

# POWER LOSSES OF THE MONOPHASE COAXIAL HIGH CURRENT BUSDUCT

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**Abstract:** In the paper are presented the calculations of the active and reactive power in the tubular screen of the monophasic single-pole high current busduct. Into account were taken skin and internal and external proximity effect. Calculations were made using the Poynting theorem and Joule-Lenz law.

**Key words:** High current busduct, tubular busbar, electromagnetic field, active power, reactive power

## INTRODUCTION

The most popular solution of the high current busduct is single-pole coaxial high current busduct fig. 1.

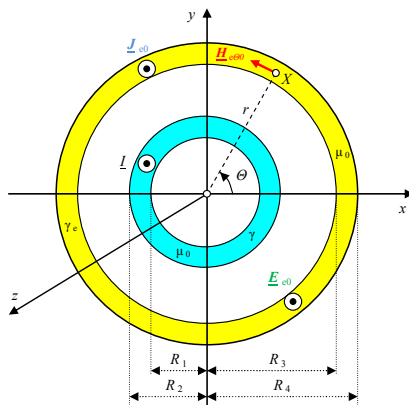


Fig. 1. Single-pole high current busduct

Design of the high current busducts on high currents and voltages causes necessity precise describing of electromagnetic, dynamic and thermal effects. Knowledge of the relations between electrodynamics and constructional parameters is necessary in the optimization construction process of the high current busducts. Mathematics analysis of electromagnetic effects in the high current busducts is rather complicated. In devices these type, the active power emits into phase conductors and screens [2].

Information about distribution electromagnetic field and power losses is a base into analysis of electrodynamics and thermal effects in the high current busducts. Correct determination of the electrodynamics parameters has huge practical meaning into analysis of work of the every types of electrical devices. Determination of the power losses into high current busducts let calculate the temperature these devices, which is a basic constructional parameter.

In the analysis of the electromagnetic effects in the high current busducts should be taken the shape of the phase conductors and screen. Besides into account should be taken all mutual couplings between conductors and screen. Into account should be the taken eddy currents induced on the screens, skin and proximity effect and coupling of the electromagnetic field with temperature [2].

## 1. ELECTROMAGNETIC FIELD

Let us consider the electromagnetic field in the tubular screen with conductivity  $\gamma_e$ , with internal radius  $R_3$  and external radius  $R_4$  parallel to the coaxial internal conductor with conductivity  $\gamma$ , internal radius  $R_1$  and external radius  $R_2$  with complex rms current  $\underline{I}$  - fig. 1.

The alternating magnetic field generated by phase current  $\underline{I}$  induces on the screen the eddy currents about density  $\underline{J}_{e0}$  - proceeds the internal proximity

effect. On account of cylindrical symmetry of the system, this current density has one component along  $Oz$  axis and is function  $r$  variable of the cylindrical coordinate system, thus  $\underline{J}_{e0}(r) = \mathbf{1}_z J_{e0}(r)$ . This current density fulfills the Helmholtz's equation, which in the cylindrical coordinate system becomes the Bessel's equation [2]

$$\frac{d^2 \underline{J}_{e0}(r)}{dr^2} + \frac{1}{r} \frac{d\underline{J}_{e0}(r)}{dr} - \underline{\Gamma}_e^2 \underline{J}_{e0}(r) = 0 \quad (1)$$

where the complex propagation constant of electromagnetic wave in the screen  $\underline{\Gamma}_e = \sqrt{2j} k_e$  in which the attenuation constant  $k_e = \sqrt{\frac{\omega \mu_0 \gamma_e}{2}} = \frac{1}{\delta_e}$ .

From the above formula we obtain

$$\underline{J}_{e0}(r) = \underline{C}_0 I_0(\underline{\Gamma}_e r) + \underline{D}_0 K_0(\underline{\Gamma}_e r) \quad (2)$$

where  $\underline{C}_0$  and  $\underline{D}_0$  are integral constants.

