Efficient Linear Local Features of Digital Signals and Images: Computational and Qualitative Properties

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ABSTRACT

The paper presents the analysis of efficiency of two original approaches to the construction of the sets of linear local features (LLF), which are used for digital signal and image processing. The first approach is based on generating of LLF set, which consists of separately constructed efficient LLFs, each of which has its own algorithm for feature calculation. The second approach assumes the construction of an efficient LLF set, which has a single algorithm for joint simultaneous computation of all features. The analysis is carried out by several indicators that characterize the computational and qualitative properties of the constructed LLFs.

Keywords

Features, digital images and signals, computational complexity, processing quality.

1. INTRODUCTION

Feature creation is one of the main stages of visual data processing systems development and it affects the final quality of the system. A local feature of a digital signal is usually a numerical characteristic - the result of a transformation of digital signal/image samples, which belong to a local analysis area [1]. For linear local features (LLF) this transformation is linear with constant parameters. Taking into account, that calculation of LLF values can be made in different ways (direct algorithms or fast convolution, recursive algorithms, etc.), a specific LLF is characterized by two components – a linear convolution kernel (we call it as *LLF's kernel*) and an algorithm for calculation of the convolution of the input signal/image and this kernel (we call it as *LLF's algorithm* or *algorithm for LLF values calculation*). Moreover, if LLF's kernel determines *qualitative characteristics* of the specific LLF, the algorithm for LLF values calculation characterizes *computational complexity* of the feature. Sets of features, which have not just one but several feature values for the same analysis area of a digital signal, are usually used to solve practical problems. It is essential, that calculation of the corresponding feature values in a set can be produced by several independent algorithms as well as a general algorithm that executes jointly simultaneously calculations for all the values of features in a set. In the latter case we speak about a *set of jointly computed features*. Qualitative indicators (for sets of jointly and independently calculated LLFs) are determined by a set of corresponding kernels. The general formulation of the *problem of constructing an efficient (set of) LLFs* implies the constructing LLFs (or set of LLFs) with the best quality indicator and with specified computational complexity [2-4]. Despite the seeming simplicity of the presented formulation, we should accept the problem of constructing features and their sets extremely complex.

In the author's paper [2] the formal approach for efficient LLFs construction has been proposed, and in the papers [3, 4] this approach has been extended to the case of constructing an efficient set of jointly calculated LLFs. These approaches allow us to design an efficient LLF (or efficient set of LLFs) for the most applied problems. The term "*efficiency of LLF*" refers to the satisfaction of *two basic requirements*:

– algorithm for LLF values calculation has a predetermined computational complexity value;
– LLF's kernel(s) is(/are) the best matched to a given quality indicator.

Under the preceding requirements efficient LLFs enable us to establish a reasonable balance between two opposing groups of features:

– features, which are optimal in the sense of some quality criteria and do not have suitable or fast computation algorithm (e.g., features, obtained using Karhunen-Loeve transform);
– features, which are obtained by using fast algorithms and are not related to the content of the

problem and relevant quality indicators (e.g., features, obtained using fast Fourier transform algorithm).

According to the information of author, the only alternative approach of the feature construction, that satisfies all requirements mentioned above, exists. It was proposed by Prof. V.Labunets in 2013 and was denoted as «multiparametric wavelet transforms» [12,13]. Unfortunately, these papers do not provide the method of solving the efficient LLFs construction problem, they only show that multiparametric (or adaptive) wavelets exist and can be constructed.

The *main purpose of this paper* is to analyze/compare the author's two approaches to constructing sets of LLFs. The first approach constructs a set of features by constructing a set of efficient LLFs, each of which has its own algorithm for feature calculation. The second approach constructs an efficient set of LLFs, in which there is a single algorithm for computing all features jointly. Short description of these approaches is presented in the Section 2, where the known information is collected. New results on analytical and experimental analysis of these approaches are presented in Sections 3 and 4.

2. SETS OF JOINTLY AND INDEPENDENTLY CALCULATED LINEAR LOCAL FEATURES OF DIGITAL SIGNALS: BACKGROUND

This Section presents short reference information on the efficient linear local features of the digital signals: basic definitions, equations and construction methods. Full description may be found in the papers [2-4].

