Skin effect in conductor of rectangular cross section

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Abstract **: Two approximate formulae for skin effect in conductor of rectangular cross section were derived in the last time. They differ in the current distribution not only in the conductor volume but also on its surface. Since the current distribution in the conductor cannot be measured, we are limited to the measurement of effects on conductor surface and its environment. The potential drop was measured on the surface, but its sensitivity and precision is too low. Therefore flux density of magnetic field generated by current in the conductor was calculated for skin effect and uniform distribution. Then the magnetic field was measured. Unfortunately, at frequency of 1200 Hz where the skin effect was evident the exciting current was to low, to find good agreement with theoretical prediction. However it ii only technical limitation, that will be eliminated in the future.**

Keywords **skin effect, massive conductor, three phase net, Hall probe, eddy currents, distribution point.**

I. INTRODUCTION

Analytical formulae of skin effect are in textbooks [1] only for very simple geometry. Circular cross section is the most complicated shape. In last time two approximate theories appeared in literature, [2] from present authors and another [3]. They are based on different approaches. In the paper basics of both theories are outlined, and theoretical results are compared. Experiment for the decision, which of theories is in better agreement with reality is presented and its results are discussed

II. THEORY

Let us consider infinite conductor of width 2*a* in the direction of *X* axis and height 2*b* along *Y* axis. Wire material has permeability μ and conductivity γ. The current f angular frequency ω flows along the *Z* axis. Each of the theories [2] and [3] apply the same differential equation. The difference is in boundary conditions. Theory in Ref. [2] supposes constant current density i_o on the surface of the conductor; it will be termed constant current theory. The second theory [3] uses the total current *I* in the conductor as the basic boundary condition. Then the surface current density differs and we term the theory as variable current theory

The constant current theory leads the formula for current density *i* in the conductor. The formula contains two linear terms and one nonlinear product term, in the form of product of linear terms

$$\hat{i}(x,y) = i_O \left[-\frac{\cosh(\hat{\delta}x)\cosh(\hat{\delta}y)}{\cosh(\hat{\delta}a)\cosh(\hat{\delta}b)} + \frac{\cosh(\hat{\delta}x)}{\cosh(\hat{\delta}a)} + \frac{\cosh(\hat{\delta}y)}{\cosh(\hat{\delta}b)} \right], \qquad (1)$$

where

$$\hat{\delta} = (1+\mathrm{j})\sqrt{\frac{\omega\mu\gamma}{2}}, \qquad (2)$$

is a complex attenuation constant.

The variable current theory [3] results in simpler formula in the low frequency range

$$\hat{i}(x,y) = I\,\frac{\hat{\delta}^2}{4}\,\frac{\cosh(\hat{\delta}x)\cosh(\hat{\delta}y)}{\sinh(\hat{\delta}a)\sinh(\hat{\delta}b)}, \qquad (3)$$

where

$$\hat{\delta}_x = (1+\mathrm{j})\sqrt{\frac{\omega\mu\gamma}{2}}\sqrt{\frac{2b}{a+b}}, \qquad \hat{\delta}_y = (1+\mathrm{j})\sqrt{\frac{\omega\mu\gamma}{2}}\sqrt{\frac{2a}{a+b}}. \qquad (4)$$

It can be shown [4] that formula (3) is identical with nonlinear them in (2) for the square wire cross section. Also, if we put $\hat{\delta}_x = \hat{\delta}_y = \hat{\delta}$ in general, we get the same result.

III. EXPERIMENT

The main problem of experimental verification is that we cannot measure the current density in the conductor. We are limited to the conductor surface or its environment. In order to verify the assumption of constant surface current density the voltage drop on the conductor vertical surface at different positions was measured. In the conductor environment the magnetic flux density was measured, since is depends on the current distribution in the conductor.

The measuring apparatus contains three conductors for three phase excitation, but only centre conductor was fed from one phase source. Back currents flow in other conductors. All the measurements were made in time domain, and were fully automated. Processing of experimental results was made by originals scripts in MATLAB. The details are in Ref. [4].

IV. RESULTS

The investigated conductor has length of 2 m, rectangular cross section with the width of 5 mm and height of 40 mm. First of all we compare results from both the theories. The surface current density on vertical conductor surface is in Fig. 1. The variable current theory states that the current flows only near edges and in a part of surface its direction is opposite. In the central part it has practically zero value. A similar behaviour, but not so strictly limited, is for the narrower conductor side. Unfortunately, the surface voltage measurement had not enough sensitivity and precision to verify this prediction. If we use longer distance between probes, we do not measure the surface voltage.

