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APPROXIMATION OF STRONG ELECTRIC FIELD

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Abstract: *Since strong electric field is used now in many areas, simple methods for exact calculation of electric field should be found. For the apparatus, where the strong electric field is produced by thin straight conductor over conducting body, several approximating formulae (that use conducting plane instead of the body) were derived. Using the formulae, the field strength has been calculated in detail and typical results are presented. The comparison of all the methods shows that in the area near the conductor (near zone) any of the formulae can be used. In the vicinity of the conductor a difference exists.*

Key words: *Electrostatic field of thin wire, mirror method, field of finite length conductor, conducting plane, electrostatic probe*

INTRODUCTION

At the present time a strong electric field is used in many applications in science, design, industry etc - from dust separators or electrostatic industrial painting systems, through nano-fibres production in textile science [1, 8], to highly sophisticated devices in particle physics. Therefore, the task of the electrostatic field modeling is very important.

From the all possible sources it is clear, that this part of physics is unjustly neglected. The possible reason may be the difficulties both in theoretical and experimental research. It is a shame, because it could lead to many practical applications of this effect.

There is an active research at Technical University in Liberec in the field of the asymmetrical capacitors - e.g. thin wire over a metal plate electrode. To find a formula describing the force on the asymmetrical capacitor [2, 3, 4] connected to high voltage (tens of kilovolts) we need to obtain a description of the electrostatic field. In this particular configuration the exact 3D description of this electrostatic field is not possible without using the finite element method (FEM), but for our research the approximate description would be sufficient – the geometry is simple enough and the application of basic equations of electrostatic theory leads to acceptable results. This paper deals with several methods of strong electrostatic field approximation.

1 THEORY

The problem can be formulated like this: Over a conductive orthogonal plane of negligible thickness (length a, width b) and in distance d there is a wire of the same length a and radius R suspended in a parallel way to the plane. The wire is connected to DC voltage U_0, the plane is grounded. The wire radius R is very small compared to the distance d. Our task is to find a formula describing the strength of the electric field on the axis perpendicular to the plane and passing through the middle of the wire. From the theoretical point of view the problem can be described as electrostatic field of charged conductors. It is possible to solve this problem using potential φ, which can be in turn used to find the electric field strength:

$$\vec{E} = -grad\,\varphi \,. \qquad (1)$$

Potential has to fulfill the following conditions:

1. The Laplace equation is fulfilled around the conductor

$$\Delta\varphi = 0 \,. \qquad (2)$$

2. The potential in the infinity equals zero.

$$\lim_{r\to\infty}\varphi(r) = 0 \,. \qquad (3)$$

3. The potential has unknown but constant value φ_0 on the surface of the conductor.
4. The normal component of the electric field strength on the surface of the conductor is equal to surface charge density, which can be written in integral form thus:

$$\int_{(S)} E_n \, dS = -\int_{(S)} grad_n \varphi \, dS = Q, \qquad (4)$$

where Q is charge on the conductor and the S symbol shows us, that we are integrating over the surface of the conductor.

So the following rules must apply to electric field strength:

1. Inside the conductor it equals zero
2. On the surface of the conductor its tangent component is equal to zero (otherwise there would be a strong current flow inside and on the surface of the conductor)
3. On the surface there is only a normal component of the field strength – see formula (4)

Formulated this way the problem cannot be solved analytically. Typically this problem could be approximately solved using finite element method (FEM). In simplified form the FEM is applied on the Laplace equation (2). The boundary conditions are the specified values of potential φ_0 on conductors. But there is a problem with the zero value of the potential in infinity. The last condition – the total charge of the system of conductors is equal to zero - can be used to verify the results. But because we need the formula as a starting point for further derivations, it is better to solve the problem analytically, though only with approximate results – that is, by using simplified but sufficient models.

The problem can be analytically solved only if we assume that the plane could be substituted by infinite conductive surface. Another simplification uses the fact, that the wire radius is very small, so we can replace it with charged line segment. Linear charge density is considered to be constant.

So for the analytical solution we can use these four gradually more accurate models:

1. Infinite single wire
2. Infinite wire and conductive plane
3. Finite length single wire
4. Finite length wire and conductive plane

Orthogonal system of coordinates is orientated so, that its zero is in the middle of the wire axis, X-axis is parallel to the wire axis and it is orientated from left to right and Z-axis is perpendicular to the charged plane. If we are to use cylindrical symmetry we will be using the radial distance r instead of coordinate z. Following formulae are valid only for the field on the plane crossing the wire axis and perpendicular to the plane – that is in the XZ plane. Then the system of coordinates is shown on Fig. 1. It is possible to derive the analytical formulae for the 3D field, but they are much more complicated.

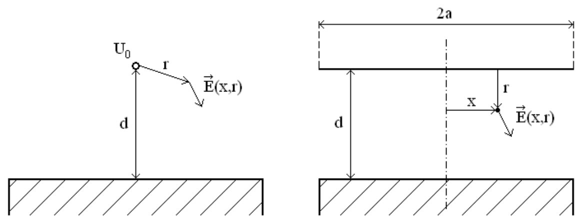

Fig.1: Geometry of the experimental arrangement.

