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# SELECTED PROBLEMS OF COOPERATION BETWEEN ASYMMETRIC RECEIVERS AND SYNCHRONOUS MACHINE IN A LOW-VOLTAGE NETWORK OF AN INDUSTRIAL PLANT

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**Abstract**: *The paper presents an analysis of the effect of a group of asymmetric receivers of R,L type on the battery of static capacitors and a synchronous machine connected to a common low-voltage network of an industrial plant, supplied from an own transformer. In result of the analysis a proposal of a limit of the degree of the receiver asymmetry is formulated with a view to prevent harmful effect on the synchronous machine and the battery of capacitors. The analysis considers the influence of the asymmetry on the currents flowing in the transformer, the synchronous machine, and the capacitor.*

**Key words**: electrical machines, synchronous generator, asymmetry of currents, asymmetric loads of generators

## INTRODUCTION

In low-voltage power networks of industrial plants the unsymmetrical loads usually occur, both in one and two phases. Any unsymmetrical load is conductive not only to voltage asymmetry of the net but also asymmetric current propagation in all the devices connected to it. As the asymmetrical conditions usually disadvantageously affect operation of the equipment, the regulations in force constrain the allowable level of the asymmetry degree both with regard to voltage and current. Diagram of the supplying circuit of RLC-type receivers and a synchronous machine in an industrial plant is shown in Fig. 1.

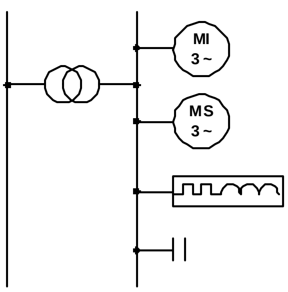


*Fig. 1. Diagram of the supplying circuit of RLC-type receivers and a synchronous machine in an industrial plant*

Taking into account the expected particularly single-phase load asymmetry, the transformers supplying such networks should include only the connection systems of low susceptibility to asymmetric single-phase load. Although the transformers of appropriate connection arrangement well resist the asymmetric loads, they give rise to voltage asymmetry in their secondary circuit. This, in turn, is conducive to further growth of asymmetry, due to the load asymmetry. Consequences of the voltage asymmetry are particularly disadvantageous for three-phase AC motors, especially the synchronous ones. Such motors are recommended to be used in industrial networks because of their higher usefulness for reactive power compensation as compared to commonly used capacitor batteries. The capacitor batteries poorly resist higher voltage harmonics and, at the same time, reply with large alterations of reactive power to voltage deviations from the rated level. In case of large voltage asymmetry the capacitor batteries are additionally endangered to the currents of the third harmonic components, that may be ignored in case of symmetry of the voltage. The paper considers a case of supplying a low-voltage network including a synchronous motor and a group of R,L,C receivers with a medium/low voltage transformer. The C-type receiver composed of the battery of capacitors is a symmetrical three-phase receiver supplied without a neutral conductor. A synchronous motor is similarly supplied without a neutral conductor. On the other hand, the R- and L-type receivers are connected in asymmetric arrangement with a neutral conductor. It was assumed that the medium/low voltage transformer is supplied with symmetric three-phase voltage without the neutral conductor. Since the asymmetric R,L receivers impose asymmetric voltage in a low-voltage network, the allowable degree of asymmetry of the R,L receiver should be investigated accordingly, with a view to prevent exceeding the voltage asymmetry degree allowable for a synchronous machine. The degree is determined by the negative- to positive-sequence voltage ratio $U_2/U_1$ that should not exceed 0.02 or even 0.01. Moreover, it was checked whether for the allowable degree of voltage asymmetry the factors of negative- to positive-sequence current ratio $I_2/I_1$ and negative- to rated current ratio $I_2/I_1$ remain in the range allowable for the synchronous machine. This ratio must must not exceed 0.1.

