Statistical solution of 3D transformation problem

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ABSTRACT

Obtaining the 3D model of an object is currently one of the most important issues that image processing is dealing with. Measurement of the points on 3D objects requires different scans from different positions in different coordinate systems. At our disposal are measured coordinates of an identical point, which can be obtained from a laser 3D scanner, depth sensor, or any motion input device as Microsoft Kinect. A point whose coordinates are known in both coordinate systems is called an identical point. Data transformation of identical points from one coordinate system to another coordinate system is therefore required. The aim of this contribution is to present a possible approach on how to estimate the unknown transformation parameters by regression models in a special transformation problem. This transformation in its standard version has been derived under the assumption that non-negligible random errors occur at points of that coordinate system into which the transformation is performed. Points of the inverse image coordinate system are assumed to be errorless.

Keywords

Transformation of coordinates, estimators of transformation parameters, Helmert transformation, nonlinear regression model, linearization

1 INTRODUCTION

The problem of obtaining a precise 3D digital model of some object today is a very actual theme. It is used for example in geodesy, civil engineering, 3D printing, etc. Measuring of points of 3D objects requires different scans from different sites. Measuring can be performed by terrestrial laserscanning, photogrammetry, using Microsoft Kinect etc. Coordinates of measured points on an object are given in different coordinate systems.

For a full object description, transformation of coordinates to common coordinate system is needed. Such a problem is connected with an estimation of parameters of transformation between different coordinate systems. Relatively large potential for the use of such models arise there, see [PPZN05],[SB05], [Sha06], [ZK03], [JSS15].

For simplicity, let us use case where we have two different systems of coordinates. We will assume that a measurement error appears only in the second system. The transformation problem includes an estimation of translational parameters and an estimation of a rotation matrix between the two coordinate systems. Studied transformation problem leads to solving the overdetermined system of equations. These equations are often solved using numerical methods, rely on linearization, approximate starting values and iterations. The rotation matrix can be expressed by Cardan angles, Euler angles, angles yaw, pitch and roll, cf. [GA03], [LCSG13], [SMH14]. Large class of statistical problems and different approach for solutions of transformation problem arise there. Literature offer solutions based on Gauss-Jacobi algorithm, Procrustes algorithms, etc. In some cases, where the initial starting parameters are far from true values of unknown parameters, iteration proceses may fail to converge. In geodesy an geoinformatics, is often succesfully used constraints on orthogonality of rotation matrix. Therefore, the problem we describe by a model with constraints. A detailed inspection of regression model with constraints is provided in [Kub88], [Kub93], [KKV95], [KM04], [KM04].

2 TRANSFORMATION PROBLEM

Our problem is an estimation of the transformation coefficients $\beta_1, \ldots, \beta_{12}$ specifying the transformation between the systems of coordinates.

Let us suppose that $i = 1,\dots,n$ are identical points at our disposal for the transformation. The formulated transformation problem can be written in the following form

$$\begin{aligned} \mu_i &= \begin{pmatrix} \mu_{i1} \\ \mu_{i2} \\ \mu_{i3} \end{pmatrix} = \phi(\beta,\nu) = \gamma + \mathbf{T}\nu_i \qquad (1) \\ &= \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \beta_4, & \beta_5, & \beta_6 \\ \beta_7, & \beta_8, & \beta_9 \\ \beta_{10}, & \beta_{11}, & \beta_{12} \end{pmatrix} \begin{pmatrix} \nu_{i1} \\ \nu_{i2} \\ \nu_{i3} \end{pmatrix}, \\ i &= 1,\dots,n. \end{aligned}$$

Instead of vectors of actual coordinates $\nu_{I,i}$ (System I), $i = 1,\dots,n$, and $\mu_{II,i}$ (System II), $i = 1,\dots,n$, estimators of them only are at our disposal.