Let **N** be a set of natural numbers, **K** be a commutative ring with unity, $\{x(n)\}_{n=0}^{N-1}$ be an input signal of length *N* over the ring **K**.

Definition 1. A *linear local feature (LLF) of length M over the ring* **K** is a pair $\left(\{h(m)\}_{m=0}^{M-1}, A\right)$, where $\{h(m)\}_{m=0}^{M-1}$ is a linear convolution kernel of length *M*, which is determined as a finite sequence over the ring **K** and satisfies the constraint $h(m) \neq 0, h(M-1) \neq 0$, and *A* is an algorithm for calculating a linear convolution (1) of an arbitrary input signal over the ring **K** with the kernel $\{h(m)\}_{m=0}^{M-1}$:

$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m), \quad n = \overline{M-1, N-1}. \tag{1}$$

A *set of R independently calculated LLF* of length M over the ring **K** is a further set of LLFs:

$$\left\{\left(\left\{h_r^{ind}(m)\right\}_{m=0}^{M-1}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}.$$

Definition 2. A *set of R jointly calculated LLFs over the ring* **K** is a pair $\left(\{h_r(m)\}_{\substack{m=\overline{0,M-1},\\ r=\overline{0,R-1}}}, A\right)$, where $\{h_r(m)\}_{\substack{m=\overline{0,M-1},\\ r=\overline{0,R-1}}}$ is a set of *R* kernels, each of which is determined as a finite sequence over the ring **K** and satisfies the following constraints:

$$h_0(0) \neq 0; \quad \forall r \in \overline{0,R-1} \quad \exists m \in \overline{0,M-1} \quad h_r(m) \neq 0;$$
$$\exists\, r \in \overline{0,R-1} \quad h_r(M-1) \neq 0;$$

and *A* is an algorithm for joint calculation of a set of linear convolutions of an arbitrary input signal $\{x(n)\}_{n=\overline{0,N-1}}$ $(M < N)$ over the ring **K** with a set of kernels:

$$y_r(n) = h_r(n) * x(n) = \sum_{m=0}^{M-1} h_r(m)x(n-m), \quad n = \overline{M-1, N-1}, \quad r = \overline{0, R-1}. \tag{2}$$

To distinguish the elements of sets of independently calculated LLFs from jointly calculated LLFs the last will be denoted as follows:

$$\left(\left\{h_r^{set}(m)\right\}_{\substack{m=\overline{0,M-1},\\ r=\overline{0,R-1}}}, A^{set}\right) \square$$

In author's papers [2-4] we proposed a method for construction of the sets of independently and jointly calculated LLFs, based on designing (sets of) sequences of kernel's samples in the form of *linear (mutual) recurrent sequences* (LRS or LMRS, respectively) [5,6,9]. For these (sets of) sequences, called NMC-(sets) sequences[1], the computational complexity of calculating linear convolutions (1) or (2) is minimal. For fixed parameters of *linear (mutual) recurrent relations* (LRR or LMRR, respectively) these sets of NMC sequences or NMC-sets of sequences form a collection of sequences, denoted, respectively $\wp(M, K, \bar{c})$ or $\wp(R, M, T, K, \bar{a})$. Here *K* is an order of LRR for samples of a sequence, *R* is a number of sequences in a set, *T* is an order of mutual recurrence (for sets), $\bar{c}$ and $\bar{a}$ are LRR's or LMRR's coefficients respectively. As it has been shown in papers [2-4], the powers of these collections satisfy the relations:

$$\begin{aligned}
&\forall M > K \geq 1,\ \bar{a}(a_K \neq 0) \quad \left|\wp(K, M, \bar{c})\right| \leq C_{M+K-2}^{K-1}, \\
&\forall M > K \geq 1 \quad R \geq T \geq 1 \\
&\left|\wp_{(b,c,d)}(R, M, T, K, \bar{a})\right| \leq C_{R(M+K)-1}^{RK} - C_{R(M+K-1)-1}^{RK}.
\end{aligned} \tag{3}$$

