A three-dimensional model of direct heat treatment of a sample surface with a moving laser has been established utilizing the finite element method. Attention is devoted to the preparation of complex boundary conditions of a moving heat source. Boundary conditions of material heat treatment are defined in the form of the heat transfer coefficient with consideration of several effects. Those include general distribution of energy in the laser beam, laser motion velocity, laser axis position outside the sample, and utilization of multiple laser motion tracks over the sample. Various arrangements of sample heat treatment are proposed and computer simulated. Different velocities of laser motion, multiple motion over the same track, and simple motion over a number of tracks are investigated. The temperature distribution in the sample and the depths of material heat treatment are evaluated. The simulation model can be used for temperature prediction during laser surface treatment of materials.

**OCIS codes:** (000.3860) Mathematical methods in physics; (120.6780) Temperature; (140.3390) Laser materials processing; (140.6810) Thermal effects; (160.3900) Metals; (350.3390) Laser materials processing.

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### 1. INTRODUCTION

Laser beams are usually used for surface heat treatment. A laser beam with a certain diameter moves over the surface in proposed tracks to influence the whole surface of the chosen region.

The laser beam is partly reflected; the rest is absorbed to a small depth that is dependent on the absorption coefficient of the material. In the case of metals, the surface absorption of heat power can be assumed.

The application of material heating using a moving heat source has attracted attention for many years. Analytical solutions [1–4] can be obtained under limited conditions. Numerical methods for task solution are used to achieve results for more complex geometries and boundary conditions [5–10].

The authors of [3] deal with an analytical solution of the temperature increase in the material due to stationary/moving bodies. They limit their studies to half-space and half-plane geometries and the integral and differential equations are derived. In [4], the authors utilize an analytical solution for a simple geometrical arrangement.

Numerical simulation of heat transfer in a sample caused by a moving heat source is described in [5]. It considers planar sample geometry, all heat losses to the surroundings are ignored and an ideal heat source with a Gaussian shape is used. The exemplary results include only temperatures in the heat source axis of the trajectory for various velocities of heat source motion.

The authors of [6] deal with heat transfer in a sample with a moving heat source and try to determine the size of the melt area at the sample surface. They consider a hyperbolic heat transfer equation and compare the results with a classic diffusion heat transfer equation. The phase change is already solved in the model. Knowledge of the precise thermal properties of the material dependence on temperature is fundamental in these models. These data are not always available, nor is it possible to measure them. This problem is also solved numerically in [7], using the authors’ own method based on the finite difference method. The energy distribution in the heat source is Gaussian-shaped, and all of the material properties and boundary conditions are simplified and assumed to be constants. Also, the planar symmetry of the sample is considered.

The authors of [8] also use their own numerical solution of the heat transfer equation based on the finite difference method. Their numerical solution is compared with an analytical one. Unlike the other works, they use the cluster of heat sources that is used to heat the sample material.
An adaptive mesh scheme of the finite element method (FEM) is used in [9] to solve heat conduction in a solid heated by a moving heat source. A mathematical description of the FEM method including the mathematical procedure of the adaptive mesh scheme that is used for mesh refinement in the areas of large gradients is provided. The results obtained show the functionality of the developed model.

The authors of [10] deal with the effect of heat source beam geometry on the temperature distribution in the material that is assumed to be a half-space. The hyper-elliptical geometry of the heat source beam considered covers a wide range of heat source shapes, including elliptical, rectangular with round corners, rectangular, circular, and square. The effects of the heat source speed, aspect ratio, and other factors are investigated using a general solution of a moving point source on a half-space and superposition of the beam shape.

The majority of the authors use a uniform or Gaussian shape of energy distribution in the heat source and some of the authors use sample material in the shape of a half-space or half-plane. In this paper, a general energy distribution in the laser beam is utilized, because it can reflect the real application of laser heat treatment more precisely than widely used uniform or Gaussian energy distributions.