The estimator of μ has the form of random vectors $\mathbf{Y}_{II} \sim (\mu_{II}, \Sigma_{\mathbf{Y}_{II}})$; the mean value of the random vector $\mathbf{Y}_{II}$ is $E(\mathbf{Y}_{II}) = \mu_{II} = (\mu_{II,1},\dots,\mu_{II,n})'$ and its covariance matrix $\mathrm{var}(\mathbf{Y}_{II}) = \Sigma_{Y_{II}})$.

We will assume $\Sigma_{\mathbf{Y}_{II}}$ is a positive definite (i.e. regular).

The problem in determining the optimum estimators of the unknown transformation parameters and of the transformed coordinates of the identical points simultaneously is caused by the fact that they are not errorless, which leads to application of the least squares method. Note that there are corrections of the coordinates within the System II.

2.1 Orthogonality condition

We assume that the systems of coordinates are orthogonal. The transformation coefficients should satisfy the condition

$$\begin{aligned} \mathbf{h}(\beta) &= (h_1(\beta), h_2(\beta), h_3(\beta), h_4(\beta), h_5(\beta), h_6(\beta))' \\ &= (1,0,0,1,0,1)', \qquad (2) \end{aligned}$$

where

$$\begin{aligned} h_1(\beta) &= \beta_4^2 + \beta_5^2 + \beta_6^2, \\ h_2(\beta) &= \beta_4\beta_7 + \beta_5\beta_8 + \beta_6\beta_9, \\ h_3(\beta) &= \beta_4\beta_{10} + \beta_5\beta_{11} + \beta_6\beta_{12}, \\ h_4(\beta) &= \beta_7^2 + \beta_8^2 + \beta_9^2, \\ h_5(\beta) &= \beta_7\beta_{10} + \beta_8\beta_{11} + \beta_9\beta_{12}, \\ h_6(\beta) &= \beta_{10}^2 + \beta_{11}^2 + \beta_{12}^2. \end{aligned}$$

3 REGRESSION MODEL WITH CONSTRAINTS AND ITS LINEARIZATION

If we know good approximations $\mu_{I,0}$ of the vector μ_I and β_0 of the parameters $\beta_1,\dots,\beta_{12}$, respectively, such that the vectors $\delta\mu_I$ and $\delta\beta$ can be neglected ($\delta\mu_I = \mu_I - \mu_{I,0}$), then the linearized version of the transformation problem (1) with constraint (2) can be used. After linearization, we can write our model in form of linear model with linear constraint

$$\begin{pmatrix} \widehat{\mu}_1 - \mathbf{X}_1\beta_0 \\ \dots \\ \widehat{\mu}_I - \mathbf{X}_I\beta_0 \end{pmatrix} \sim_{3I} \left[\begin{pmatrix} \mathbf{X}_1 \\ \dots \\ \mathbf{X}_I \end{pmatrix} \delta\beta, \begin{pmatrix} \Sigma_1 & \dots & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & \dots & \Sigma_I \end{pmatrix} \right], \qquad (3)$$

$$\mathbf{b} + \mathbf{B}\delta\beta = 0 \qquad (4)$$

Let consider $\Sigma_i = \sigma^2\mathbf{V}$, the value of σ can be adapted from the documentation protocol of the measurement device. In our case, $\Sigma_i = (0.03\text{ m})^2 \cdot \mathbf{I}_{3\times 3}$.

The best linear unbiased estimator of unknown parameter is (see [KKV95], [K13])

$$\begin{aligned} \hat{\widehat{\delta\beta}} &= \left\{ \mathbf{I} - \mathbf{CB}' \left[\mathbf{BCB}'\right]^{-1} \mathbf{B} \right\} \times \\ &\times\ \mathbf{C}(\mathbf{X}_1'\Sigma_1^{-1}, \mathbf{X}_2'\Sigma_2^{-1}, \dots, \mathbf{X}_I'\Sigma_I^{-1}) \times \\ &\times \begin{pmatrix} \hat{\mu}_1 - \mathbf{X}_1\beta_0 \\ \vdots \\ \hat{\mu}_I - \mathbf{X}_I\beta_0 \end{pmatrix} - \mathbf{CB}' \left[\mathbf{BCB}'\right]^{-1} \mathbf{b}, \\ \mathbf{C} &= \left(\sum_{i=1}^{I} \mathbf{X}_i'\Sigma_i^{-1}\mathbf{X}_i \right)^{-1}, \qquad (5) \\ \mathbf{X}_i &= \mathbf{1}_n \otimes \gamma + (\mathbf{I}_n \otimes \mathbf{T})\,\nu_i, \qquad (6) \end{aligned}$$