[1] NMC - normalized with minimal complexity

Each sequence from the collection, along with its parameters, is also characterized by a set Θ of additional independent parameters – *degrees of freedom*. The powers of degrees of freedom sets Θ are determined in the following way [2-4]:

$$\left|\Theta_{\wp(M,K,\bar{c})}\right| = K, \quad \left|\Theta_{\wp(R,M,T,K,\bar{a})}\right| = RK\ . \qquad (4)$$

The computational complexity of algorithms A^{ind} and A^{set} for calculating relevant features or sets of features for all NMC sequences or NMC sets of sequences from collections $\wp(M,K,\bar{c})$ and $\wp(R,M,T,K,\bar{a})$ is determined by these equations [2-4]:

$$u\left(A^{ind}\right) \le 2K \frac{N}{N-M+1}\ , \qquad (5)$$

$$u\left(A^{set}\right) \le \frac{N}{N-M+1}\begin{pmatrix} R(K-1)-(R-1)\xi_{add} + \\ +(K+1)T\left(R-\frac{T-1}{2}\right)\end{pmatrix}.\ (6)$$

The *problems* of construction of an *efficient (set of) LLF(s)* are defined as follows [2-4]. A *particular problem of construction of an efficient set of LLFs* is defined as a problem of searching in a predefined collection $\wp(R,M,T,K,\bar{a})$ of such a set (with its corresponding algorithm of joint calculation of LLFs A^{set}), for which the minimum condition for a problem-specific objective function $\Psi : \mathbf{K}^{RM} \to \mathbf{R}$ is fulfilled:

$$\Psi\left(h_0(0),\ldots,h_0(M-1),\ldots,h_{R-1}(0),\ldots,h_{R-1}(M-1)\right) \to \min_{\left\{h_r^{set}(m)\right\}_{m=\overline{0,M-1},\ r=\overline{0,R-1}} \in \wp(R,M,T,K,\bar{a})}. \qquad (7)$$

For *a particular problem of construction of an efficient LLF* the drafting changes are related to a collection $\wp(M,K,\bar{c})$ and an objective function $\Psi : \mathbf{K}^{M} \to \mathbf{R}$.

The difference in the solutions of these problems lies in the fact that in the first case a set of jointly calculated LLFs is formed $\left(\left\{h_r^{set}(m)\right\}_{m=\overline{0,M-1},\ r=\overline{0,R-1}}, A^{set}\right)$ and in the second case there is only one LLF constructed $\left(\left\{h(m)\right\}_{m=0}^{M-1}, A\right)$. Note that using a particular problem of constructing an efficient LLF it is possible to construct a set of independently calculated features $\left\{\left(\left\{h_r^{ind}(m)\right\}_{m=0}^{M-1}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}$, for example by their consequent construction with appropriate modification of objective functions for each of particular problems.

The computational complexity of calculation of the sets of LLFs and the number of their degrees of freedom can be used as indicators or constraints in the analysis of constructed sets of jointly and independently calculated LLFs. Additionally, for further analysis we can introduce a formalized notion of collections “comparability” of jointly and independently calculated LLFs as follows.

Let’s consider a set of LMRS $\left\{h_r(m)\right\}_{m=\overline{0,M-1}}$ $\left(r=\overline{0,R-1}\right)$, which belongs to collection $\wp(R,M,T,K,\bar{a})$ and satisfies a LMRR [3,4]:

$$h_r(m) = \begin{cases} b_{rm}, \quad r=\overline{0,T-1},\ m=\overline{0,K-1}, \\ \sum\limits_{k=1}^{\min(K,m)} a_{0k}^r h_r(m-k) + \\ + \sum\limits_{t=1}^{\min(r,T-1)} \sum\limits_{k=0}^{\min(K,m)} a_{tk}^r h_{r-t}(m-k) + \varphi_r(m), \\ \qquad r \ge T \vee m \ge K. \end{cases} \qquad (8)$$

In case, when $\varphi_r(m) \equiv 0$, LMRS and LMRR are called *homogeneous* [5,6,9]. The following lemma defines characteristics of the sequences in this set.