In the above formulas  $I_n(\underline{\Gamma}_e r)$  and  $K_n(\underline{\Gamma}_e r)$  are modified Bessel's functions accordingly first and second kind, of  $n$ -th order.

The magnetic field has only one component, i.e.  $\underline{H}_{e0} = \mathbf{1}_\theta \underline{H}_{e0\theta}$ , which on the basis the second Maxwell's equation, given is by the formula

$$\underline{H}_{e0\theta}(r) = \frac{1}{\underline{\Gamma}_e} \frac{d\underline{J}_{e0}(r)}{dr} = \frac{1}{\underline{\Gamma}_e} [\underline{C}_0 I_1(\underline{\Gamma}_e r) - \underline{D}_0 K_1(\underline{\Gamma}_e r)] \quad (3)$$

Constants  $\underline{C}_0$  and  $\underline{D}_0$  we will determine from boundary conditions

$$\underline{H}_{e0\theta}(r = R_1) = \frac{I}{2\pi R_1} \quad \text{and} \quad \underline{H}_{e0\theta}(r = R_2) = \frac{I}{2\pi R_2} \quad (4)$$

and then we obtain the current density

$$\underline{J}_{e0}(r) = \frac{\underline{\Gamma}_e I}{2\pi R_3} j_{e0}(r) = J_{e0}(r) \exp[\varphi_{J_{e0}}(r)] \quad (5)$$

where

$$j_{e0}(r) = \frac{b_0 I_0(\underline{\Gamma}_e r) + c_0 K_0(\underline{\Gamma}_e r)}{d_0}$$

$$b_0 = \beta_e K_1(\underline{\Gamma}_e R_3) - K_1(\underline{\Gamma}_e R_4)$$

$$c_0 = \beta_e I_1(\underline{\Gamma}_e R_3) - I_1(\underline{\Gamma}_e R_4)$$

and  $d_0 = I_1(\underline{\Gamma}_e R_4) K_1(\underline{\Gamma}_e R_3) - I_1(\underline{\Gamma}_e R_3) K_1(\underline{\Gamma}_e R_4)$

and parameter  $\beta_e = \frac{R_3}{R_4}$  where  $(0 \leq \beta_e \leq 1)$ .

The electric field strength  $\underline{E}_{e0}(r) = \frac{1}{\gamma_e} \underline{J}_{e0}(r)$ , is a function of variable  $r$  the cylindrical coordinates system and has one component along  $Oz$  axis, thus  $\underline{E}_{e0}(r) = \mathbf{1}_z \underline{E}_{e0}(r)$ , where

$$\underline{E}_{e0}(r) = \frac{\underline{\Gamma}_e I}{2\pi \gamma_e R_3} j_{e0}(r) = E_{e0}(r) \exp[\varphi_{E_{e0}}(r)] \quad (6)$$

From formula (5)

$$\underline{H}_{e0\theta}(r) = \frac{I}{2\pi R_3} h_{e0}(r) = H_{e0}(r) \exp[\varphi_{H_{e0}}(r)] \quad (7)$$

where

$$h_{e0}(r) = \frac{b_0 I_1(\underline{\Gamma}_e r) - c_0 K_1(\underline{\Gamma}_e r)}{d_0}$$

## 2. ACTIVE AND REACTIVE POWER IN THE SCREEN

Stream of the complex power penetrates into area of the tubular screen through his border is expresses by formula

$$\underline{S}_{e0}(r) = \underline{E}_{e0}(r) \times \underline{H}_{e0}^*(r) = \mathbf{1}_z \underline{E}_{e0}(r) \times \mathbf{1}_\theta \underline{H}_{e0\theta}^*(r) \quad (8)$$

Complex apparent power of the screen

$$\underline{S}_{e0} = -\oint_S [\underline{E}_{e0}(r) \times \underline{H}_{e0}^*(r)] \cdot d\mathbf{S} = P_{e0} + j Q_{e0} \quad (9)$$