Using the Gauss law from electrostatics [5], [6] we can derive the following formula for potential of a single line with the linear charge density η:

$$\varphi(r) = -\frac{\eta}{2\pi\varepsilon_0}\ln r + A, \qquad (5)$$

where ε_0 is permittivity of vacuum and r is the distance from the line, where the field is calculated. Constant A could be written like this:

$$A = \frac{\eta}{2\pi\varepsilon_0}\ln r_n, \qquad (6)$$

where the new constant r_n must fulfill the following condition: $r_n > 0$. When we apply (6) on (5) we get a physically more illustrative general formula for potential:

$$\varphi(r) = -\frac{\eta}{2\pi\varepsilon_0}\ln r + \frac{\eta}{2\pi\varepsilon_0}\ln r_n = \frac{\eta}{2\pi\varepsilon_0}\ln\frac{r_n}{r}. \qquad (7)$$

Constant r_n represents the distance of the point, where the potential is equal to zero. More precisely, it is the radius of a coaxial grounded conductive cylindrical surface. Applying a standard procedure based on the formula for the potential of the point charge and superposition principle [7] we get formula (7).

If we replace the charged line with the infinite charged wire of a circular cross-section and radius R, due to cylindrical symmetry the electrostatic field will remain the same. On the wire surface there will be constant potential $\varphi(R)$. Between this wire and the cylindrical conductive surface in the distance r_n there will be the voltage U_0, which can be, using formula (7), defined as:

$$U_0 = \varphi(R) = \frac{\eta}{2\pi\varepsilon_0}\ln\frac{r_n}{R}. \qquad (8)$$

As the linear charge density η is difficult to define, we can replace it, using (8), with voltage U_0 between electrodes or conductors:

$$\eta = U_0\frac{2\pi\varepsilon_0}{K}, \qquad (9)$$

where dimensionless constant K is

$$K = \ln\frac{d}{R}. \qquad (10)$$

It gives the relation between the geometrical properties defining the electrical field. The roughest approximation of electric field resides in our replacing the outer cylindrical conductive surface with zero potential with conductive plane in the distance d. That is $r_n = d$. After using (9) on (7) we get a practical approximative formula for potential:

$$\varphi(r) = \frac{U_0}{\ln\frac{d}{R}}\ln\frac{d}{r} = \frac{U_0}{K}\ln\frac{d}{r}\,. \qquad (11)$$

For the approximative electric field strength of single infinite wire in the distance r from its axis we can use the Gauss law to get a similar formula:

$$E_r(r) = \frac{U_0}{K}\left[\frac{1}{r}\right]. \qquad (12)$$

When we apply the mirror method, which replaces the infinite conductive plane, after twice applying (12) we get a following formula for strength:

$$E_r(r) = \frac{U_0}{K}\left[\frac{1}{r} - \frac{1}{2d-r}\right]. \qquad (13)$$

Axial component of the electric field strength (in direction of X-axis) is due to the symmetry (caused by the infinite wire length and infinite conductive surface) in both cases equal to zero. This also warrants the fulfillment of the boundary condition on the conductive surface, which requires the tangent component of the electric field to be equal to zero.

For the single wire of finite length (assuming the linear charge density on the wire is constant) we can derive the following formula for the radial component of the electric field strength:

$$E_r(r,x) = \frac{U_0}{2K}\frac{1}{r}\left[\frac{a_p + x}{\sqrt{(a_p + x)^2 + r^2}} + \frac{a_p - x}{\sqrt{(a_p - x)^2 + r^2}}\right], \qquad (14)$$

where the constant

$$a_p = \frac{a}{2} \qquad (15)$$

represents half of the wire length and constant K was defined above (10).

The centre of the system of coordinates (zero value) is in the middle of the wire axis, the x coordinate represents the position on the wire axis and r represents the distance from the wire axis. So the field strength now depends on both coordinates.

If we consider a wire of finite length suspended in a parallel way above a conductive plane, the formula (14) for radial component of the electric field strength changes to:

$$E_r(r,x) = \frac{U_0}{2K}\frac{1}{r}\left[\frac{a_p + x}{\sqrt{(a_p + x)^2 + r^2}} + \frac{a_p - x}{\sqrt{(a_p - x)^2 + r^2}} - \frac{a_p + x}{\sqrt{(a_p + x)^2 + (2d-r)^2}} - \frac{a_p - x}{\sqrt{(a_p - x)^2 + (2d-r)^2}}\right]. \qquad (16)$$

In the case of finite length wire there also exists an axial component of the electric field strength – in the direction of wire axis (or the X-axis) in the examined plane XZ, see Fig. 1. For the single wire we can derive the following formula for the axial component of strength:

$$E_r(r,x) = \frac{U_0}{2K}\left[\frac{1}{\sqrt{(a_p + x)^2 + r^2}} - \frac{1}{\sqrt{(a_p - x)^2 + r^2}}\right]. \qquad (17)$$

The x coordinate represents the position on wire axis; the centre of the system of coordinates (zero value) is in the middle of the wire. It is clear, that the tangent component of the strength is equal to zero only in the point $x = 0$, that is in the centre of the wire axis projection.