$\mathbf{I}$ is an identical matrix, (7)

$\mathbf{1}$ is a matrix with entries equal to 0, (8)

$\otimes$ is the Kronecker product. (9)

Matrix $\mathbf{B} = \frac{\partial \mathbf{h}}{\partial \beta}$ is of the form

$$\mathbf{B} = (\mathbf{0}_{6,3}, \mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3) \qquad (10)$$

$$\mathbf{B}_1 = \begin{bmatrix} 2\beta_{0,4} & 2\beta_{0,5} & 2\beta_{0,6} \\ \beta_{0,7} & \beta_{0,8} & \beta_{0,9} \\ \beta_{0,10} & \beta_{0,11} & \beta_{0,12} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} 0 & 0 & 0 \\ \beta_{0,4} & \beta_{0,5} & \beta_{0,6} \\ 0 & 0 & 0 \\ 2\beta_{0,7} & 2\beta_{0,8} & 2\beta_{0,9} \\ \beta_{0,10} & \beta_{0,11} & \beta_{0,12} \\ 0 & 0 & 0 \end{bmatrix},$$

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$$\mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{0,4} & \beta_{0,5} & \beta_{0,6} \\ 0 & 0 & 0 \\ \beta_{0,7} & \beta_{0,8} & \beta_{0,9} \\ 2\beta_{0,10} & 2\beta_{0,11} & 2\beta_{0,12} \end{bmatrix}$$

and $\mathbf{b}$ is of the form

$$\mathbf{b} = \begin{pmatrix} \beta_{04}^2 + \beta_{05}^2 + \beta_{06}^2 - 1 \\ \beta_{04}\beta_{07} + \beta_{05}\beta_{08} + \beta_{06}\beta_{09} \\ \beta_{0,4}\beta_{0,10} + \beta_{0,5}\beta_{0,11} + \beta_{0,6}\beta_{0,12} \\ \beta_{0,7}^2 + \beta_{0,8}^2 + \beta_{0,9}^2 - 1 \\ \beta_{0,7}\beta_{0,10} + \beta_{0,8}\beta_{0,11} + \beta_{0,9}\beta_{0,12} \\ \beta_{0,10}^2 + \beta_{0,10}^2 + \beta_{0,11}^2 + \beta_{0,12}^2 - 1 \end{pmatrix}, \quad (11)$$

$$\mathbf{X}_i = (\mathbf{I}, \mathbf{X}_i^1, \mathbf{X}_i^2, \mathbf{X}_i^3), \quad (12)$$

$$\mathbf{X}_i^1 = \begin{pmatrix} v_{i,1} & v_{i,2} & v_{i,3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{X}_i^2 = \begin{pmatrix} 0 & 0 & 0 \\ v_{i,1} & v_{i,2} & v_{i,3} \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{X}_i^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ v_{i,1} & v_{i,2} & v_{i,3} \end{pmatrix}.$$

3.1 Conversion of coordinates

For conversion of the non-identical points to another system of coordinates we have the formula

$$\hat{\mu}_j = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \beta_4 & \beta_5 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \\ \beta_{10} & \beta_{11} & \beta_{12} \end{pmatrix} v_j, \quad (13)$$

or formula

$$\hat{v}_j = -\begin{pmatrix} \beta_4 & \beta_5 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \\ \beta_{10} & \beta_{11} & \beta_{12} \end{pmatrix} \left[\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} - \hat{\mu}_j \right]. \quad (14)$$

4 EXAMPLE

Let us think about the task in Figure 1.

The points in the left picture were measured in coordinate system S_1 from position 1. The number of measured points is 27. Let us sign them with symbol μ.

