET 2003, Warsaw, Poland, Vol. II, pp. 421–424 (2003).

[11] VANDE SANDE H., HAMEYER K.: 'Comparison of neutral network and polynomial models for the approximation of nonlinear and anisotropic ferromagnetic materials'. *IEE Proc. Sci. Meas. Technol.*, 2002, **149**, (5), pp. 214-217

[12] DE GERSEM H., HAMAYER K.: 'Harmonic boundary conditions for circular inclusions and their coupling to electrical circuits'. *IEE Proc. Sci. Meas. Technol.*, 2001, **148**, (6), pp. 257-262

[13] LINDELL I.V., HÄNNINGEN J.J., NIKOSKINEN K.I.: 'Electrostatic image theory for an anisotropic boundary'. *IEE Proc. Sci. Meas. Technol.*, 2004, **153**, (3), pp. 188-194

[14] BENHAMA A., WILIAMSON A.C., REECE A.B.J.: 'Virtual work approach the computation of magnetic force distribution from finite element field solutions'. *IEE Proc. Electr. Power Appl.*, 2000, **147**, (6), pp. 437-442

[15] VANDEVELDE L., MELKEBEEK J.A.A.: 'Computation of deformation of ferromagnetic material'. *IEE Proc. Sci. Meas. Technol.*, 2002, **149**, (5), pp. 222-226

[16] HARRISON D.J.: 'Characterization of cylindrical eddy-current probes in terms of their spatial frequency spectra'. *IEE Proc. Sci. Meas. Technol.*, 2001, **148**, (4), pp. 183-186

[17] ZHU Z.Q., NG K., SCHOFIELD N., HOWE D.: 'Improved analytical modeling rotor eddy current loss in brushless machines equipped with surface-mounted

permanent magnets'. *IEE Proc. Electr. Power Appl.*, 2004, **151**, (6), pp. 641-650
[18] SAGEDHI S.H.H, SALEMI A.H.: 'Electromagnetic field distributions around conductive slabs, produced by eddy-current probes with arbitrary shape current-carrying excitation loops'. *IEE Proc. Sci. Meas. Technol.*, 2001, **148**, (4), pp. 187-192
[19] SPAŁEK D.: 'Spherical induction motor with anisotropic rotor - analytical solutions for electromagnetic field distribution, electromagnetic torques and power losses'. *International Compumag Society*. Testing Electromagnetic Analysis Methods (T.E.A.M.) - problem no. 34.
http://www.compumag.co.uk/team.html (2007)

Dariusz Spałek
Electrical Engineering Faculty,
Silesian University of Technology, Poland
ul. Akademicka 10, 44-100 Gliwice
Dariusz.Spalek@polsl.pl
http://www.elektr.polsl.pl/dspalek/