## 2. BASIC EQUATIONS SERVING FOR ANALYZING THE SYSTEM OPERATIONAL CONDITIONS

Analysis of the problem is carried out with the method of symmetrical components. In order to solve the problem the knowledge of voltage symmetrical components of the secondary transformer side $U_{t1}$, $U_{t2}$, and $U_{t0}$ is required. In order to calculate the components three groups of equations have been generated: i.e. the transformer, the synchronous machine, and the receiver equations.

The group of the transformer equations serves for calculation of symmetrical components of the secondary side voltage, i.e. the positive sequence component

$$U_{tl} = U_{tp} - Z_z I_{tl}$$

where $U_{tp}$ is the transformer supply voltage (the upper voltage side) converted to the lower voltage side.

- the negative sequence component

$$U_{t2} = -Z_z I_{t2}$$

and the negative sequence component

$$U_{t0} = -Z_\mu I_{t0}$$

In these equations $Z_z$ and $Z_\mu$ are transformer impedances, while $I_{t1}$, $I_{t2}$, and $I_{t0}$ are symmetrical components of the transformer currents, calculated as the sums of symmetrical current components of the synchronous machine $I_{m1}$, $I_{m2}$, and $I_{m0}$ and the symmetrical current components of the load $I_1$, $I_2$, and $I_0$. The currents of symmetrical components of the synchronous machine have been calculated from the following formulae:

- positive-sequence voltage

$$I_{m1} = \frac{(U_{t1} - E_w)}{Z_{g1}}$$

- negative-sequence voltage

$$I_{m2} = \frac{-U_{t2}}{Z_{g2}}$$

- zero-sequence voltage

$$I_{m0} = \frac{-U_{t0}}{Z_{g0}}$$

In these expressions $Z_{g1}$, $Z_{g2}$, and $Z_{g0}$ denote impedances of the synchronous machine for the symmetrical components of positive, negative, and zero sequence components, respectively.

$E_w$ denotes the electromotive force induced in the armature winding at the exciting current $I_w$ may be calculated based on the formula:

$$E_w = U_{tp} + jZ_{g1} I_s$$

The electromotive force at the rated excitation current is calculated from the formula:

$$E_{wn} = U_{tp} + jZ_{g1} I_{ns}$$

The symmetrical components $I_1$,$I_2$ and $I_0$ of asymmetric currents of the receiver load have been calculated from the formula (1)

$$\begin{pmatrix} I_1 \\ I_2 \\ I_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{10} \\ M_{21} & M_{22} & M_{20} \\ M_{01} & M_{03} & M_{t0} \end{pmatrix} \cdot \begin{pmatrix} U_{t1} \\ U_{t2} \\ U_{t0} \end{pmatrix} \cdot \frac{1}{D} \qquad (1)$$

in which the matrix M is filled with given values, $Z_1$, $Z_2$, $Z_0$ being symmetric components of the impedance of the receiver load $Z_{zu}$, $Z_{zv}$, and $Z_{zw}$.

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} Z_{zu} \\ Z_{zv} \\ Z_{zw} \end{pmatrix}$$

$$\begin{pmatrix} Z_{zu} \\ Z_{zv} \\ Z_{zw} \end{pmatrix} = \begin{bmatrix} \left(R_u + jX_{lu} - \frac{1}{jX_{cu}}\right) e^{j\phi_u 2\frac{\pi}{3}} \\ \left(R_v + jX_{lv} - \frac{1}{jX_{cv}}\right) e^{j\phi_v 2\frac{\pi}{3}} \\ \left(R_w + jX_{lw} - \frac{1}{jX_{cw}}\right) e^{j\phi_w 2\frac{\pi}{3}} \end{bmatrix}$$