Authors who deal with the solution of problems of heat transfer in a material that is thermally loaded by a moving heat source use several fundamental possibilities of energy distribution in the heat source: (1) uniform, (2) Gaussian, (3) parabolic, and (4) general. The great majority (4/5) of publications use a uniform distribution of energy in the heat source in order to simplify the solution of the heat transfer process. Some of the publications use a Gaussian energy distribution because it is more precise, and can be easily used, especially when the heat transfer is solved by analytical methods. Only a few authors try to compare different energy distributions and they usually utilize both uniform and parabolic distributions. In the available literature, only one publication has been found that uses a general energy distribution in the heat source [11]. The authors use only a mathematical point of view for the solution of the heat transfer process and do not take into account any real application.

This research is focused on a general distribution of energy in the laser beam because that reflects the real applications of laser heat treatment. Real heat treatment applications employ, instead of laser beams with uniform or Gaussian distributions, laser beams whose energy decreases with increasing distance from the axis of the laser beam with a general shape. When optimization of laser heat treatment of material is utilized with respect to more uniform treatment of the material surface, laser beams with a very specific distribution are taken into account (certain value of heat flux in the center of the beam increases toward the edge of the beam and then rapidly decreases near the beam edge).

A two-dimensional numerical simulation model [12] has been developed for heat transfer during coating deposition in the authors’ laboratory. Since then, this model has been improved and widely used for modeling of the dynamic behavior of thermal barrier coatings during thermal shocks [13–15]. The model was also compared with the stochastic solution method [16]. Major improvement of the model has been made to enable heat transfer simulation in 3D sample geometry [17,18].

In this paper, a simulation model of material heat treatment using a moving laser beam and considering 3D geometry with respect to a spatial non-homogeneous profile of the laser beam is investigated. Attention is focused on a description of complex boundary conditions. Results of the individual tasks are shown with respect to variable laser motion velocity, the variable number of movements across the sample using the same track, and the case of several tracks over the sample’s front surface.

2. SIMULATION MODEL

A. Model Characteristics

Simulation models of 3D direct non-stationary tasks using the finite element method are prepared. A characteristic feature of the task is the complex boundary conditions of the heat-treated sample surface. A computer model of non-stationary heat transfer is created using the commercial computational system Cosmos/M. The model describes heat treatment using a moving laser beam.

The idea is not to develop a new numerical computational system for the solution of heat transfer processes, but to utilize existing commercial computational systems. When non-standard processes (such as material heating using a moving laser beam) are simulated, the aim is to develop and use a mathematical description of complex processes in the simulation model created in the commercial computational system. This is the reason why only mathematical differential equations of diffusion heat transfer with the additional constraint conditions containing initial and boundary conditions are used in the following text. Moreover, the description and preparation of complex boundary conditions of moving laser beam heating are discussed in detail.

The simulation model is created in the commercial computational system Cosmos/M, which is now part of SolidWorks software. The commercial computational system enables the creation of a simulation model of the heat transfer process (to define the geometry of the model, the physical process to be modeled, initial conditions, boundary conditions, material properties, the computational mesh with the types of finite elements, parameters for the simulation, etc.). Then the system provides the numerical solution of the equations and finally, the system has capabilities for evaluation of the results of the simulation.

The energy distribution in the laser beam is not simplified to a uniform or Gaussian distribution as other authors usually use, but the dependence of energy density on the distance from the laser axis can have a general shape. It is described by an independent user-defined curve, as is discussed in Subsection 2.D.

The sample material is assumed to be homogeneous and isotropic; initial temperature is constant in the sample volume. On the front surface of the sample, the boundary condition of heat convection representing the thermal effect of moving beam source heating \( q_{bh} \) and the boundary condition of radiation cooling \( q_{cr} \) are used (Fig. 1(a)). Lateral sample sides are considered thermally isolated; the back side of the sample has the boundary conditions of free convection cooling \( q_{cv} \) and radiation cooling \( q_{rc} \), see Fig. 1(a). The simplified
two-dimensional task is solved in the sample cross section in the $xz$ plane.

The laser beam moves over the sample in certain straight tracks in the $x$-axis direction [Fig. 1(b)], between the reversal points that are outside the sample. In the simplest case, the beam moves over only one track. When heat treatment fills up the larger surfaces, the beam motion in the $y$-axis direction [Fig. 1(c)] is made between reversal points outside the sample. The laser beam is circular, with maximum power in the center. Power density declines with increasing distance from the beam axis.