If we take into account formulas (8) and (9) then

$$\underline{S}_{e0} = \frac{\underline{\Gamma}_e l I^2}{2\pi \gamma_e R_3} \times \frac{b_0 [I_0(\underline{\Gamma}_e R_4) - I_0(\underline{\Gamma}_e R_3)] + c_0 [K_0(\underline{\Gamma}_e R_4) - K_0(\underline{\Gamma}_e R_3)]}{d_0} \quad (10)$$

In the above formula we can not distinguish of real part (active power) and imaginary part (reactive power) because this equation contains complex Bessel's functions and complex propagation constant. Therefore we will calculate the active power from the formula

$$P_{e0} = \iiint_V \frac{1}{\gamma_e} \underline{J}_{e0}(r) \underline{J}_{e0}^*(r) dV \quad (11)$$

From the above formula we obtain

$$P_{e0} = \frac{\underline{\Gamma}_e^* l I^2}{4\pi \gamma_e \beta_e^2 R_4} \frac{a_0}{d_0 d_0^*} \quad (12)$$

where

$$a_0 = b_0 b_0^* \left\{ I_0^*(\underline{\Gamma}_e R_4) I_1(\underline{\Gamma}_e R_4) + j I_0(\underline{\Gamma}_e R_4) I_1^*(\underline{\Gamma}_e R_4) - \right. \\ \left. - \beta_e [I_0^*(\underline{\Gamma}_e R_3) I_1(\underline{\Gamma}_e R_3) + j I_0(\underline{\Gamma}_e R_3) I_1^*(\underline{\Gamma}_e R_3)] \right\} - \\ - c_0 c_0^* \left\{ K_0^*(\underline{\Gamma}_e R_4) K_1(\underline{\Gamma}_e R_4) + j K_0(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_4) - \right. \\ \left. - \beta_e [K_0^*(\underline{\Gamma}_e R_3) K_1(\underline{\Gamma}_e R_3) + j K_0(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_3)] \right\} - \\ - c_0 b_0^* \left\{ I_0^*(\underline{\Gamma}_e R_4) K_1(\underline{\Gamma}_e R_4) - j K_0(\underline{\Gamma}_e R_4) I_1^*(\underline{\Gamma}_e R_4) - \right. \\ \left. - \beta_e [I_0^*(\underline{\Gamma}_e R_3) K_1(\underline{\Gamma}_e R_3) - j K_0(\underline{\Gamma}_e R_3) I_1^*(\underline{\Gamma}_e R_3)] \right\} + \\ + b_0 c_0^* \left\{ I_1(\underline{\Gamma}_e R_4) K_0^*(\underline{\Gamma}_e R_4) - j I_0(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_4) - \right. \\ \left. - \beta_e [I_1(\underline{\Gamma}_e R_3) K_0^*(\underline{\Gamma}_e R_3) - j I_0(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_3)] \right\}$$

$$\underline{b}_0^* = \beta_e K_1^*(\underline{\Gamma}_e R_3) - K_1^*(\underline{\Gamma}_e R_4)$$

$$\underline{c}_0^* = \beta_e I_1^*(\underline{\Gamma}_e R_3) - I_1^*(\underline{\Gamma}_e R_4)$$

and  $\underline{d}_0^* = I_1^*(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_3) - I_1^*(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_4)$

The reactive power emitted on the internal reactance of the tubular screen we determine from formula (9), yielding

$$Q_{e0} = -j \frac{l I^2}{2 \pi \gamma_e R_3 \underline{d}_0} \times \left\{ \underline{\Gamma}_e \left[ \frac{\underline{b}_0 (I_0(\underline{\Gamma}_e R_4) - I_0(\underline{\Gamma}_e R_3)) + \underline{c}_0 (K_0(\underline{\Gamma}_e R_4) - K_0(\underline{\Gamma}_e R_3))}{2 \beta_e \underline{d}_0^*} \right] \right\} \quad (13)$$

The active (12) and reactive (13) power are real numbers although they are expressed by complex numbers and complex Bessel's functions. We can prove this using properties of Bessel's functions or making numerical calculations.