The M matrix includes the elements referred to in the equation (2)

$$\begin{bmatrix} (Z_z + Z_0)\cdot(Z_0 + Z_{\mu0}) - Z_1 \cdot Z_2 & Z_1^2 - Z_2 \cdot (Z_{\mu0} + Z_0) & Z_2^2 - Z_1 \cdot (Z_0 + Z_z) \\ Z_2^2 - Z_1 \cdot (Z_0 + Z_{\mu0}) & (Z_0 + Z_z)\cdot(Z_0 + Z_{\mu0}) - Z_1 \cdot Z_2 & Z_1^2 - Z_2 \cdot (Z_0 + Z_z) \\ Z_1^2 - Z_2 \cdot (Z_0 + Z_z) & Z_2^2 - Z_1 \cdot (Z_0 + Z_z) & (Z_0 + Z_z)^2 - Z_1 \cdot Z_2 \end{bmatrix} \qquad (2)$$

Particular elements of the M matrix are denoted by the symbols used in the formulas describing the currents and voltages of an unsymmetrical receiver.

$$M_{11} = (Z_z + Z_0)(Z_0 + Z_{\mu0}) - Z_1 Z_2$$

$$M_{12} = Z_1^2 - Z_2(Z_{\mu0} + Z_0)$$

$$M_{10} = Z_2^2 - Z_1(Z_0 + Z_z)$$

$$M_{21} = Z_2^2 - Z_1(Z_0 + Z_{\mu0})$$

$$M_{22} = (Z_0 + Z_z)(Z_0 + Z_{\mu 0}) - Z_1 Z_2$$

$$M_{20} = Z_1^2 - Z_2(Z_0 + Z_z)$$

$$M_{01} = Z_1^2 - Z_2(Z_0 + Z_z)$$

$$M_{02} = Z_2^2 - Z_1(Z_0 + Z_z)$$

$$M_{00} = (Z_0 + Z_z)^2 - Z_1 Z_2$$

M matrix determinant denoted by D may be computed from the formula:

$$D = D_1 + D_2 + D_3$$

in which:

$$D_1 = (Z_0 + Z_z)(Z_0 + Z_z)(Z_0 + Z_{\mu 0})$$

$$D_2 = -Z_1 Z_2 [3Z_0 + (Z_z + Z_z + Z_{\mu 0})]$$

$$D_3 = Z_z^3 + Z_2^3$$

Moreover, the D determinant may be calculated from the formula:

$$D = (Z_0 + Z_z)^2 \left( Z_0 + 0.5Z_z + \frac{Z_{\mu 0} 0.5Z_z}{Z_{\mu 0} + 0.5Z_z} \right) +$$

$$- Z_1 Z_2 [2(Z_z + Z_0) + \left( Z_0 + 0.5Z_z + \frac{Z_{\mu 0} 0.5Z_z}{Z_{\mu 0} + 0.5Z_z} \right)] + (Z_1^3 + Z_2^3)$$

Symmetric components of transformer secondary voltage is given by the relationships (3):

$$U_{t1} = \frac{\left( U_{tp} - \frac{Z_z}{Z_{g1}} E_{wn} \right)}{M_{ia1}}$$

$$U_{t2} = U_{t1} \frac{B}{W}$$

$$U_{t0} = U_{t1} \frac{-Z_{\mu 0}}{KD} \left( M_{01} + M_{02} \frac{B}{W} \right) \qquad (3)$$

where:

$$M_{ia1} = C + \frac{M_{12}}{DW} B Z_z - Z_z \frac{M_{10}}{D} Y$$

$$C = 1 + Z_z \frac{M_{11}}{D} - \frac{Z_z}{Z_{g1}}$$

$$B = -Z_z \frac{M_{21}}{D} + Z_z M_{20} M_{01} \frac{Z_{\mu 0}}{KD^2}$$

$$A = 1 - \frac{Z_z}{Z_{g2}} + Z_z \frac{M_{22}}{D}$$

$$K = 1 - \frac{Z_{\mu 0}}{Z_0} + Z_{\mu 0} \frac{M_{03}}{D}$$

$$W = 1 + Z_z \left( -1 + \frac{M_{22}}{D} - Z_{\mu 0} M_{02} \frac{M_{20}}{KD^2} \right)$$

$$Y = \frac{-Z_{\mu 0}}{DK} \left( M_{01} + M_{02} \frac{B}{W} \right)$$

Phase voltages of the transformer lower voltage side are computed from the formulas:

$$\begin{pmatrix} U_{tu} \\ U_{tv} \\ U_{tw} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} U_{t1} \\ U_{t2} \\ U_{t0} \end{pmatrix}$$