The effect of the moving laser beam is described as a time- and space-dependent surface heat convection on the front sample surface. The basis is the heat transfer coefficient dependence on the distance from the beam axis. The external beam temperature (external temperature for convection) and heat transfer coefficient express heat convection as a boundary condition on the thermally loaded front sample side. The value of external temperature for convection is constant; the heat transfer coefficient is considered to be temperature independent.

Provided that a region of a material heats up above some specific temperature, it is a matter of material heat treatment. When the specific temperature reaches the so-called hardening temperature $T_h$, and the heating process is followed by rapid cooling of the material, the overall process is called material hardening. The hardening temperature $T_h$ is approximately 800°C. Provided a laser beam is the heating source, it is called surface laser hardening. For surface laser hardening, a high-intensity heat flux in the laser spot is characteristic, which results in very rapid heating of the surface layer of the material and subsequent rapid cooling due to heat transfer further into the material. There is a change of phases and transformation of the surface layer to high hardness due to the rapid cooling of the heated material. Especially the speed of the heating and cooling processes, the formation of a high temperature gradient, and the absence of a liquid cooling medium are three fundamental advantages of this process.

### B. Model Mathematical Description

The partial differential equation for diffusion heat transfer in the sample material without inner heat sources has the form

$$\nabla (\lambda(x, y, z, t) \nabla T(x, y, z, t)) = c(x, y, z, t) \rho(x, y, z, t) \frac{\partial T(x, y, z, t)}{\partial t}, \quad (1)$$

where $x, y, z$ are spatial coordinates; $t$ is the time of the process; $T(x, y, z, t)$ is the temperature of the sample; $\lambda(x, y, z, t)$, $c(x, y, z, t)$, and $\rho(x, y, z, t)$ are spatial and time-dependent thermal conductivity, specific heat capacity, and density.

The set of additional constraints involves the initial condition and several types of boundary conditions. The initial condition is in the form

$$T(x, y, z, 0) = T_{ini}(x, y, z), \quad (2)$$

where $T_{ini}(x, y, z) = T_{ini}$ is the initial sample temperature that is assumed to be constant for the whole sample volume.

The heat flux boundary condition is in the form

$$-\lambda(x, y, z, t) \frac{\partial T(x, y, z, t)}{\partial n} = q_p(x, y, z, t), \quad (3)$$

where $n$ is the normal vector to the surface in the position $x, y, z$. Partial derivative $\frac{\partial T}{\partial n}$ expresses the derivative of the temperature in the direction perpendicular to the sample surface. The vector quantity $q_p(x, y, z, t)$ denotes the prescribed value of heat flux at the sample boundary. Boundary conditions of this type are used for lateral sample sides [see Fig. 1(a)]. Prescribed surface heat flux is equal to zero $q_p(x, y, z, t) = 0$.

The convective heat transfer boundary condition is used in the form

$$-\lambda(x, y, z, t) \frac{\partial T(x, y, z, t)}{\partial n} = \alpha_{cc}(x, y, z, t)(T(x, y, z, t) - T_{cc}(x, y, z, t)), \quad (4)$$

where $\alpha_{cc}(x, y, z, t)$ is the prescribed heat transfer coefficient for convection cooling, $T_{cc}(x, y, z, t)$ is the prescribed external temperature for convection cooling, and $T(x, y, z, t)$ is the sample surface temperature, because the equation is valid only for the positions on the sample boundary. The equation expresses the linear relation between the sample surface temperature and its gradient. A boundary condition of this type is utilized for free convection cooling at the sample back side [Fig. 1(a)].
Convective heat transfer for moving beam heating at the front side of the sample is described using the total heat transfer coefficient \( \alpha_T(x, y, z, t) \), computed from Eqs. (8) or (10) and the external temperature, called the laser beam temperature \( T_b \),

\[-\lambda(x, y, z, t) \frac{\partial T(x, y, z, t)}{\partial n} = \alpha_T(x, y, z, t)(T(x, y, z, t) - T_b(x, y, z, t)), \]

where the total heat transfer coefficient \( \alpha_T(x, y, z, t) \) is defined so as to include all the sample heating by the moving laser beam, which means both convective and radiative parts of the heating from the laser beam [Fig. 1(a)].