For the finite length wire above the conductive plane the following formula for the axial component of the strength can be derived:

$$E_r(r,x) = \frac{U_0}{2K}\left[\frac{1}{\sqrt{(a_p + x)^2 + r^2}} - \frac{1}{\sqrt{(a_p - x)^2 + r^2}} - \frac{1}{\sqrt{(a_p + x)^2 + (2d-r)^2}} + \frac{1}{\sqrt{(a_p - x)^2 + (2d-r)^2}}\right]. \qquad (18)$$

Also in this case the tangent component of the strength is equal to zero only in the point $x = 0$, that is in the centre of the wire axis projection.

These simple models of electric field were derived to compute the weak dynamic force on asymmetrical capacitor with air dielectric. Positive ions are accelerated by strong electric filed in the vicinity of the wire. Because of the conservation of momentum law the accelerated particles exert a force on the wire. As the field is due to the conducting plane strongly asymmetrical, the individual forces are not nullified. (Free electrons move in the opposite direction.) In the present time we are preparing the theory of the force originating on the asymmetrical capacitor and the electrical field description will be its main part.

2 EXPERIMENTS

Measuring the electric field strength is one of the most complicated measurements, so it is really difficult to verify our theoretical results. There are several types of measuring probes, but they are all at least 20 x 20 mm in size, so they do not measure the point values, but only average value over certain area. That is why for our geometry those probes are simply too big and useless.

Because of the size of the probes we would have to use the experimental setup at least ten times bigger and maybe higher voltage too. Even in this case we would have to consider the size of the probe, eliminate other parasitic effects and maybe consider the conductive probe changing the shape of the field. We did not succeed to get a suitable probe, but we plan a direct experimental verification for the near future.

As was mentioned above our research is concerned with forces on asymmetrical capacitors. To find a formula describing relation between the force originating thereon and the electrical strength around the electrodes, we first needed a method of describing the electrical field. The above mentioned formulae were used for that purpose. We used them to theoretically compute the force on the asymmetrical capacitor (wire diameter 0.1 mm, larger electrode: 100 x 50 x 10 mm, wire distance 30 mm, see Fig. 2) and we compared this value with the one we experimentally measured. Measuring of this weak force is very complicated. However to decide between the individual models - not only for dynamic description of the force, but also for the electric field models - we need to reach high accuracy of measurement.

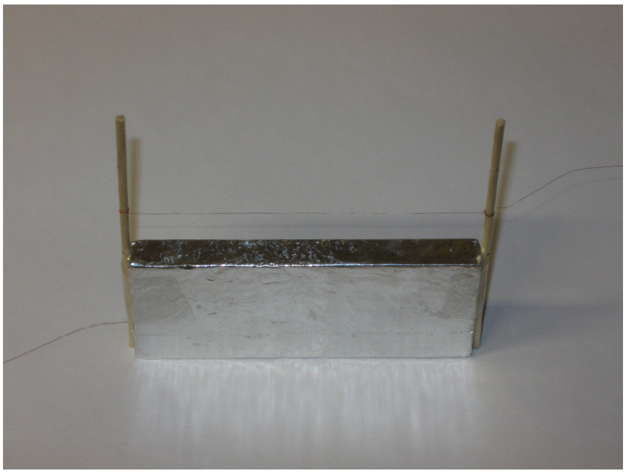

Fig.2: Model of Asymmetrical capacitor.

3 RESULTS

The calculations were made on the model for conductor with diameter of 0.1 mm, total length of 100 mm and distance from conducting plane of 30 mm. The applied voltage was 2 kV. The field was calculated only in the area containing the wire axis and normal to the conducting plane, since the key information is contained in this area. Standard graphs are used for field component description in the direction normal to the conducting plane and parallel to it. Logarithmic scales are often used, since the distance and field strength varies by several orders. Only graphs of physical or technical importance, or graphs with typical or illustrative contents are presented here, no systematic presentation of a lot of calculations is chosen.

The effect of conducting plane in the case of infinite length wire is in Fig. 3. The radial component changes only near the plane, but the change is very fast. Near the wire, the radial component of field strength decreases according to the law 1/*r*, where *r* is the distance from wire axis.

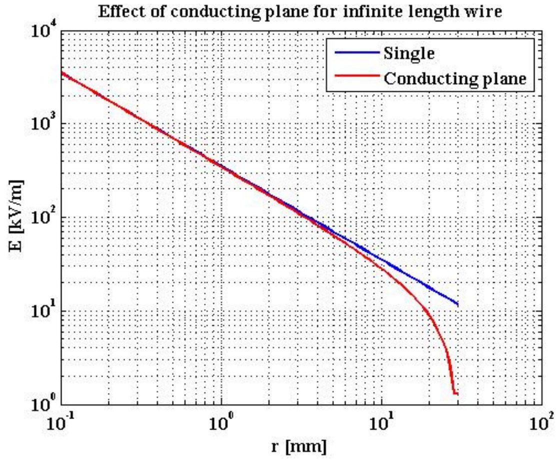

Fig.3: Effect of conducting plane on the field of infinite length wire.