The points in the right picture were measured in coordinate system S_2 from position 2. The number of measured points is 24. Let us sign them with symbol v.

In both scans I identical points were polarized. In our case on the picture, the number of identical points is 14.

The first three points were the corners of the building and the fourth was the top of the tower. The coordinates are given at Table 1 and 2.

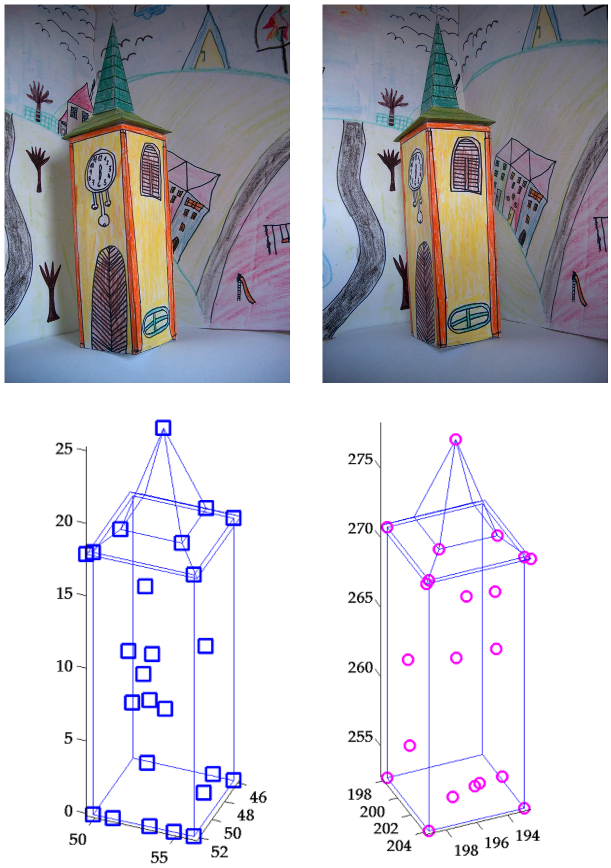

Figure 1: 3D object.

By transformation of formula (1) we get the non-consistent system of equations (in our case 42 equations with 12 unknowns). To get a solution, the method of least squares can be used. Another way is to get the solution using the first 4 identical points.

Using the method of least square we get the solution:

i	v_i		
1	199.0011	197.7993	252.8010
2	198.9996	203.9991	252.8037
3	192.7959	204.0007	252.7993
4	195.8986	200.8992	278.2588
5	199.0003	197.8015	270.8577
6	199.0032	204.0042	270.8596
7	192.8020	203.9973	270.8624
8	197.7968	202.7980	272.0078
9	193.9998	202.8021	272.0113
10	198.9980	201.1369	257.1768
11	198.9975	200.9067	263.2470
12	195.9970	204.0031	255.2122
13	197.4546	204.0014	254.8240
14	197.2252	204.0039	264.8107

Table 1: Measured points — first scan

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i	μ_i		
1	51.9913	49.7667	-0.1975
2	52.0058	55.9771	-0.1762
3	45.8238	55.9992	-0.1935
4	48.8862	52.9172	25.2851
5	52.0035	49.7963	17.8745
6	51.9882	56.0437	17.8573
7	45.8023	56.0213	17.8612
8	50.7981	54.7834	19.0159
9	46.9733	54.8143	19.0425
10	51.9681	53.1109	4.1871
11	51.9920	52.9199	10.2655
12	49.0106	56.0258	2.2254
13	50.4793	55.9760	1.8276
14	50.2220	55.9679	11.8168

Table 2: Measured points — second scan

$$\hat{\beta}^0 = \begin{pmatrix} \beta_1^0 & \beta_4^0 & \beta_5^0 & \beta_6^0 \\ \beta_2^0 & \beta_7^0 & \beta_8^0 & \beta_9^0 \\ \beta_3^0 & \beta_{10}^0 & \beta_{11}^0 & \beta_{12}^0 \end{pmatrix} =$$

$$\begin{pmatrix} -146.7812 & 0.9984 & 0.0015 & -0.0008 \\ -148.4136 & -0.0020 & 1.0022 & 0.0013 \\ -252.6847 & -0.0008 & -0.0010 & 1.0002 \end{pmatrix}.$$

