A transmission line model of the human cardiovascular system

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Abstract **Methods of computer modelling and simulations represent a strong tool in the technical practice. It is widely utilized because of its effectiveness and variability. Applied to modelling of biological system it allows to study a body under physiological conditions or evaluate the effect of pathologic changes in the organism and possible impact of therapy without any invasion. A computer simulation of the electro-mechanical analogy of human cardiovascular system is developed for the purpose of haemodynamic relationships investigation.**

Keywords **blood pressure, blood flow, pulse wave, electro-mechanic analogy, cardiovascular system.**

I. Introduction

The planning of the treatment of disease depends mainly on diagnostic data for definition of the actual patient state, statistical data of prior treatments for similar patients, and the judgement of the expert to decide on a preferred treatment. Yet, the selfhood of each patient system is such that this approach is insufficient to prognosticate the effect of a chosen treatment for an individual patient. Computer simulations of biological system models are promising solution for the evaluation of possible outcome for any individual. However, biological systems are rather complex structures. In a closer view it could be divided into several subsystems. For example we can recognize subsystems: mechanic, hydrodynamic, thermodynamic, electric, chemical etc. All these subsystems are interconnected and influencing each other and are described with equations which can be taken as dual. Thus the problem can be transferred from one physical environment to the other, computationally easily handled one.

Cardiovascular illnesses represent one of the serious global problems especially in developed countries. The vascular system of human body is complex and the blood flow is influenced by many factors. Therefore the complete and detail model is quite challenging task. The paper deals with the numerical modelling of hemodynamics in human vascular system.

Many models of vascular system are based on the electromechanical analogy. The recent models use an analogy of the blood vessel with an electrical transmission line with distributed parameters. On the basis of theoretical analysis [1] we have developed model of the human arterial system in the MATLAB environment [2]. This model allows to simulate different phenomena taking place in the arterial system [7].

II. Theory

A. Model formulation

Exact mathematical model of phenomenon in blood vessels is hindered by complexity of blood rheology and nonlinearity of blood vessels' wall properties. Therefore, for the model development some simplifications are taken:

1. Only mechanical interaction between blood flow and vessel wall is considered.
2. The blood is considered as non compressible Newtonian fluid with constant density.
3. The vascular system is considered as the system without sources and is described using continuity equation $div\,\bar{v} = 0$.

Utilization of electro-mechanic analogy assumes the set of mechanical quantities assigned to corresponding electrical quantities, Table 1.

TABLE I
Analogical electric and mechanic quantities

	Mechanic quantities		Electric quantities
P	Pressure	*P*	Potential
Pt	Transmural pressure	*U*	Voltage
I	Volume flow	*I*	Current
V	Volume	*Q*	Load
W_k	Kinetic energy of blood flow	W_m	Energy of magnetic field
W_e	Energy of blood vessel wall elastic deformation	W_e	Energy of electric field

The theoretical model resulting from the electro–mechanical analogy describes the laminar flow of blood taken as a viscous liquid in a vessel taken as an elastic tube. The tube with liquid is modelled by an electrical transmission line [2], the elementary segment (Fig. 1) of which can be substituted by the longitudinal impedance $\mathbf{Z}_\mathrm{L}$ and transverse admittance $\mathbf{Y}_\mathrm{T}$ given by

$$\mathbf{Z}_\mathrm{L} = -\frac{\mathrm{j}\omega\rho\,\mathrm{J}_0(\mathbf{a}r_0)}{\pi r_0^2\,\mathrm{J}_2(\mathbf{a}r_0)}\ ,$$

$$\mathbf{Y}_\mathrm{T} = \mathrm{j}\omega\frac{2\pi r_0}{k_w\,(E_w + j\,\omega\eta_w)}\ , \qquad (1)$$

where $\mathbf{a} = \sqrt{-\frac{\mathrm{j}\,\omega\,\rho}{\eta}}$ is the blood parameter, ρ the blood density, η the blood viscosity, ω the angular frequency of the transmitted wave, r_0 the tube internal radius, E_w and η_w the tube wall elastic modulus and tissue viscosity and k_w the dimension factor. J_0 and J_2 are the Bessel functions of the 0–th and 2–nd orders. Longitudinal impedance $\mathbf{Z}_\mathrm{L}$ represents conservative and dissipative component, transversal admittance $\mathbf{Y}_\mathrm{T}$ describes cross elasticity, inter-wire capacity and cross losses.