The conductive plane removes the axial symmetry of the field and makes the field strongly asymmetric. It is confirmed in Fig. 4 that presents the radial component of the field strength on opposite wire sides. The curve "Down" shows the radial component between conductor and plane. The curve "Up" is for the radial field strength component on the opposite side of conductor, from conductor to empty space. In the opposite side the field strength decreases approximately according to the law 1/*r*.

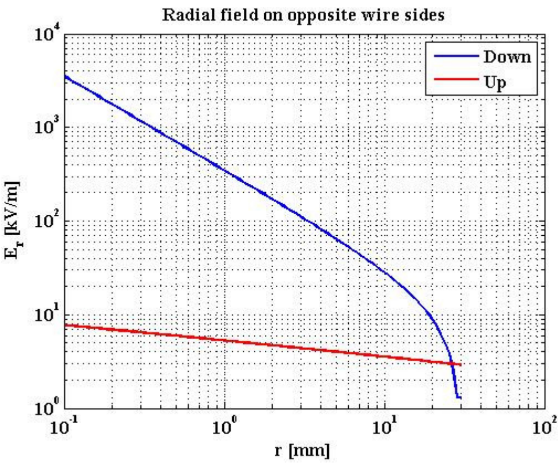

Fig.4: Comparison of radial fields on opposite conductor sides.

Effect of conducting plane is really very strong in the zone near the conductor. The difference is almost 3 orders at the conductor surface. This is very important to explain the workings of some special devices (e.g. asymmetrical capacitor).

In the case of infinite wire only a single coordinate, the distance *r* from wire axis, is used in the basic investigated area, see Fig. 1. The description of the field strength of finite length wire requires two coordinates, the distance *r* from wire axis in vertical direction and the distance *x* from wire centre in horizontal direction. A popular approach is to use surface graphs. Unfortunately, they exhibit only qualitative information. Therefore we prefer graphs with several parametric curves. For the field strength function in radial direction the relative distance of vertical line from wire centre is used. At the wire centre the relative position is 0 and at the wire end this parameters achieves value 1.

Typical results for the case of finite length conductor are in Fig. 5. The difference is only at the wire edge and in the zone near the conductive plane.

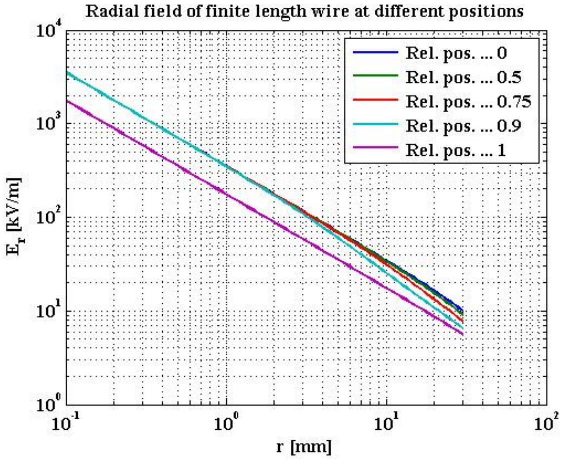

Fig.5: Radial component of single finite length conductor.

Comparison of the field of single infinite and finite length conductors is in Fig. 6. The fields are compared fields at the conductor edge. The extended radial distance is used in order to show differences.

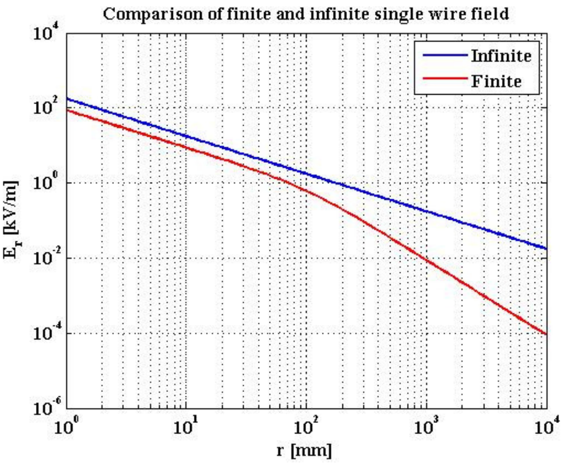

Fig.6: Comparison of radial field of finite and infinite length single conductor

It is evident from Fig. 6 that the slope of radial field strength for finite length conductor is different at different distances. The distance from conductor centre can be divided into three zones:

1. Near zone, in which the slope is the same and constant.
2. Middle zone, where the slope of field strength of finite length wire varies.
3. Far zone with constant slope but of value different from each other.

According to Fig. 6 the middle zone starts approximately at position r = 30 mm and stops at r = 200 mm.

Comparison of fields in the experimental equipment in Fig. 7 shows that in the experiment there is the near zone only. Therefore the definition of middle zone is acceptable. The single conductors are compared in Fig. 7.