The problem is that our solution does not satisfy the orthogonality condition $\mathbf{h}(\beta_0) = 0$ because

$$\mathbf{h}(\hat{\beta}_0) = \begin{pmatrix} 0.9969 \\ -0.0005 \\ -0.0016 \\ 1.0045 \\ 0.0003 \\ 1.0004 \end{pmatrix}.$$

In our case, $\Sigma_i = (0.03\ \mathrm{m})^2 \cdot \mathbf{I}_{3\times 3}$. Now using the formula (5), we get the estimate

$$\delta\hat{\hat{\beta}} = \begin{pmatrix} \hat{\hat{\beta}}_1 & \hat{\hat{\beta}}_4 & \hat{\hat{\beta}}_5 & \hat{\hat{\beta}}_6 \\ \hat{\hat{\beta}}_2 & \hat{\hat{\beta}}_7 & \hat{\hat{\beta}}_8 & \hat{\hat{\beta}}_9 \\ \hat{\hat{\beta}}_3 & \hat{\hat{\beta}}_{10} & \hat{\hat{\beta}}_{11} & \hat{\hat{\beta}}_{12} \end{pmatrix} =$$

$$\begin{pmatrix} -0.5992 & 0.0016 & 0.0012 & 0.0002 \\ 0.6167 & -0.0007 & -0.0022 & -0.0001 \\ -0.1496 & 0.0014 & -0.0001 & -0.0004 \end{pmatrix}.$$

The final result is obtained using the formula

$$\hat{\hat{\beta}} = \hat{\beta}^0 + \delta\hat{\hat{\beta}} =$$

$$\begin{pmatrix} -147.3804 & 1.0000 & 0.0027 & -0.0006 \\ -147.7969 & -0.0027 & 1.0000 & 0.0012 \\ -252.8344 & 0.0006 & -0.0011 & 0.9998 \end{pmatrix}.$$

and the constraints (2) is now of the form

$$\mathbf{h}(\hat{\hat{\beta}}) = \begin{pmatrix} 1.0000 \\ -0.0000 \\ 0.0000 \\ 1.0000 \\ 0.0001 \\ 0.9996 \end{pmatrix}.$$

Let us try to recalculate the coordinates of a point that matches the top of the clock, which can be found in Fig. 1. In the system S_2, the point has coordinates

$$\mu = \begin{pmatrix} 52.0000 \\ 53.0000 \\ 16.2963 \end{pmatrix}.$$

By using the formula (14), where we put estimate $\hat{\hat{\beta}}$ to $\beta_1, \ldots, \beta_{12}$, we get $\hat{\nu} = \begin{pmatrix} 199.7611 \\ 200.5815 \\ 268.9756 \end{pmatrix}$.

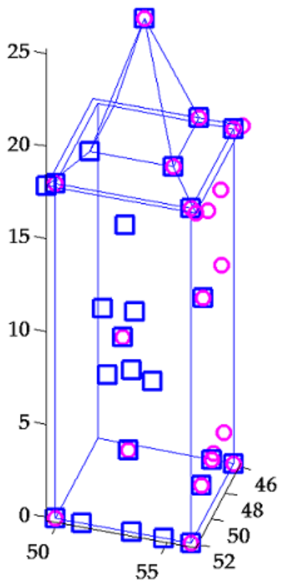

Figure 2: Points after transformation.

5 CONCLUDING REMARKS

In this paper, the problem to determine the optimum estimators of the unknown transformation parameters and of the transformed coordinates of the identical points simultaneously is solved.

A rotation transformation matrix and shift parameters can be estimated using a regression model, where rotation parameters fulfill the orthogonality constraints. This constraints is barely fulfilled for the initial solution/average solution.

To illustrate the whole process of estimation, we presented the results of an algorithm in an example.

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