The radiation heat transfer boundary condition has the form

\[-\lambda(x, y, z, t) \frac{\partial T(x, y, z, t)}{\partial n} = \varepsilon_x(x, y, z, t)\sigma_0(T^4(x, y, z, t) - T_b(x, y, z, t)), \]

and expresses the radiative cooling of the sample. The quantity \( \varepsilon_x \) denotes the prescribed emissivity of the sample surface, \( \sigma_0 \) is the Stefan–Boltzmann constant, and \( T_{ec} \) is the external temperature for sample radiation cooling. The prescribed sample surface emissivity is assumed as a constant value, but the model created enables the utilization of a temperature-dependent value of sample surface emissivity. A boundary condition of this type is assumed at front and back sides of the sample for radiation cooling [Fig. 1(a)].

Because the convective heat transfer for moving beam heating is used only on the front side of the sample, where the position \( z = 0 \) holds, the full expression \( \alpha_T(x, y, 0, t) \) is substituted by the simplified form \( \alpha_T(x, y, t) \) in the following text.

### C. Characteristics of Complex Boundary Conditions

For the computation of time dependence of the total heat transfer coefficient \( \alpha_T \) for a certain position (certain computational node) on the thermally loaded sample side, it is necessary to know the basic heat transfer coefficient \( \alpha_y \) dependence on the distance from the beam axis \( l_{x,offset} \) in the \( x \)-axis direction, the actual position of the beam axis \( x_{axis} \) in the \( x \)-axis direction, the actual distance of the beam axis from the sample side \( l_{x,offset} \) in the \( x \)-direction, the dependence of the reduction coefficient \( c_{a_x} \) on the distance of the beam axis from the sample side \( x_{min} \) and \( x_{max} \) in the \( x \)-axis, and also \( c_{a_y} \) in the \( y \)-axis, directions, the dependence of the rotational symmetry of the laser spot. The advantage is to preserve the rotational symmetry of the laser beam.

### 2. Full Description of Boundary Conditions for the 3D Model

This approach gives a full precise description of the 3D task. It is used when commercial computation software enables a sufficient number of time curves. A full description of boundary conditions consists of the definition of time curves for all sample surface computational nodes.

The advantage is to preserve the rotational symmetry of the laser spot. A small disadvantage is the slight disruption of the rotational symmetry of the laser spot.

From the mathematical point of view, the dependence of the total heat transfer coefficient \( \alpha_T \) on the \( y \)-axis is replaced with the normalized heat transfer coefficient \( \alpha_{N} \). The normalized heat transfer coefficient is dependent on the distance from the beam axis in the \( y \)-axis direction and the actual position of the beam axis \( y_{axis} \) in the \( y \)-axis direction.

Characteristic courses of the basic heat transfer coefficient, reduction coefficients, actual positions of the beam axis, and the normalized heat transfer coefficient are schematically illustrated in Fig. 2(a) (input courses for model). The aim is to evaluate the dependence of total heat transfer \( \alpha_T(x, y, t) \), see Fig. 2(c). The time dependence of \( \alpha_T \) for certain values of \( x, y \) (positions on the loaded sample side) defines the heat transfer coefficient for individual computational nodes. These time dependencies for individual nodes can be directly loaded to the computational system during simulation model preparation.

The advantage of this model is a simpler evaluation of boundary conditions and the ability to utilize a small number of time curves. A small disadvantage is the slight disruption of the rotational symmetry of the laser spot.

### 1. Simple Description of Boundary Conditions for the 3D Model

This approach expresses a simple description of a 3D task. It is used when commercial computation software enables only a limited number of time curves. A simple description of boundary conditions expresses the definition of time curves only for computational nodes on the laser track at the sample surface. The times curves for other nodes at the sample surface are computed from time curves of laser track nodes using the multiplication coefficient called normalized heat transfer coefficient \( \alpha_{N} \). The simple 3D simulation model can be assumed to be an enhancement of the 2D model.

The advantage of this model is a simpler evaluation of boundary conditions and the ability to utilize a small number of time curves. A small disadvantage is the slight disruption of the rotational symmetry of the laser spot.