The symmetric-component currents of an unsymmetrical receiver (load) are defined by the expressions:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{10} \\ M_{21} & M_{22} & M_{20} \\ M_{01} & M_{02} & M_{00} \end{pmatrix} \begin{pmatrix} U_{t1} \\ U_{t2} \\ U_{t0} \end{pmatrix} \frac{1}{D}$$

The receiver phase currents are defined by the expressions:

$$\begin{pmatrix} I_u \\ I_v \\ I_w \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_0 \end{pmatrix}$$

The phase currents of an asymmetric machine are defined by the expressions:

$$\begin{pmatrix} I_{mu} \\ I_{mv} \\ I_{mw} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_{m1} \\ I_{m2} \\ I_{m0} \end{pmatrix}$$

Division of the phase currents of an asymmetric machine by its rated current provides relative values of the asymmetric machine currents: $I_{mu}/I_{ng}$, $I_{mv}/I_{ng}$ and $I_{mw}/I_{ng}$.

The transformer phase currents may be conveniently computed by previous summing of the symmetrical components of the unsymmetrical receiver and the symmetrical components of the synchronous machine. The symmetrical components of the transformer currents are given by the expressions:

$$I_{t1} = I_{m1} + I_1$$

$$I_{t2} = I_{m2} + I_0$$

$$I_{t0} = I_{m0} + I_0$$

The transformer phase currents are calculated from the formula:

$$\begin{pmatrix} I_{tu} \\ I_{tv} \\ I_{tw} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_{t1} \\ I_{t2} \\ I_{t0} \end{pmatrix}$$

Division of the transformer phase currents by its rated current provides relative values of the transformer currents: $I_{tu}/I_{dn}$, $I_{tv}/I_{dn}$ and $I_{tw}/I_{dn}$.

The relative currents and voltages enable easy assessment of the load degree of particular circuits.

In the considered circuit the output (secondary) voltages of the transformer and the synchronous machine as well as their symmetrical components are equal.

Nevertheless, in order to estimate the important operational load indices of the transformer and synchronous machine circuits the computation of the relative values of currents and phase voltages of the synchronous machine and the transformer is insufficient. Moreover, the ratios of symmetrical components of the negative- and zero-sequence voltages and currents to the positive-sequence ones should be calculated too. In case of a synchronous machine calculation of the ratio of negative- and zero-sequence current to rated current is

also required. Finally, the following indices are necessary:
for a transformer: $U_{t2}/U_{t1}$, $U_{t0}/U_{t1}$ and $I_{t2}/I_{t1}$, $I_{t0}/I_{t1}$
while for a synchronous machine $U_{m2}/U_{m1}$, $U_{m0}/U_{m1}$ and $I_{m2}/I_{m1}$, $I_{m0}/I_{m1}$
and, additionally, $I_{m2}/I_{ng}$.

Knowledge of the above mentioned indices is important as their maximal values are constrained by the regulations. The excess of allowable levels of the indices $I_{m2}/I_{m1}$, $I_{m0}/I_{m1}$ and $I_{m2}/I_{ng}$ gives rise to hazard, particularly in case of a synchronous machine.

## 3. EXAMPLE RESULTS OF THE CALCULATION

Diagram of the circuit considered in the present paper is shown in Fig. 1. In order to justify usefulness of the analysis presented in the paper some example results of the computer simulation (results of computation) carried out with the use of the presented equations are included. The calculation has been performed for the parameters of an 800kVA transformer and a 200kVA synchronous machine. Parameters of the RLC receiver have been chosen so as to prevent transformer overload and to assists compensation of reactive power of the synchronous machine by the C capacitor. The synchronous machine used in the system is a synchronous motor designed mainly for driving a definite industrial device. The motor is, as far as possible, used for reactive power compensation with a view to prevent excessing of the power index value cosφ (with tanφ remaining below 0.4).