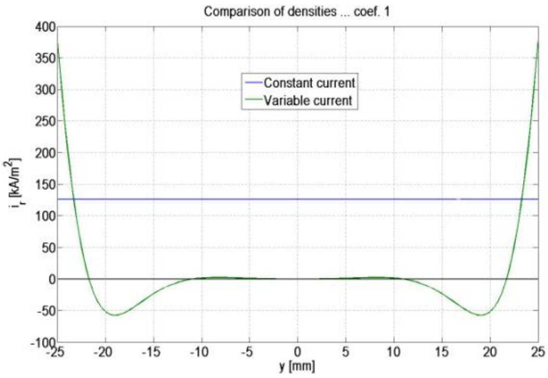

Fig 1: Current density on vertical conductor side

To verify the current distribution in the conductor volume its outer magnetic field was calculated and measured. The details are in Ref. [4]. The comparison between theory and experiment is in Fig. 2 for the horizontal component B_x of magnetic flux density and in Fig. 3 for the vertical component B_y. Frequency was 1200 Hz. The conductor positions are sketched for better understanding.

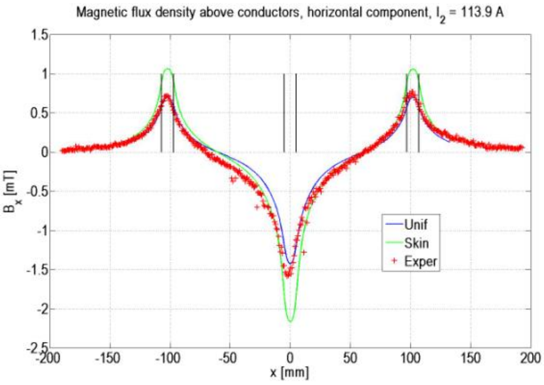

Fig. 2. Horizontal component of magnetic flux density

In both the figures the results of theoretical calculations for uniform current distribution and no uniform one using formula (1) for skin effect are shown. As for the horizontal component in Fig. 2, the difference is only above conductors. Lower value of measured points can explained by averaging effect of 3D Hall probe sensors, due their area, since the flux density changes rapidly there.

In the comparison for horizontal component in Fig. 3, the experimental points agree approximately with skin effect curve in the left part from the centre conductor. In the opposite part the agreement is better for uniform distribution. While the theoretical curve exhibit anti-symmetry, the experimental points do not respect this condition. It can be explained by other effects, probably by eddy currents. In our one phase experiment, alternating back currents flow in the side conductor. Since the distance between conductors is small from technical reasons, the effect of eddy currents probably non-negligible.

V. DISCUSSION AND CONCLUSIONS

Although two experimental methods were used to find which of theories is closer to reality, none of them was successful. The measurement of very small surface potential drop is difficult. On the other hand by the measurement at higher distance between probes we proved that the conductor resistance increases with frequency. Furthermore we found that there is a phase shift between surface voltage and total current.

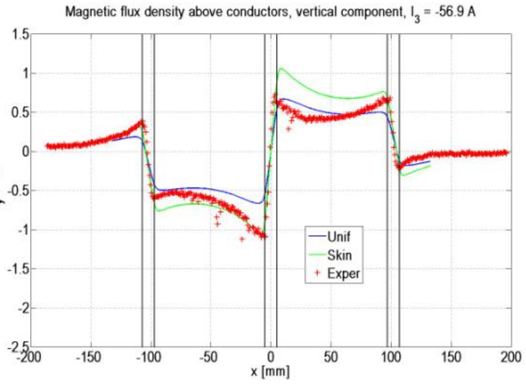

Fig.3: Vertical component of magnetic flux density.

The complication of magnetic field approach was due to the very small exciting currents at frequency of 1200 Hz which had lead to low precision of experimental points. Low exciting current due to the error in the design of power transformers is the technical problem only. Higher currents can be achieved in the future.

In the calculation (by numeric integration) of magnetic filed field we have considered only the skin effect. In the experimental set up eddy current take a place, probably. Therefore their effect should be taken into account. In this case Finite Element Method (FEM) must be used, since the task is of dynamic nature. The bet approach is to use back conductor in large distance from active one, but at present time, this solution is difficult technically.

We can conclude that after the changes in experimental setup and improvement of calculations, there will be possibility to distinguish between two theories for skin effect.

VI. ACKNOWLEDGMENTS

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