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Characteristic courses of the basic heat transfer coefficient, reduction coefficients, actual positions of the beam axis, and the normalized heat transfer coefficient are schematically illustrated in Fig. 2(a) (input courses for model) and Fig. 2(c) (final output curve). These time dependencies for individual nodes can be directly loaded to the computational system during preparation of the simulation model.
D. Mathematical Description of Complex Boundary Conditions

Mathematical equations of boundary conditions for the thermally loaded front side of the sample depend on the simplicity or complexity of their descriptions.
from the sample edge are 10 mm. The laser beam moves along a track passing over the center of the loaded surface; motion begins at the right reversal point and finishes at the same position.

Sample absorptivity \( a \) is equal to emissivity \( \varepsilon \) and emissivity for radiation cooling \( e_{\varepsilon} \) \((a = \varepsilon = e_{\varepsilon} = 0.7)\). External temperatures for radiation and convection coolings are equal to sample initial temperature \( T_{rc} = T_{cc} = T_{in} = 20^\circ C\).

Space distribution of the basic heat transfer coefficient \( \alpha_b(r) \) dependent on the distance from the beam axis is assumed according to Fig. 2(a). The quantity course is described by the values \( \alpha_b(r) = \alpha_{b,max} \) for \( r = 0 \), \( \alpha_b(r) = \alpha_{b,max}/2 \) for \( r = r_b/2 \), and finally \( \alpha_b(r) = 0 \) for \( r = r_b \). For the maximum of the basic heat transfer coefficient, the following holds:

\[
P = \varepsilon S_{red} \alpha_{b,max}(T_b - T_S),
\]

where \( S_{red} \) is the reduced surface of the laser spot \( S_{red} = \pi r_b^2 \), \( r_b = r_b/2 \), and \( T_S \) is the sample surface temperature. Beam temperature \( T_b \) is set to a specific value. Absolute values of the basic heat transfer coefficient and specific temperature of the laser beam are linked together and give the value of power \( P \).

Generally, the boundary condition coefficients, such as the emissivity \( \varepsilon \) and the basic heat transfer coefficient \( \alpha_b \), are assumed to be constant in the model. The constant value of the basic heat transfer coefficient corresponds with reality (when the surface does not melt), because the value does not change with the surface temperature nor with the state of the surface. However, the value of the emissivity undergoes small changes during the laser heat treatments even without surface melting. The precise values of the emissivity during the treatment process are not known. Therefore, a constant value of the emissivity is assumed. On the other hand, the simulation model created enables the utilization of temperature-dependent emissivity, if it is known.

**F. Simulated Cases of Laser Treatment**

Several simulation models have been created in order to compare heat distribution in the sample during various thermal laser processing procedures. A typical technological example of laser surface heat treatment is laser surface hardening [19,20]. Generally, the field of laser material treatment is a continually evolving area [21–23].

- **First simulation case.** A comparison of laser beam motion velocity is provided for three velocities 17.14, 24,
dimensionless times equal to 0.25 and 0.75, the heat source position is over the sample center. Using motion velocity 24 mm \( \cdot \) s\(^{-1}\), the surface temperature in the center of the track is 730°C during the first movement of the laser beam and approximately 740°C for the second beam movement. In the case of 40 mm \( \cdot \) s\(^{-1}\) motion velocity, the surface center temperature has its maximum about 870°C during the first beam movement, and 920°C during the second movement. These temperatures exceed the ones for the material heat treatment. In accordance with expectations, with the lowest motion velocity of 17.14 mm \( \cdot \) s\(^{-1}\), the surface center temperature maximum is higher than in the previous case. The surface center temperature maximum is approximately 1000°C for the first, and 1050°C for the second laser beam movement.

Figure 4 shows spatial courses of temperature in the \( x \)-direction passing the sample center. Dimensionless time is a parameter of temperature curves. At the dimensionless time \( \Theta = 0.75 \), the heat source is directly under the sample center during the second movement. The velocity value has a great effect on temperature spatial courses. Surface temperatures are high in the range 650°C–900°C, but they rapidly decrease with depth increase. The temperature is below 200°C at the 3 mm depth. At the dimensionless time \( \Theta = 1.0 \), the heat source is back at the right reversal point. The depth temperature profile is more balanced than at the time \( \Theta = 0.75 \). This denotes fast temperature equalization in the sample material. The different beam motion velocity has only a small influence on spatial courses of temperature at dimensionless time \( \Theta = 1.0 \).