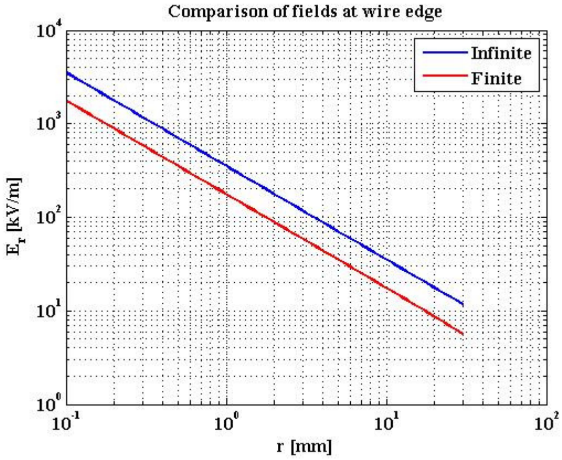

Fig.7: Effect of single wire length at the edge.

The comparison of field in the near zone and conductor centre is in Fig. 8. In this case the finite length has practically no effect.

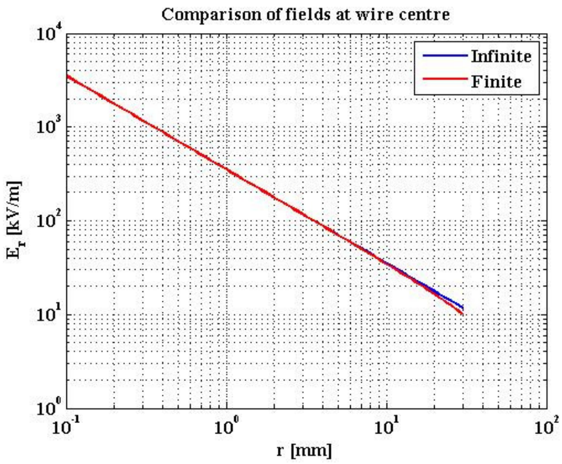

Fig.8: Effect of single wire length at the centre.

The effect of conducting plane of the finite length wire is very similar to that on infinite wire shown in Fig. 3. Therefore we omit the details here.

Comparison of all the approximations in the line going through conductor centre is in Fig. 9. The only different one among them is the approximation, representing the conducting plane of the finite wire length.

The same case for the wire end is in Fig. 10. The effect of wire length is in the entire near zone and the effect of conducting plane is only near its vicinity.

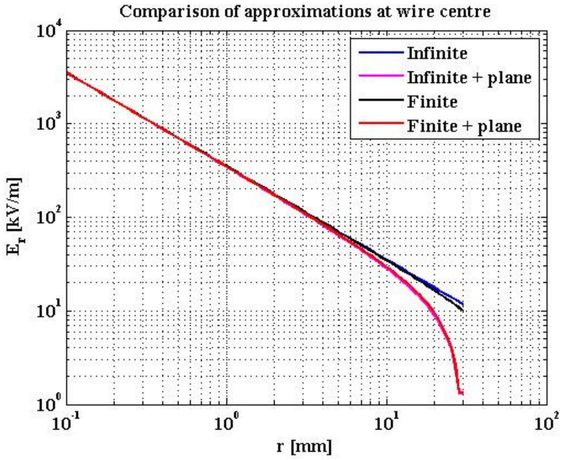

Fig.9: Comparison of all approximating methods for conductor centre.

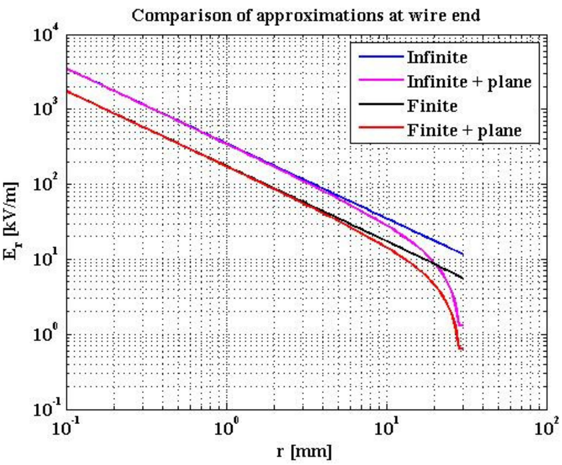

Fig.10: Comparison of all approximating methods for conductor centre.

In previous graphs (Fig. 3 to 10) the dependence of electric field strength on radius was shown and the distance *x* from conductor centre was the parameter. In order to get full information in next graphs the field strength on lines parallel to conductor axis will be presented and the distance from conductor axis will be the parameter.

Several curves exhibiting the field strength of finite length single wire are in Fig. 11. Very close to the conductor axis the field radial component is uniform along almost all the conductor length and then decreases rapidly. At large distances the field component is not uniform.

In Fig. 12 there is the comparison of single infinite and finite length conductor at relatively large distance from the conductor axis and near to the conducting plane. At large distance from conductor axis the effect of finite length is very large.