If we introduce the parameter binding frequency, conductivity and transverse dimensions of the tubular screen, i.e.  $\alpha_e = \frac{R_4}{\delta_e} = k_e R_4$  then the active power with taking into account the internal proximity effect is expressed by formula

$$P_{e0} = \frac{\sqrt{-2j} \alpha_e l I^2}{4 \pi \gamma_e \beta_e^2 R_4^2} \frac{\underline{a}_0}{\underline{d}_0 \underline{d}_0^*} \quad (14)$$

and reactive power

$$Q_{e0} = -j \frac{\sqrt{2j} \alpha_e l I^2}{2 \pi \gamma_e \beta_e R_4^2 \underline{d}_0} \times \left\{ \begin{aligned} & \left[ \frac{\underline{b}_0 [I_0(\sqrt{2j} \alpha_e) - I_0(\sqrt{2j} \alpha_e \beta_e)] + \underline{c}_0 [K_0(\sqrt{2j} \alpha_e) - K_0(\sqrt{2j} \alpha_e \beta_e)]}{2 \beta_e \underline{d}_0^*} \right] + j \frac{\underline{a}_0}{2 \beta_e \underline{d}_0^*} \end{aligned} \right\} \quad (15)$$

and complex apparent power

$$\underline{S}_{e0} = \frac{\sqrt{2j} \alpha_e l I^2}{2 \pi \gamma_e \beta_e R_4^2} \times \left\{ \begin{aligned} & \left[ \frac{\underline{b}_0 [I_0(\sqrt{2j} \alpha_e) - I_0(\sqrt{2j} \alpha_e \beta_e)] + \underline{c}_0 [K_0(\sqrt{2j} \alpha_e) - K_0(\sqrt{2j} \alpha_e \beta_e)]}{\underline{d}_0} \right] + j \frac{\underline{a}_0}{\underline{d}_0} \end{aligned} \right\} \quad (16)$$

If we introduce the reference power

$$P_{0ew} = \frac{l I^2}{\pi \gamma_e (R_4^2 - R_3^2)} = \frac{l I^2}{\pi \gamma_e (1 - \beta_e^2) R_4^2} \quad (17)$$

Then the active power lost in the screen as a result of the internal proximity effect, we can express as coefficient

$$k_{e0}^{(P)} = \frac{P_{e0}}{P_{0ew}} = \frac{\sqrt{-2j} \alpha_e (1 - \beta_e^2)}{4 \beta_e^2} \frac{\underline{a}_0}{\underline{d}_0 \underline{d}_0^*} \quad (18)$$

Dependence of the above coefficient on parameter  $\alpha_e$  for different values of the relative thickness  $\beta_e$  wall of the tubular screen presents figure 2.

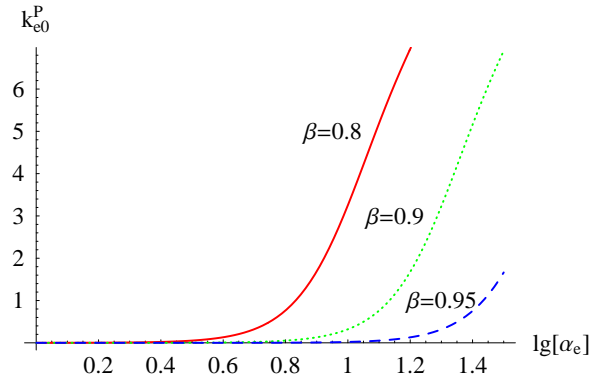


Fig. 2. Dependence of relative active power lost in the tubular screen on parameter  $\alpha_e$