The spatial profile of surface temperature in the direction perpendicular to the beam track is in Fig. 5. The parameter of the curves is dimensionless time again. At the dimensionless time \( \Theta = 0.75 \), the beam is directly under the sample center during the second movement. The width of the heat-affected zone has only slight differences, but the temperatures obtained in this zone vary for tested motion velocities. At the dimensionless time \( \Theta = 1.0 \), surface temperature profiles in the \( y \)-axis direction gradually flatten, similarly as in the \( z \)-axis direction (Fig. 4).

**B. Effect of Multiple Movements across the Same Track**

The sample temperature during the laser treatment with a number of movements along the same track is described in this section. The laser beam motion velocity is 24 mm \( \cdot \) s\(^{-1}\), which corresponds to 5 s of travel time between the opposite reversal points of the track. Four movements are done in total. The time courses of temperature at the center of the sample surface and several positions below are displayed in Fig. 6. During the first movement of the laser beam over the sample, the sample temperature at the center of the track reaches over 800°C. Increasing the number of movements, this temperature slightly increases to nearly 950°C during the fourth movement. Taking the depths of 1 and 2 mm, the maximum temperature at the track center decreases. The temperature of 600°C at the depth of 1 mm is exceeded until the third laser movement.

Figure 7 shows spatial courses of temperature both in the \( z \)-direction going through the sample center (lower x-axis in the graph, solid line) and in the \( y \)-axis direction (higher x-axis in the graph, dotted line). The first two time levels shown are 7.5 and 17.5 s (when the laser is passing through the center of the track).
in the second and fourth movements). The temperature rapidly decreases with increasing depth from the surface. The times when the laser is in the reversal position after the second and fourth movements (10 and 20 s) are characterized by more balanced temperature profiles.

The surface temperature profiles in the $y$-axis direction (dotted lines in Fig. 7) present the decrease of temperature with increasing distance from the track. At the times of 7.5 and 17.5 s, the width of the heat-affected zone is clearly visible in the figure. When the laser beam is outside the sample, the temperature profiles in the $y$-axis direction flatten similarly as in the $z$-axis direction. The maximum surface temperature is about 240°C at the end of the laser treatment.

C. Effect of One Movement across Multiple Tracks

When the laser treatment of a certain area should be done, a number of tracks are used to provide full coverage of this area, and the tracks are separated by some distance. In this section, three tracks in the $x$-axis direction, each separated by the distance of 20 mm, are used to test the simulation model created.

The laser beam motion starts at the A position (Fig. 8). Each track takes 5 s to travel, while $y$-axis motions take 1 s. The surface treatment ends when the laser beam reaches the B position.

Figure 9 shows time courses of surface and subsurface (depth 1 mm) temperatures in the center of each track. The red line shows the temperatures in the center of the first track. As the laser beam comes to the center of the track, the temperature increases and the maximum value is achieved when the laser spot is a small distance after the center of the track. Then the sample temperature at the track center rapidly decreases and the surface and subsurface (at the depth of 1 mm) temperatures equalize. The temperature courses at the center of tracks II and III have a similar character, only time shifted.

Temperature spatial profiles perpendicular to the laser tracks and passing their centers are shown in Fig. 10. The red line shows the temperature profile at the time 2.5 s, when the laser beam is over the center of track I. The temperature profiles at the times of 8.5 and 14.5 s have similar peaks. The peak values of temperature profiles shown are about 760°C, while their maximum values of approximately 870°C are attained several tenths of a second later.

The distance between the laser tracks in this sample heat treatment is too wide. Considering the temperature curves in Fig. 10, in order to achieve a uniform surface heat treatment, the distance between the laser tracks should be reduced. The temperature profiles at subsurface positions would have a similar trend, but distinctly lower values of temperature.
D. Sample Temperature Distribution and the Possibilities of Depth Evaluation of Laser Treatment

Figure 11 gives the image of spatial temperature distribution in the sample that undergoes thermal treatment using a moving laser beam. The figure shows the temperature state at the time 2.5 [Fig. 11(a)] and 17.5 s [Fig. 11(b)], when the laser beam is over the center of the track during the first and fourth movement. The maximum surface temperatures are 874°C and 964°C, respectively, on the heat-loaded sample surface. Transversal cross sections passing the sample center, Fig. 11 (x = 50 mm), indicate the shape of the heat-affected zone in the sample. Beam motion velocity 24 mm \cdot s^{-1} is considered in the simulation.