The paper presents only the most interesting calculation results. The results are also presented as relative values with reference to their rated levels.

In the load impedances the resistance and reactance have been independently changed so as to enable variation of the angle of load impedances of particular phases. Characteristic transformer and synchronous machine parameters are as follows:

The transformer: rated power $S_{nt}$=800kVA, rated voltage 15kV/0.4kV, rated current of lower voltage side 1155A, short circuit voltage 5%, short circuit impedance $Z_z$=0.011, $Z_{\mu 0}$=0.00462Ω. The reference impedance corresponding to the transformer rated current at the rated voltage $Z_{odn}$=0.21.

The synchronous machine: rated power $S_{ng}$=200kVA, $\cos\varphi_n$=0.8 (at overexcitation), rated voltage $U_{np}$=400V, rated current $I_{ng}$=289A. Impedances of the synchronous machine: for the positive-sequence current $Z_{g1}$=0.15Ω, for the negative-sequence current $Z_{g2}$=0.09Ω, for the positive-sequence current $Z_{g0}$=0.07Ω.

Table 1 specifies the calculation results – the Example 1 – the results obtained for two various load impedance values (of the receiver), and the Example 2 – under various excitation current values (various $E_{wx}$ values). The electromotive force $E_{wx}$ amounted to $0.95E_{wx}$ and $1.05E_{wx}$, respectively. Relative values (e.g. of voltage and current) are denoted by small letters.

Fig. 2 shows the dependence of the symmetrical component of positive-sequence voltage of transformer output on the synchronous machine excitation current (i.e. the electromotive force $E_{wx}$).

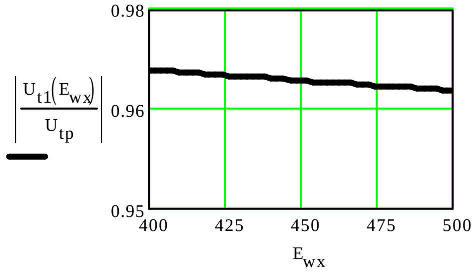


*Fig. 2. The dependence of the symmetrical component of positive-sequence voltage of the transformer output on the excitation current*

Fig. 3 shows the dependence of the symmetrical component of negative-sequence voltage of transformer output on the synchronous machine excitation current (i.e. the electromotive force $E_{wx}$). At the same time, the relationship depicts the symmetrical component of negative-sequence current of the synchronous machine currents.

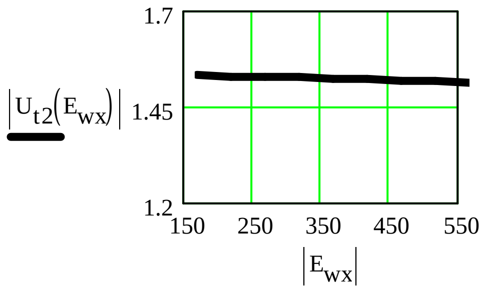


*Fig. 3. The dependence of the symmetrical component of negative-sequence voltage of the transformer output on the excitation current*

Fig. 4 shows the dependence of the symmetrical component of positive-sequence current of the synchronous machine on its excitation current (i.e. the electromotive force $E_{wx}$).

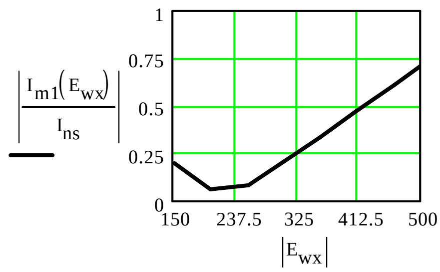


*Fig. 4. The dependence of the symmetrical component of positive-sequence current of the synchronous machine on its excitation current (i.e. the electromotive force $E_{wx}$)*