In the paper [1], author derived a formula on the internal inductance  $\mathcal{L}_{0ew}$  of the tubular conductor without taking into account skin effect. From here the internal reactance of the tubular screen  $\mathcal{X}_{0ew} = \omega \mathcal{L}_{0ew}$  and then we can introduce the reactive power of reference

$$Q_{0ew} = \mathcal{X}_{0ew} I^2 = \omega \frac{\mu_0 l}{2 \pi} \left[ \frac{R_3^4}{(R_4^2 - R_3^2)^2} \ln \frac{R_4}{R_3} - \frac{1}{4} \frac{3R_3^2 - R_4^2}{R_4^2 - R_3^2} \right] I_1^2 = \quad (19)$$

$$= \frac{\alpha_e^2 l I^2}{\pi \gamma_e R_4^2} \left[ \frac{\beta_e^4}{(1 - \beta_e^2)^2} \ln \frac{1}{\beta_e} - \frac{1}{4} \frac{3\beta_e^2 - 1}{1 - \beta_e^2} \right]$$

Then the reactive power we can express as a relation

$$k_{e0}^{(Q)} = \frac{Q_{e0}}{Q_{0ew}} \quad (20)$$

Dependence of the above coefficient on parameter  $\alpha_e$  for different values of the relative thickness  $\beta_e$  wall of the tubular screen presents figure 3.

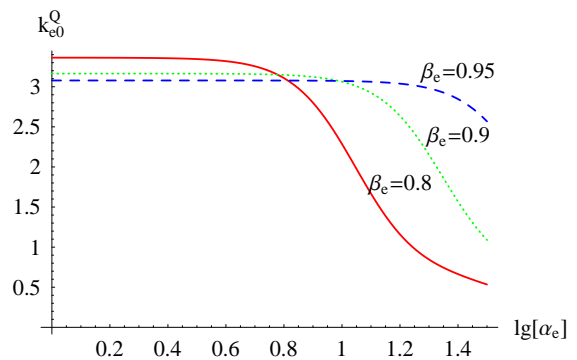


Fig. 3. Dependence of relative reactive power lost in the tubular screen on parameter  $\alpha_e$

Internal inductance of the tubular screen has a form [1]

$$\underline{e}_{ew} = \frac{\mu_0 l}{2\pi} \left\{ \frac{1}{\sqrt{2j} \alpha_e \underline{b}_{ew}} \left[ K_1(\sqrt{2j} \alpha_e \beta_e) I_0(\sqrt{2j} \alpha_e) + I_1(\sqrt{2j} \alpha_e \beta_e) K_0(\sqrt{2j} \alpha_e) \right] - \frac{\underline{a}_{ew}}{2 \underline{b}_{ew}^*} \right\} \quad (21)$$

where

$$\begin{aligned} \underline{a}_{ew} &= K_1(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_3) \left[ \frac{I_0(\underline{\Gamma}_e R_4) I_1^*(\underline{\Gamma}_e R_4) -}{-j I_1(\underline{\Gamma}_e R_4) I_0^*(\underline{\Gamma}_e R_4)} \right] - \\ &- I_1(\underline{\Gamma}_e R_3) I_1^*(\underline{\Gamma}_e R_3) \left[ \frac{K_0(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_4) -}{-j K_1(\underline{\Gamma}_e R_4) K_0^*(\underline{\Gamma}_e R_4)} \right] + \\ &+ I_1(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_3) \left[ \frac{K_0(\underline{\Gamma}_e R_4) I_1^*(\underline{\Gamma}_e R_4) +}{+j K_1(\underline{\Gamma}_e R_4) I_0^*(\underline{\Gamma}_e R_4)} \right] - \\ &- K_1(\underline{\Gamma}_e R_3) I_1^*(\underline{\Gamma}_e R_3) \left[ \frac{I_0(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_4) +}{+j I_1(\underline{\Gamma}_e R_4) K_0^*(\underline{\Gamma}_e R_4)} \right] \\ \underline{b}_{ew} &= I_1(\underline{\Gamma}_e R_4) K_1(\underline{\Gamma}_e R_3) - I_1(\underline{\Gamma}_e R_3) K_1(\underline{\Gamma}_e R_4) \end{aligned}$$

$$\text{and } \underline{b}_{ew}^* = I_1^*(\underline{\Gamma}_e R_4) K_1^*(\underline{\Gamma}_e R_3) - I_1^*(\underline{\Gamma}_e R_3) K_1^*(\underline{\Gamma}_e R_4)$$