These temperature data can be further processed in order to evaluate the maximum temperature $T_{\text{max}}(x, y, z)$ at each sample position $(x, y, z)$ during the entire laser treatment process:

$$T_{\text{max}}(x, y, z) = \max\{T(x, y, z); t \in [t_i, t_f]\}, \tag{21}$$

where $t_i, t_f$ are the initial and final times of the process.

Taking the hardening temperature $T_h$ (Subsection 2.A), the region of the material where the laser hardening has been performed can be defined by the equation

$$T_{\text{max}}(x, y, z) \geq T_h. \tag{22}$$

The depth of hardening $d_h$ is the thickness of the laser hardening region and can be defined as

$$d_h(x, y) = z, \quad \text{where} \quad T_{\text{max}}(x, y, z) = T_h. \tag{23}$$

The cases of multiple movements across the same track and one movement across multiple tracks have been selected for evaluation of the depths of hardening. The hardening depth $d_h$, dependent on the distance from the laser track in the case of multiple movements, can be observed in Table 2. The evaluation is done in the $y$-axis direction from the center of the sample surface ($x = 50$ mm) for positions that are 0, 1, and 2 mm from the track. For positions that are farther than 2 mm, the number of laser movements necessary for hardening of a small subsurface layer increases. The depths of hardening from 0.2 or 0.3 mm are used for real ordinary applications. In the results from the simulation model, the hardening depth of 0.3 mm is achieved in the laser track (0 mm from the laser track) after the second laser movement. Considering the third laser movement, a hardening depth of 0.3 mm is obtained at the position 1 mm from the track. Commercial laser hardening is performed at slower velocities of about 10 mm \cdot s^{-1}, approximately; thus the depth of hardening in the laser track can reach 1 mm and the hardening depth of 0.3 mm can be found several millimeters from the laser track.

Table 3 is evaluated from the simulation of one movement across multiple tracks; only the positions on the tracks are presented. A small hardening depth of about 0.2 mm is achieved on laser tracks II and III. In the case of real hardening with slower laser motion velocity, the laser-treated zone gets both deeper into the sample material and further from the laser tracks.

<table>
<thead>
<tr>
<th>No. of Tracks/Distance from Track (mm)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.168</td>
<td>0.201</td>
<td>0.207</td>
</tr>
<tr>
<td>II</td>
<td>0.207</td>
<td>0.227</td>
<td>0.070</td>
</tr>
<tr>
<td>III</td>
<td>0.167</td>
<td>0.076</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*The depths of hardening $d_h'(mm)$ reached in the center of individual tracks.

Table 2. Multiple Movements across the Same Track*

<table>
<thead>
<tr>
<th>No. of Movements/ Distance from Track (mm)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.167</td>
<td>0.076</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.302</td>
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<td>3</td>
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<td>0.305</td>
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</tr>
<tr>
<td>4</td>
<td>0.434</td>
<td>0.364</td>
<td>0.217</td>
</tr>
</tbody>
</table>

*The depths of hardening $d_h'(mm)$ reached in the track and at the perpendicular distance of 1 and 2 mm from the track.

4. CONCLUSIONS

The established three-dimensional model of sample heat transfer during surface heat treatment using a moving laser...
Multiple laser beam movements across the sample utilizing 1049°C. The great power of the established simulation model lies in the possibility of prediction of sample maximum temperatures during the entire heat treatment process. Using maximum temperature evaluation, the isotherm surfaces in the sample volume can be constructed. When a temperature value of the isotherm is set to the laser hardening temperature, the appropriate isotherm would predict the laser hardening region in the sample. Using the established simulation model, it is possible to investigate the effect of process parameters (laser beam motion velocity, heat source power, thermal properties of processed material, etc.), or to perform process optimization without the necessity of doing a series of experiments.

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REFERENCES

Queries

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