## 4. SUMMARY OF THE CALCULATION RESULTS AND THE CONCLUSIONS

The formulas presented in the paper enable analyzing of the operation of electric mains of an industrial plant, where various RLC receivers are supplied with single- or three-phase voltage. The formulas may be used for analyzing the cases in which the three-phase supplied receivers are symmetrical or unsymmetrical ones. The analyses have been made with or without connection of the receiver neutral wire with the neutral wire of the

Table 1. Specification of the calculation results

| **Parameter** | | **Example I** | **Example II** |
|---|---|---|---|
| Receiver impedances | Ω | $Z_{zu}$ =1. 0.21.expj0.307.120<br>$Z_{zv}$ =1.42.0.21.expj0.375.120<br>$Z_{zw}$ =1.85.0.21.expj0.417.120 | $Z_u$ =1. 0.21.expj0.3069.120<br>$Z_v$ =1.15 0.21.expj0.375.120<br>$Z_w$ =1.35.0.21.expj0.7.120 |
| Impedances of the symmetrical components receiver | Ω | $Z_1 = 0.054$<br>$Z_2 = 0.056$<br>$Z_0 = 0.298$ | $Z_1 = 0.055$<br>$Z_2 = 0.075$<br>$Z_0 = 0.229$ |
| Electromotive force $E_{wx}$ | V | $1.05.E_{wn}$ | $E_{wx} = 0.95\ E_{wn}$. |
| Symmetrical components at the transformer output terminals | V/- | $U_{t1} = 222.6$, $u_{t1} = 0.975$<br>$U_{t2} = 2.2$ $u_{t2} = 0.975$<br>$U_{t0} = 2.4$ $u_{t0} = 0.975$ | $U_{t1} = 225.5$ $u_{t1} = 0.976$<br>$U_{t2} = 2.197$ $u_{t2} = 0.009$<br>$U_{t0} = 2.36$ $u_{to} = 0.01$ |
| Transformer output voltage values | V | $U_{tu} = 222.6$ $u_{tu} = 0.964$<br>$U_{tv} = 224.3$ $u_{tv} = 0.971$<br>$U_{tw} = 229.9$ $u_{tw} = 0.995$ | $U_{tu} = 222.5$ $u_{tu} = 0.963$<br>$U_{tv} = 224.2$ $u_{tv} = 0.971$<br>$U_{tw} = 229.8$ $u_{tw} = 0.995$ |
| Voltage asymmetry index | % | $U_{t2}/U_{t1} = 0.61$<br>$U_{to}/U_{t1} = 0.667$ | $U_{t2}/U_{t1} = 0.971$<br>$U_{to}/U_{t1} = 1.049$ |
| Symmetrical component currents of the receiver | A | $I_1 = 773.7$<br>$I_2 = 148.5$<br>$I_0 = 134.0$ | $I_1 = 857$<br>$I_2 = 217$<br>$I_0 = 231$ |
| Phase currents of the receiver | A | $I_u = 1020$<br>$I_v = 738$<br>$I_w = 574.9$<br>$I_{po} = 402.1$ | $I_u = 1020$<br>$I_v = 901$<br>$I_w = 807$<br>$I_{p0}$ =693 A. |
| Index of receiver current asymmetry | % | $I_2/I_1$ =19.2<br>$I_0/I_{nt} = 12.9$ | $I_2/I_1 = 25.4$,<br>$I_0/I_{nt} = 18.8$ |
| Symmetrical component currents of the synchronous machine | A | $I_{m1} = 301.8$ $i_{m1} = 1.046$<br>$I_{m2} = 11.7$ $i_{m2} = 0.041$<br>$I_{m0} = \sim 0$ $i_{m0} = \sim 0$ | $I_{m1} = 280.1$ $i_{m1} = 0.97$<br>$I_{m2} = 17$ $i_{m2} = 0.059$<br>$I_{m0} = \sim 0$ $i_{m0} = \sim 0$. |
| Phase currents of the synchronous machine | A | $I_{mu} = 300.9$ $i_{mu}$ =1.042<br>$I_{mv} = 312.6$ $i_{mv} = 1.083$<br>$I_{mw} = 292.5$ $i_{mw} = 1.013$ | $I_{mu}$ =264.7 $i_{mu} = 0.91$<br>$I_{mv} = 294.4$ $i_{mv} = 1.02$,<br>$I_{mw} = 281.97$ $i_{mw} = 0.977$ |
| Asymmetry index of the synchronous machine currents | % | $I_{m2}/I_{m1} = 3.88$<br>$I_{m2}/I_{ns} = 4.05$ | $I_{m2}/I_{m1} = 6.12$<br>$I_{m2}/I_{ns}$ =5.9 |
| Symmetrical component currents of the transformer | A | $I_{t1} = 989.6$ $i_{t1} = 0.857$<br>$I_{t2}$= 160.2 $i_{t2} = 0.139$<br>$I_{t0} = 134$ $i_{t0} = 0.116$ | $I_{t1} = 1088$ $i_{t1} = 0.942$<br>$I_{t2}$= 234 $i_{t2} = 0.233$<br>$I_{t0} = 231$ $i_{t0} = 0.2$. |
| Asymmetry index of the transformer currents | % | $I_{t2}/I_{t1} = 16.2$<br>$I_{t0}/I_{t1} = 13.5$ | $I_{t2}/I_{t1}$ =21.7<br>$I_{t0}/I_{t1}$ =21.3 |
| Phase currents of the transformer | A | $I_{tu} = 1215$ $i_{tu} = 1.052$<br>$I_{tv} = 968.7$ $i_{tv} = 0.839$<br>$I_{tw} = 808.1$ $i_{tw} = 0.7$ | $I_{tu} = 1195$ $i_{tu} = 1.035$<br>$I_{tv} = 1127$ $i_{tv} = 0.976$<br>$I_{tw} = 1084$ $i_{tw} = 0.939$ |
| Reactive power of the synchronous machine | Q/q | 203.5 kVAr<br>1.7 | 189500 VAr,<br>q = 1.58 |
| Active power transmitted by the transformer | P/p | 366.8 kW<br>0.458 | 343 kW<br>0.53 |