We should add that the total reactive power in the tubular screen is a sum of the above determined reactive power connected with the internal inductance and of the reactive power connected with external inductance [1]

$$\underline{e}_z = \frac{\mu_0 l}{2\pi} \left( \ln \frac{2l}{R_4} - 1 \right) \quad (22)$$

Hence the external reactive power

$$Q_z = \mathcal{R}_z I^2 = \omega \underline{e}_z I^2 = \omega \frac{\mu_0 l}{2\pi} \left( \ln \frac{2l}{R_4} - 1 \right) I^2 \quad (23)$$

and do not depend from skin effect. The above power on the unit length of conductor  $\frac{Q_z}{l}$  do not depend on length  $l$ , we can not take it, like a external inductance  $\frac{\underline{e}_z}{l}$  on the unit length, as a correct defined. Although this quantity has certain practical meaning into determination of the reactive power in the transmission lines.

This reactive power we can compare with the internal reactive power as a coefficient

$$k_{ez0}^{(Q)} = \frac{Q_z}{Q_{e0}} \quad (24)$$

Dependence of the above coefficient on parameter  $\alpha_e$  for different values of parameter  $\beta_e$  and  $\eta_e = \frac{l}{R_4}$  presents figure 4.

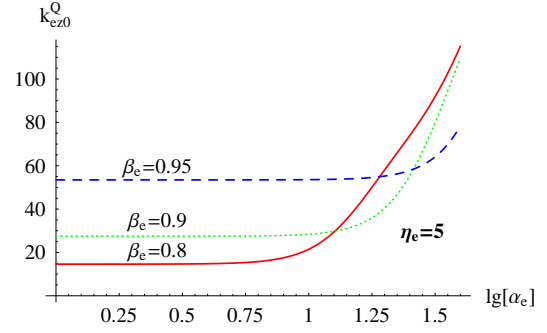


Fig. 4. Dependence of the relative external reactive power in the tubular screen on parameter  $\alpha_e$  for constant value  $\eta_e$  and different values of parameter  $\beta_e$

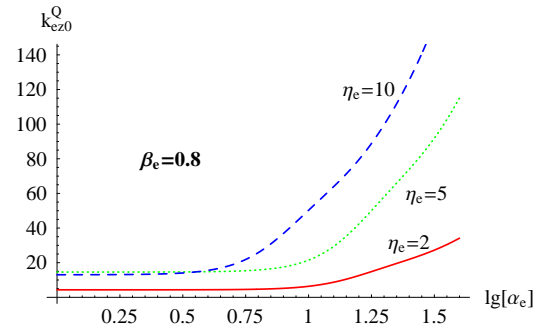


Fig. 5. Dependence of the relative external reactive power in the tubular screen on parameter  $\alpha_e$  for constant parameter  $\beta_e$  and different values of  $\eta_e$

### 3. CONCLUSIONS

Presented research show that about total active and reactive power of single-pole high current busduct decide skin and proximity effect.

Active power (fig. 2) in the coaxial screen increases with rise of parameter  $\alpha_e$ . It is caused the rise of resistance as a result of internal proximity effect. In turn the reactive power (fig 3) decreases with rise of parameter  $\alpha_e$ . It is connected with decreasing of inductance as a result of skin effect. Along with increase of parameter  $\alpha_e$  the internal inductance drops because the effective thickness of walls drops too.

Figures 4 and 5 show that the reactive power connected with mutual and external inductances is tens bigger than the reactive power connected with internal inductances.

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