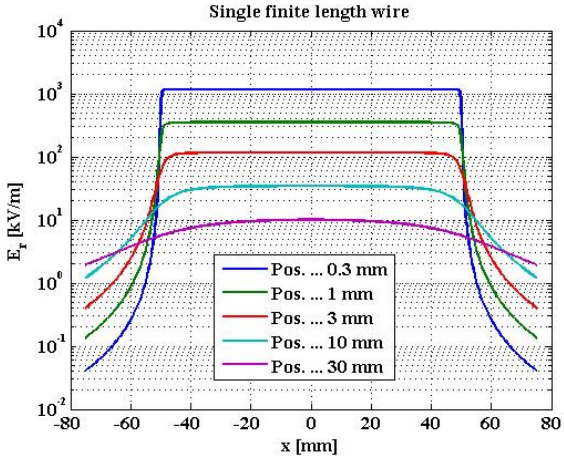

Fig. 11: Field strength of single wire of finite length.

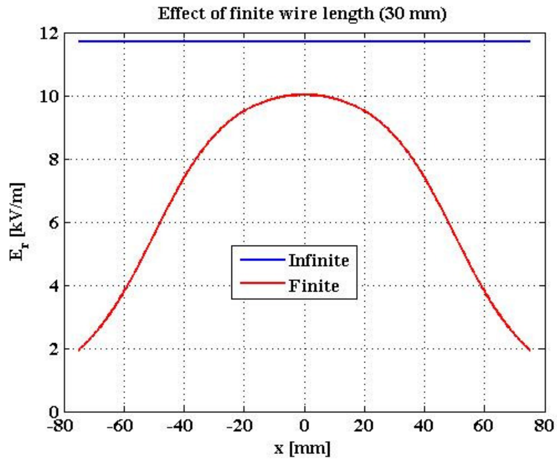

Fig. 12: Comparison of the field of single wire of finite and infinite length.

The radial field component in the presence of conducting plane is in Fig. 13. In comparison with Fig. 11 the field is more uniform and its decrease is more rapid.

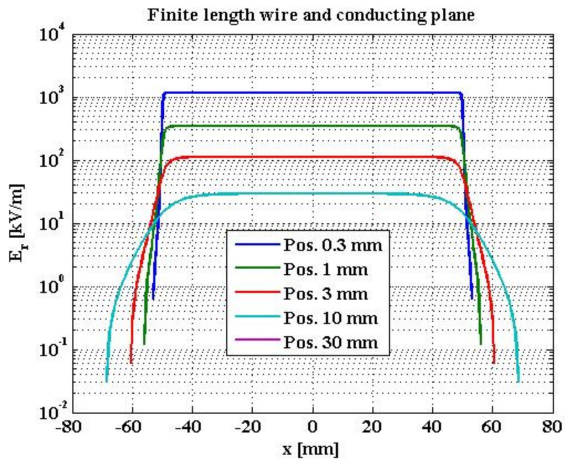

Fig. 13: Field strength of single wire of finite length parallel to the conducting plane.

The effect of conducting plane for finite length conductor is shown in Fig. 14. The field strength decreases, but the field is more uniform.

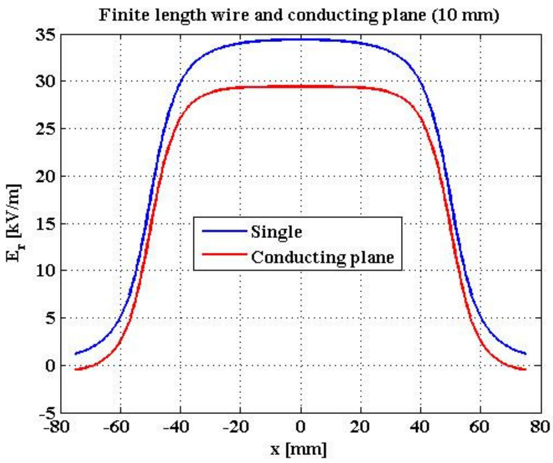

Fig. 14. The effect of conducting plane.

Comparison of all the approximations very near to the conductor is in Fig.15. Along the wire the difference is very small.

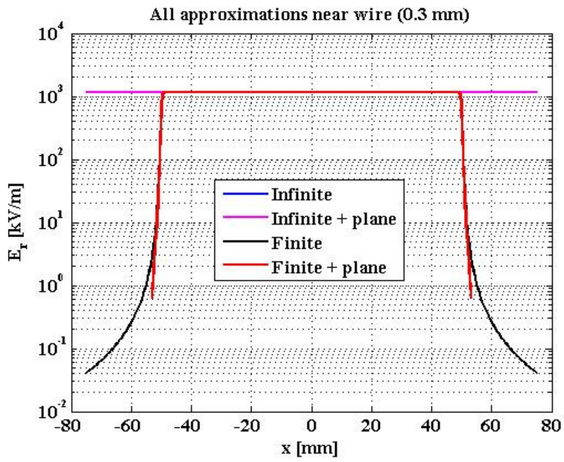

Fig. 15: Comparison of all the approximations near the conductor

Comparison of all the approximations relatively near the conducting plane is in Fig.16. In this case the differences are clearly evident.