Lemma (on solution of homogeneous LMRR). Let $T=R\ge1$ and a homogeneous LMRR of order (T,K)

$$h_r(m) = \sum_{k=1}^{K} a_{0k}^r h_r(m-k) + \sum_{t=1}^{r}\sum_{k=0}^{K} a_{tk}^r h_{r-t}(m-k), \quad r=\overline{0,R-1}$$

determines the samples of the collection of R sequences $\left\{h_r(m)\right\}_{r=\overline{0,R-1};\ m=0,1,\ldots}$ for the entire domain. Let us define matrixes $Q_r(z)$ of size $r \times r$, where each element $q_{ij}^r(z)$ is determined $\left(q_{ij}^r(z) \equiv q_{ij}^t(z) \quad \forall i,j < \min(r,t)\right)$ with an expression:

$$q_{ij}^r(z) = \begin{cases} \sum\limits_{k=1}^{K} a_{0k}^i z^{-k} - 1, & i=j, \\ 0, & i<j, \quad i,j=\overline{0,r-1}. \\ \sum\limits_{k=0}^{K} a_{(i-j)k}^i z^{-k}, & i>j, \end{cases}$$

Then every r-th sequence of the collection for the entire domain satisfies the following homogeneous LRR:

$$h_r(m) = \sum_{s=1}^{K(r+1)} c_s^{r+1} h_r(m-s), \quad r=\overline{0,R-1},$$

where the values $\left\{c_s^r\right\}_{s=1}^{Kr}$ are coefficients in the matrix $Q_r(z)$ determinant:

$$\det\left(Q_r(z)\right) = \prod_{i=0}^{r-1}\left(\sum_{k=1}^{K} a_{0k}^i z^{-k} - 1\right) = 1 - \sum_{s=1}^{Kr} c_s^r z^{-s}\ . \qquad \blacksquare$$

It is obvious, that under the lemma's conditions, the sequence of the collection with number r satisfies the homogeneous LRR with order not exceeding $K(r+1)$. This proved connection allows us to give the following definition for "*comparability*" of jointly and independently calculated LLF collections.

Definition 3. A set of collections of LRSs $\{\wp(M, K_r, \bar{c}^r)\}_{r=\overline{0,R-1}}$ and a collection of LMRRs $\wp(R, M, T, K, \bar{a})$ are called *comparable*, if these equations are valid:

$$K_r = K(r+1),$$
$$\prod_{i=0}^{r-1}\left(\sum_{k=1}^{K} a_{0k}^{i} z^{-k} - 1\right) = 1 - \sum_{s=1}^{Kr} c_s^r z^{-s}, \quad r = \overline{0, R-1}.$$

The fact of compatibility means that one can specify for at least one (homogeneous) set of sequences from $\wp(R, M, T, K, \bar{a})$ exactly the same set of sequences from $\{\wp(M, K_r, \bar{c}_r)\}_{r=\overline{0,R-1}}$. Note also that although there are more than one equal sets of sequences for comparable collections the full match of sets of sequences doesn't happen.

The results of this section allow us to make an analytical comparison of comparable sets of collections.

3. COMPUTATIONAL AND QUALITATIVE PROPERTIES: ANALYTICAL COMPARISON

3.1 Comparison of Linear Local Features Sets for Comparable Collections

Let $N,\ R, M, T, K \in \mathbf{N}$, and $\left(\{h_r^{set}(m)\}_{\substack{r=\overline{0,R-1};\\ m=\overline{0,M-1}}}, A^{set}\right)$ is an arbitrary efficient set of LLFs for a collection $\wp(R, M, T, K, \bar{a})$. Computational complexity of the algorithm of calculation of the LLF, corresponding to any set of sequences of this collection, satisfies the equation (6). From the other hand, one can construct independent efficient LLFs $\left\{\left(\{h_r^{ind}(m)\}_{m=\overline{0,M-1}}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}$ from the comparable $\wp(R, M, T, K, \bar{a})$ set of collections $\{\wp(M, K_r, \bar{c}^r)\}_{r=\overline{0,R-1}}$. Then, taking into account equations (5), computational complexity of LLF set calculation $\left\{\left(\{h_r^{ind}(m)\}_{m=\overline{0,M-1}}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}$ is determined as follows:

$$\sum_{r=0}^{R-1} u\left(A_r^{ind}\right) \le \frac{N}{N-M+1} KR(R+1). \tag{9}$$

Comparing the right part of this equation with the equation (6), one can assure of the following relation correctness:

$$\left(\begin{array}{l} R(K-1) - (R-1)\xi_{add} + \\ \qquad + (K+1)T\left(R - \dfrac{T-1}{2}\right) \end{array}\right) < KR(R+1). \tag{10}$$

Then the following statement is correct.