mains. Advantage of the presented equations consists in possible analyzing of cooperation of the single- or three-phase receivers with a synchronous machine, being usually a motor. The machine may be subject to the loads of various active power values and, at the same time, the reactive power of capacitive character may be adjusted by various excitation current levels (over excitation) that enables easier compensating of the inductive reactive power absorbed by the industrial plant from the mains. Compensation of the inductive reactive power with the help of a synchronous machine cooperating with a battery of capacitors is advantageous for several reasons. One of the reasons consists in the fact that the synchronous machine enables easier voltage stabilizing in the plant mains, since the reactive power generated by the synchronous machine only slightly depends on the voltage, as opposed to a capacitor, where the reactive power depends on voltage squared, according to the formula $Q_k(U)=Q_n \cdot U^2$. The parameter $Q_n$ is here the rated reactive power of a capacitor of capacitance $C_n$, i.e. the reactive power at rated voltage and frequency

$$Q_n = 3U_f I_f = 3U_{fn}^2 \, 2\pi f C_n$$

The formula is also important since it displays the effect of higher harmonic components of the voltage on the capacitor current, according to the formula $I_f = U_f \, 2\pi f C_n$. The higher is the rank of the harmonic component, the lower is capacitor reactance, hence, the current intensity becomes bigger. In order to delimit the currents of the so called higher harmonics reaching the capacitor, the binding regulations constrain allowable voltage levels of particular harmonics. In extreme cases choking coils must be connected to the capacitor circuit. Since the voltage harmonics contents of industrial plant mains may reach even 10 per cent, the capacitor should withstand current overload reaching 1.3 or even 1.5 of the rated current level. The synchronous machines are not so sensitive to the voltage harmonics as the capacitors and, moreover, as it has been mentioned before, their reactive power depends on voltage only to a small degree, but to much larger degree on the excitation current. The reactive power of a synchronous machine may be defined by the formula:

$$Q_s(k_u, k_w) = -3k_u \frac{U_n[(k_w U_n)(X_q \cos\beta + R_t \sin\beta) + -k_u U_n (X_q \cos\beta^2 + X_d \sin\beta^2)]}{R_t^2 + X_d X_q}$$