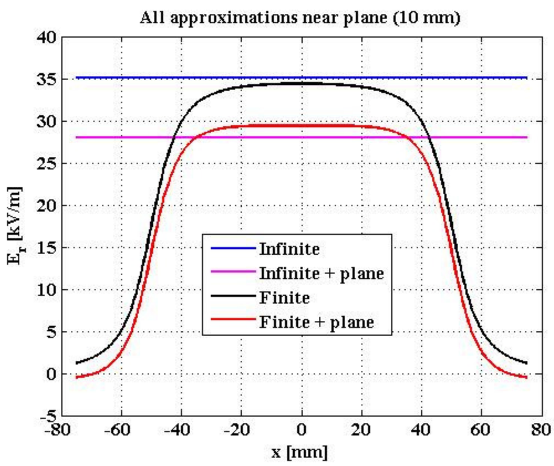

Fig. 16: Comparison of all the approximations far from the conductor

The finite length wire produces also the undesirable field with axial component. Axial (or tangential) component of the field strength along the radius is in Fig. 17 for the case of single conductor. Its value is very low near the conductor centre and achieves high value at its end.

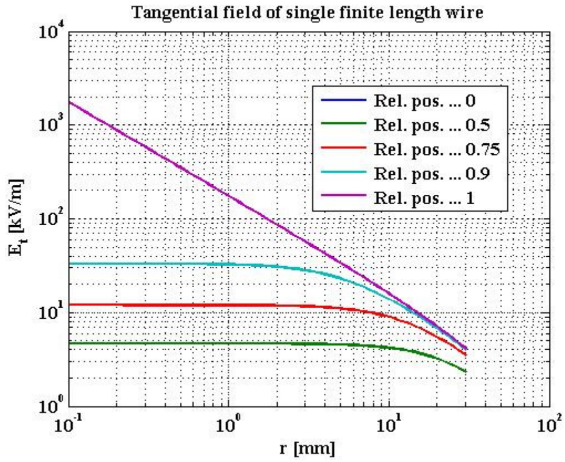

Fig. 16: Axial field strength component of single conductor along the radius.

The effect of conducting plane on the axial component is in Fig, 17. The change is visible only very near to the plane.

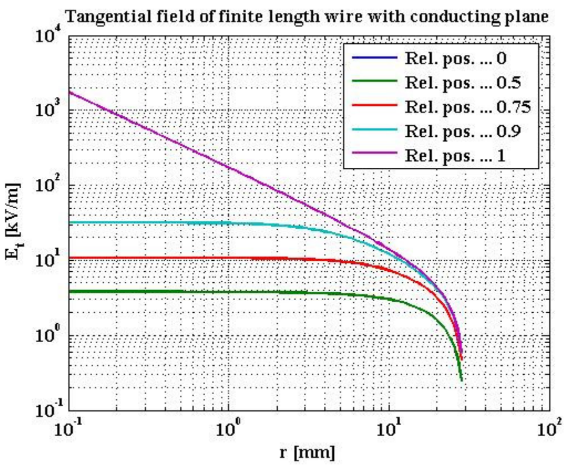

Fig. 17: Axial field strength component of conductor and plane along the radius.

The behavior of axial component on lines parallel to conductor axis is in next two Figures. In Fig. 18 there is the axial component of single conductor. The component is large only at wire ends.

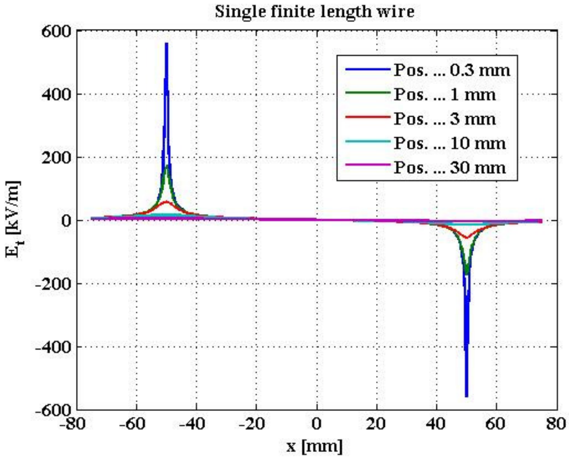

Fig. 18: Axial field strength component of single conductor on lines parallel to conductor axis.

The effect of conducting plane on the axial component is in Fig. 19. The only small change is for the curve very close to conducting plane.

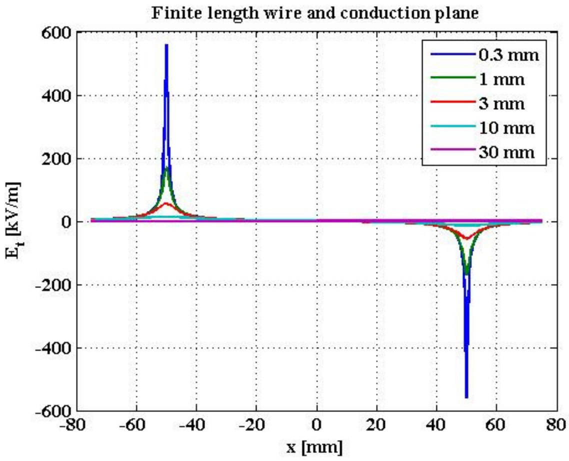

Fig. 19: Axial field strength component of conductor and plane on lines parallel to conductor axis.