Statement 1. Let $K, R, M, T \in \mathbf{N}$, $K \ge 1$, $R \ge T \ge 2$, sets of LLFs $\left(\{h_r^{set}(m)\}_{\substack{m=\overline{0,M-1},\\ r=\overline{0,R-1}}}, A^{set}\right)$ and $\left\{\left(\{h_r^{ind}(m)\}_{m=0}^{M-1}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}$ are constructed for comparable collections $\wp(R, M, T, K, \bar{a})$ and $\{\wp(M, K_r, \bar{c}_r)\}_{r=\overline{0,R-1}}$ correspondingly, while relations (5) and (6) are satisfied as equalities. Then

$$u\left(A^{set}\right) < \sum_{r=0}^{R-1} u\left(A_r^{ind}\right). \tag{11}$$

This statement makes it possible to confirm the potential *computationally* benefits of jointly calculated LLFs in comparison with sets of independently calculated efficient LLFs designed for comparable collections.

3.2 Comparison of Linear Local Features Sets with Equal Number of Degrees of Freedom

Equation (4) means that the number of degrees of freedom for the specific efficient set of LLFs from the collection $\wp(R, M, T, K, \bar{a})$ is equal to *KR*. From the other hand, one can construct $\tilde{R}$ independent efficient LLFs from collections $\{\wp(M, K_r, \bar{c}^r)\}_{r=\overline{0,\tilde{R}-1}}$ in such a way, that the overall number of degrees of freedom becomes equal *KR* too. It is easy to prove that in this case the following equality is valid:

$$\tilde{R}(\tilde{R}+1) = 2R. \tag{12}$$

Using (12) one can assure the following relation correctness ($K, R, \tilde{R}, M, T \in \mathbf{N},\ \ K \ge 1,\ \ R \ge T \ge 2$):

$$\left(\begin{array}{l} R(K-1) - (R-1)\xi_{add} + \\ \qquad + (K+1)T\left(R - \dfrac{T-1}{2}\right) \end{array}\right) > K\tilde{R}(\tilde{R}+1).$$

Statement 2. Let $K, R, M, T \in \mathbf{N}$, $K \ge 1$, $R \ge T \ge 2$, jointly and independently calculated LLFs have equal number of degrees of freedom (i.e. equation (12) is correct), while relations (5) and (6) are satisfied as equalities. Then

$$u\left(A^{set}\right) > \sum_{r=0}^{\tilde{R}-1} u\left(A_r^{ind}\right). \quad (13)$$

This statement makes it possible to confirm the potential *computational* benefits of set of independently calculated LLFs in comparison with the set of jointly calculated efficient LLFs designed for equal number of degrees of freedom.

3.3 Comparison of the Computational Complexity of Solving the Particular Problem of Features Construction

Let $\wp(R, M, T, K, \bar{a})$ and $\left\{\wp\left(M, K_r, \bar{c}^{\,r}\right)\right\}_{r=\overline{0,R-1}}$ are comparable collections of jointly and independently calculated LLFs. To compare the calculational complexities of the solving of the particular tasks of LLFs $\left(\left\{h_r^{set}(m)\right\}_{\substack{r=\overline{0,R-1};\\ m=\overline{0,M-1}}}, A^{set}\right)$ and $\left\{\left(\left\{h_r^{ind}(m)\right\}_{m=\overline{0,M-1}}, A_r^{ind}\right)\right\}_{r=\overline{0,R-1}}$ construction (see Section 2), we have to compare the number of sequences in the collections $\wp(R, M, T, K, \bar{a})$ and $\left\{\wp\left(M, K_r, \bar{c}^{\,r}\right)\right\}_{r=\overline{0,R-1}}$. In the case of the collection $\wp(R, M, T, K, \bar{a})$ the number of sequences is defined by equation (3). When we form the set of sequences from the collections $\left\{\wp\left(M, K_r, \bar{c}^{\,r}\right)\right\}_{r=\overline{0,R-1}}$, we can use two obvious strategies:

- *exhaustive search* (optimal solution)*:* in this case the number of sequences sets takes the form:

$$\prod_{r=0}^{R-1}\left|\wp\left(M, K(r+1), \bar{c}^{\,r}\right)\right|;$$

- *incremental search* (quasi-optimal solution)*:* in this case we search for the sequence of the *r*-th collection when the sequence of the (*r*-1)-th collection is found. The number of possible sets of sequences has the form: $\sum_{r=0}^{R-1}\left|\wp\left(M, K(r+1), \bar{c}^{\,r}\right)\right|$.

Taking into account equations (3), we can compare the computational complexity of solving the particular problem of LLFs construction by comparing the value $C_{R(M+K)-1}^{RK} - C_{R(M+K-1)-1}^{RK}$ with $\prod_{r=0}^{R-1} C_{M-2+(r+1)K}^{(r+1)K-1}$ (exhaustive search case) or $\sum_{r=0}^{R-1} C_{M-2+(r+1)K}^{(r+1)K-1}$ (incremental search case).

It may be done by analyzing the following ratios:

exhaustive search: $$\frac{C_{R(M+K)-1}^{RK} - C_{R(M+K-1)-1}^{RK}}{\prod_{r=0}^{R-1} C_{M-2+(r+1)K}^{(r+1)K-1}}, \quad (14)$$

incremental search: $$\frac{C_{R(M+K)-1}^{RK} - C_{R(M+K-1)-1}^{RK}}{\sum_{r=1}^{R} C_{M-2+(r+1)K}^{(r+1)K-1}}. \quad (15)$$

Using (15) we can prove the following statement.

Statement 3. Let $K, R, M, T \in \mathbf{N}$ $K \geq 1$, $R \geq T \geq 2$, $M > RK + 1$. Then

$$C_{R(M+K)-1}^{RK} - C_{R(M+K-1)-1}^{RK} > \sum_{r=0}^{R-1} C_{M-2+(r+1)K}^{(r+1)K-1}.$$

This statement makes it possible to confirm that solving of the particular problem of jointly calculated LLFs construction is more difficult than the solving of the particular problem of independent calculated LLFs. Direct numerical analysis of the ratio (15) for useful parameters range (*M*=21...32; *R*=1..4) shows that it is much more difficult: values of the ratio (15) are in the range [1, 5.7*10^9].

Unlike the situation is considered with an incremental search, it the case of exhaustive search it is not possible to make an unambiguous conclusion. Direct numerical analysis of the ratio (14) for parameters ranges mentioned above shows that it is in the range [7.2*10^-8, 3.97].

Finally, we can conclude that:

- *quasi-optimal solution* of the particular problem of independently calculated LLFs construction, based on the incremental search, is less difficult then the optimal solution of the particular problem of jointly calculated LLFs construction;
- *optimal solution* of the particular problem of independently calculated LLFs construction, based on the exhaustive search, may be radically difficult then the optimal solution of the particular problem of jointly calculated LLFs construction. So, when we are going to find optimal solution, jointly calculated LLFs are preferable.

3.4 Analytical Comparison: Conclusion

Analytical and numerical results presented in this Section above make it possible to conclude that the analytical analysis cannot provide the unambiguous answer on the question what type of LLFs (sets of independently or jointly calculated LLFs) is better. Therefore, we are trying to answer this question using experiments.

4. COMPUTATIONAL AND QUALITATIVE PROPERTIES: EXPERIMENTAL COMPARISON

In order to complete the comparison of the sets of independently and jointly calculated LLFs and to compare them with existent typical ways of linear local features calculations we will consider several illustrative tasks. In every task we will compare computational and qualitative properties of the constructed LLFs.

Despite of the illustrative character of the chosen tasks, they appear often in real applications in similar formulations, and explicit criteria and mathematical model of the processing signal is necessary only to point out the best (from typical ways of linear local features calculations) set of feature kernels.