In the formula $R_t$ is for resistance of stator wiring while $X_d$ and $X_q$ are synchronous reactances of the longitudinal and transverse axes. The $k_u$ and $k_w$ coefficients depict the effect of the voltage and excitation current changes on the level of the reactive power. In case of a synchronous machine provided with a cylindrical rotor, in which $X_d$=$X_q$ and omitting the stator wiring resistance, the formula of reactive power simplifies to the form:

$$Q_s(k_u, k_w) = \frac{3U_n k_u (k_w U_n \cos\beta - k_u U_n)}{X_d}$$

It may be easily noticed that in case of the voltage lowered down even to $0.8 \cdot U_n$, i.e. for $k_u$=0.8 and for increased excitation current, i.e. increased $k_w$ factor, a constant level of reactive power may be obtained. The constant reactive power may be obtained for growing voltage value. The required value of the $k_w$ coefficient depends on the power (load) β angle. The angle depends on the excitation current and active power of the machine, in accordance with the formula:

$$P(k_u) = 3k_u U_n^2 \frac{k_w}{X_d} \sin\beta$$

With the voltage value dropping to $0.8 \cdot U_n$ reactive power of the capacitor goes down to $0.64 \cdot Q_n$, while voltage growth up to $1.2 \cdot U_n$ results in the growth of the reactive power up to $1.44 \cdot Q_n$. In order to ensure constant value of the reactive power compensated by the capacitors under oscillation of the voltage, proper capacitor groups should be switched on or off. On the other hand, in case of a synchronous machine only the excitation current should be adjusted accordingly.

The parameters specified in Table 1 show that connection of a synchronous machine to the plant mains results in remarkable stabilization of its voltage, and in spite of significant asymmetry the phase voltages are only slightly deformed. On the other hand, in consequence of the high load asymmetry large values of current asymmetry factors occur in the windings of the synchronous machine that is a very disadvantageous phenomenon.

The formulas included in the paper may be also used for analyzing the conditions of the system operation after disconnection of the synchronous machine. In such a case the calculation should be made with the assumption of zeroing of the synchronous machine current. According to the equations presented in the paper the synchronous machine current, e.g. in the U phase, is given by the formula $I_u$=$I_{m1}$+$I_{m2}$. Substitution of the formulas:

$$I_{m1} = \frac{(U_{t1} - E_{wx})}{Z_{g1}} \qquad I_{m2} = \frac{-U_{t2}}{Z_{g2}}$$

$$U_{t1} = \frac{\left(U_{tp} - \frac{Z_z}{Z_{gl}} E_{wx}\right)}{M_{ia1}}$$

provides the electromotive force corresponding to disconnected synchronous machine

$$E_{wz} = U_{t1}\left[1 - B\frac{Z_{g1}}{(WZ_{g2})}\right]$$

In the case considered in the Example 1 the electromotive force amounts to 240.9V.

The load asymmetry is conducive to asymmetry of the transformer output voltages and, in consequence, asymmetry of the low voltage mains voltages. In the considered example the voltage deviations in particular phases are relatively small (about 10 per cent), but the asymmetry degree $U_2/U_1$ exceeds its allowable level. In result of such a situation the allowable value of current asymmetry of the synchronous machine operating in the network is significantly exceeded. In the calculation example the current asymmetry degree of the synchronous machine $I_2/I_1$ is of the order 0.4 – 0.7. This value remarkably exceeds the allowable level of the current asymmetry factor equal to 0.1. So high degree of asymmetry might easily lead to break down of the synchronous machine and, therefore, must not be admitted. The degree of the voltage and current asymmetry depends considerably on the kind of the receiver and its impedance. In case of the load considered in the present paper the voltage and current asymmetry factors are large and significantly exceed the allowable levels determined by the standards and regulations.

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