The blood propagation in blood vessels can be analogically represented as electromagnetic wave propagating along the long wire. Based on this, each particular vessel can be represented by cascade connection of four-poles (vessel segments) characterized by transfer parameters of long wire. For model purposes it is necessary to divide chosen vascular system into the system of homogenous segments. Each equivalent circuit of vessel segment can be characterized by transfer parameters – wave impedance $\dot{Z}_0$ and propagation constant $\dot{\gamma}$. Then each equivalent circuit can be described by transfer equations [2]

$$\dot{P}_1 = \cosh(\dot{\gamma} l)\dot{P}_2 + \dot{Z}_0 \sinh(\dot{\gamma} l)\dot{I}_2$$
$$\dot{I}_1 = \frac{\sinh(\dot{\gamma} l)}{\dot{Z}_0}\dot{P}_2 + \dot{Z}_0 \cosh(\dot{\gamma} l)\dot{I}_2 \qquad (2)$$

As a terminal segment is utilised a three-element Windkessel model, a serial-parallel combination of R_1, R_2 and C_2, and the terminal segment impedance is given by

$$\dot{Z}_T = R_1 + \frac{R_2\left(-jX_{C_2}\right)}{R_2 - jX_{C_2}} . \qquad (3)$$

In context of the electrical theory analogy, these parameters are derived from wave impedance of vessel equivalent circuit. We assume the output impedance of vessel equivalent circuit to match the input impedance of terminal segment.

A model of human circulatory system consisting of 128 vessel segments and 60 terminal segments is developed and studied under physiological and pathological conditions.

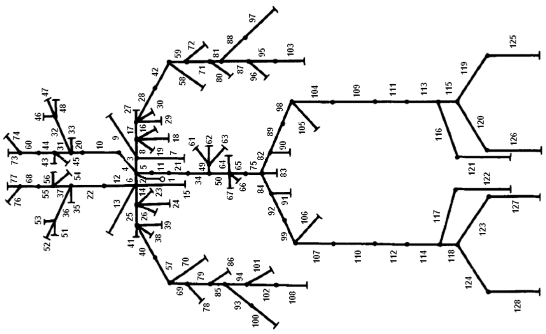

Fig. 1. Human arterial system tree [2]

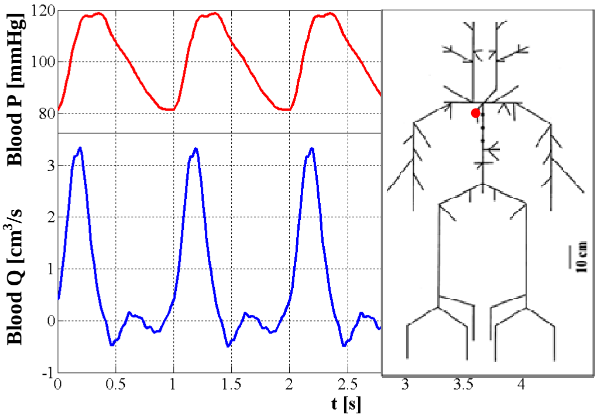

Fig. 2. blood pressure and blood flow in aorta

Figure 2 shows the propagation of blood pressure and blood flow wave in the aorta. The amplitude of blood pressure is increasing with the direction from the hearth and a second maximum of the signal occurs, Fig. 3.

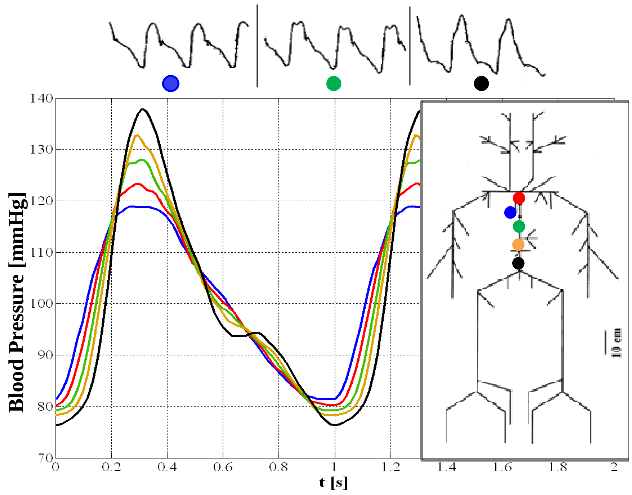

Fig. 3. Blood pressure changes in aorta measured in 5 points (color dots): • aorta ascendens, • arcus aortae, • aorta thoracica 1, • aorta thoracica 2, • aorta abdominalis

III. Conclusion

The model of human circulatory system consisting of 128 vessel segments and 60 terminal segments was developed. The model contains all the big central arteries and major peripheral arteries supplying head and extremities. The wave propagation of blood was studied in selected points of the simulated arterial tree and shows a fair agreement with real blood pressure waveforms. However the human vascular system is highly complex structure with self-regulation ability. It should be emphasized that for the comparison with physiological values the model parameters should be adjusted for the subject. All parameters for simulated arteries were obtained from the literature [2]. Therefore the results of this study should be considered as strictly theoretical.

IV. Acknowledgements

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