So, general problem statement is as follows. Let we have a digital signal that may be interpreted as a realization of the discrete stationary random process $X(n)$ with zero mean and autocorrelation function:

$$R(n) = D_x \rho^{|n|}, \quad n \geq 0\,, \tag{16}$$

here $D_x = 1, \quad \rho = 0{,}95$, for definiteness. We allow that the length of the processing signal *N* is unlimited and to perform the local analysis of the signal in the specific position n_0 we have to use *M*=33 samples of the signal (i.e. «processing window»): $X(n_0), \ldots, X(n_0 + M - 1)$. Also, we allow that the quality of the local analysis of the signal depends directly on the *quality indicator*, that is given by the following equation:

$$J_\alpha = \alpha \cdot \frac{E\left(\sum_{m=0}^{M-1}\left(\sum_{r=0}^{R-1} Y_r h_r(m) - X(m)\right)^2\right)}{E\left(\sum_{m=0}^{M-1}(X(m))^2\right)} + \\ + (1-\alpha)\frac{2}{R(R-1)}\sum_{r=0}^{R-2}\sum_{t=r+1}^{R-1}\frac{\langle h_t, h_r\rangle^2}{\|h_t\|\cdot\|h_r\|}, \quad (\alpha \in [0,1]). \tag{17}$$

Here $\{h_r(m)\}_{r=\overline{0,R-1};\ m=\overline{0,M-1}}$ is a set of kernels that is used for linear representation of the analyzed fragment of the signal, $E(\ldots)$ - the mathematical expectation operator. Obviously, the less the quality indicator the better the set of features.

In the equation (17) the first term defines relative error of the representation of the signal fragment using weighted sum of LLF's kernels, the second term shows the correlation rate of the kernels, and the denominator of the first term satisfies the equality:

$$E\left(\sum_{m=0}^{M-1} X^2(m)\right) = D_x M \quad (= 33).$$

Let define the *general problem as follows*: we have to obtain the set of kernels $\{h_r(m)\}_{r=\overline{0,R-1};\ m=\overline{0,M-1}}$ and algorithm(s) of calculation of the set of convolutions (2) of the signal with these kernels, which provide minimal value of the quality indicator (17) and satisfy certain restriction on the computational complexity of convolutions (2) calculation:

$$\begin{cases} J_\alpha \to \min \\ u(\ldots) \leq u_{\max}. \end{cases} \tag{18}$$

Bellow, we provide several ways to solve the problem (18). First and second methods (solutions, that are ordinary used in digital signal and image processing) use "optimal" kernels, that comes from Karhunen-Loewe decomposition [7] of the fragment of the discrete stationary random process (16). The only difference between these methods is the convolution algorithms. First method (*method 1*) uses the direct convolution algorithm, and the second one (*method 2*) uses the fast convolution algorithm, that is based on the Fast Fourier Transform (FFT) [8,10] and optimal sectioning of the processing signal [10]. In practice, the second method is the de facto standard for solutions of this type of problems. *Method 3* uses the set of jointly calculated LLF's, and *methods 4-7* use the sets of independently calculated LLF's (description of these methods is given bellow). It should be noted that the detail description of the problem (18) when α=1 using the set of jointly calculated LLF's was given in the paper [4]. Some useful equations, that are used here for calculation of an error of representation of the fragment of the discrete stationary random process using non-orthogonal kernels, were given in that paper too.

We analyze solutions of the problem (18) for three values of parameter α, namely:

- *group 1*: α=1,
- *group 2*: α=0,
- *group 3*: α=1/2.

Solution of the problem (18) using sets of independently or jointly calculated LLFs (methods 3-7) is performed by solving the particular problem (7) of constructing an efficient set of LLFs. This particular problem [2-4] means that the LLF's kernels are from the specific collection, and this collection is defined both by the task restrictions (the size *M* of the "processing window" and the upper bound $u_{\max}$ of the calculational complexity of features calculation), and subjective chosen parameters $T, K, \bar{a}$ and $\{\bar{c}^r\}_{r=\overline{0,R-1}}$. In our experiments, parameters are as follows:

- *method 3*: collection $\wp(R, M, T, K, \bar{a})$, parameters:

$T = 2, K = 1, \quad a_{01} = 1, a_{10} = 1, a_{11} = a_{10} = 1$;