4 Discussion

During the study basic simple analytical formulae were derived. All of them are only approximate formulae and were obtained by the use of several simplifications. The most important neglecting is in the consideration of the infinite conducting plane instead of real box of small finite dimensions. The second important neglecting is the constant linear density of charge on the finite length conductor. At the conductor end the charge density is in reality not uniform. The third neglecting is the replacement of the circular cross section conductor by the charged line. The finite diameter of conductor and presence of conducting plane leads to different surface charge density on the side near to plane and the opposite one. We must say that the models that allow analytical solution are the simplest first order models.

More accurate solution can be obtained by two ways. The simple way is the use of Finite Element Method. However, there are two disadvantages at least as it has been already mentioned in the part devoted to the theory. The first complication is in the difficult formulation of boundary conditions. The second difficulty is in the analysis of the results. To get graphs similar to the ones presented above needs a detailed knowledge of the FEM system.

The second improvement is to change the surface charge density on both the conductor surface and the finite dimension plane to get more exact solution. Unfortunately, due to finite dimensions of plane and the non-uniform charge density, the solution must be made only numerically, using the numerical integration. Numerical integration can be programmed relatively easily (e.g. in MATLAB) and the computing is very fast on modern computers. So this is not a problem. But the problem lies in the fact, that there is no straightforward way how to find the correct charge density.

Irrespective of approximate solution, the achieved results are interesting and important both from the physical and practical point of view. For praxis the results presented in the form of graphs help the better understanding the function of real devices.

The above mentioned formulae derived for all types of models revealed that especially for the area close to the wire the results are practically the same. Therefore the simplest model of single wire is fully sufficient and thus the formula (7) or (8) can be used. The model can be used with reasonable accuracy in other cases too.

The results obtained from the models in region close to the conducting plane are not in perfect agreement with reality, if we focus to the axial component of electric field. This component must be zero on the conducing plane. Fig. 19 shows that there is small but nonzero axial component at distance of 30 mm, e.g. at the conducting plane. It is due the model simplicity. Further improvements require more correct charge distribution, which is very difficult to make. The numerical integration is necessary in this case.

Very important fact for the praxis is the strong asymmetry of radial component on opposite sides of the wire, as it follows from Fig. 4. The field strength on the area between wire and plane is higher by several orders. The arrangement realizes the asymmetrical capacitor. If we consider the force due to the acceleration of ions, the force acting on the wire is much higher in the area between wire and plane. The weight of the capacitor decreases. The force was measurable in our experiments.

Graphs on the Fig. 6 are important from the physical point of view. It was already mentioned that three zones of the field of finite length conductor exist. In near zone very close to the conductor the field decreases by the first negative power of distance, $1/r$, which is the field of infinite length wire. Really, in the near zone the conductor can be treated as an infinite one. In the far zone the radial component of the field decreases by the second negative power of distance, $1/r^2$. It corresponds to the field of point charge. At large distance from the finite length wire its dimensions are negligible and it acts as a point charge. In the middle zone no approximation is valid.

The boundary of zones depends on the wire length. Fig. 6 gives distances for the used arrangement. In our experiment only near zone is to be considered.

It has been already mentioned that in special cases (computing the intensity near the ends of the wire) we have to turn to more complicated models. But as there is always some parasitic effect present in the application of electrostatic field, it is hardly ever decidable whether the deviations are due to the parasitic effect or the model.

There are at least two reasons, why the experimental confirmation of the theory is difficult. Many disturbing effects make the measurement accuracy low. The size of the electrostatic field probe is relatively big; therefore, the measurement on actual arrangement is not possible. At least 10 times larger experimental setup should be made in order to perform reliable measurement in principle. Even after the dimension increase of the experimental model the integrating effect of the probe should be considered. The experimental arrangement is now in the design stage but there is a complication to get a proper probe for the relatively accurate electrostatic field measurement.

5 CONLUSIONS

Several very simple analytical models were prepared for the modeling of experimental device. Simple formulae derived from the models allowed illustrative and detailed description of the field. The qualitative agreement with reality is perfect and in important areas also a good quantitative agreement exists especially in the area very close to the conductor. Since experimental verification is difficult, further refinement of the models is not necessary in present time.

Analytical formulae were used in the derivation of the simple approximated formula for the force acting on the wire in asymmetric capacitor. The formula was verified by preliminary experiments.

We were unable to directly verify the formulae experimentally, but we have found an agreement between the theoretically computed force and experimentally measured force on the asymmetrical capacitor. Thus we have a reason to believe that the above derived formulae are correct and they can be used in formulae for the experimental device. Our next work is heading to complete the simple but sufficient model of dynamic force on the asymmetrical capacitor and to find more precise experimental methods of measuring the force. This in turn can be used to verify the dynamic force model and also electric field models.

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