- *methods 4-7*: collections $\{\wp(M, K_r, \bar{c}^r)\}_{r=\overline{0,R-1}}$, parameters:

- quasi-polynomial (*method 4*):

$c_k^r = (-1)^{k+1} C_{(r+1)K}^k, \quad \left(K = 1, \quad k = \overline{1,(r+1)K}\right)$;

- quasi-exponential (*method 5*):

$c_k^r = ((r+1)K)^{-1}\rho^k, \quad \left(K = 1, \quad k = \overline{1,(r+1)K}\right)$;

- quasi- Fibonacci (*method 6*):

$R = 1,2: \quad c_1^1 = \; c_1^2 = c_1^1 = 1;$

$R = 3: \quad c_1^3 = 1/2, \quad c_2^3 = 3/2, \quad c_3^3 = 1/2;$

$R = 4: \quad \ldots$

- quasi-harmonic (*method 7*):

$R = 1: \; c_1^1 = 1,$

$R = 2: \; c_2^1 = 2\cos(\omega), \quad c_2^2 = -1;$

$R = 3: \; c_3^1 = \cos(\omega), c_3^2 = 2\cos^2(\omega) - 1, c_3^3 = -\cos(\omega);$

$R = 4: \quad \ldots$

Presented collection names are derived from the names of the sequences ($R \geq 2$), that satisfy the homogeneous LMRS (8) with the same parameters.

The calculational complexity of the independently and jointly calculated LLFs is defined by equations (5)-(6), that were used as equalities.

Figures 1,3-5 present the obtained results, that show the dependence of the quality indicator J_α of the constructed features on the computational complexity of the features calculation $u(\ldots)$. These results lead to the following conclusions.

- For the first group of the tasks (α=1, Fig.1) quality indicators for the sets of independently and jointly calculated LLFs (methods 3-4) are significantly less (i.e. the quality is significantly higher) then the quality indicators obtained for «optimal» kernels (obtained using Karhunen-Loewe decomposition) and direct (method 1) or fast (method 2) convolution algorithms. Particularly, when the calculational complexity of the features calculation satisfies $u_{\max} = 40$ the *quality* of the set of jointly calculated LLFs *is six time higher* (vs method 2)! For this particular case, Fig.2 shows four constructed kernels for the jointly calculated LLFs. It is easy to see that these kernels are similar to the «optimal» kernels (sinusoids of different phases and frequencies), that may be obtained using Karhunen-Loewe transform.
- For the first group of the tasks (α=1, Fig.3) quality indicators for the set of jointly calculated LLFs is less (i.e. the quality is higher) then the quality indicators for the sets of independently calculated LLFs.
- For the 2nd and 3rd groups of the tasks (α<1, Figs.4-5) quality indicator for all types of LLFs depends significantly on the collection parameters. Therefore, changing these parameters we can obtain different answers which type of feature sets (set of jointly or set of independently calculated LLFs) is better. In practice, the best type of LLFs may be found using global optimization methods: genetic algorithms, simulated annealing, etc.

The obtained experimental results allow us to make two conclusions:

- the proposed efficient LLFs have advantage in comparison with the traditional way of solving such a type of problems, even when the "optimal" kernels/bases exist;
- jointly and independently calculated efficient LLFs have comparable efficiency, i.e. neither of two approaches has clear advantages.

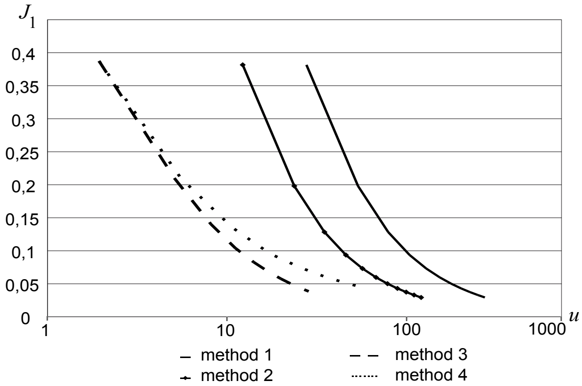

Figure 1. Comparison of the proposed efficient LLFs (methods 3-4) with traditional way of features construction (methods 1-2); task group 1: α=1.

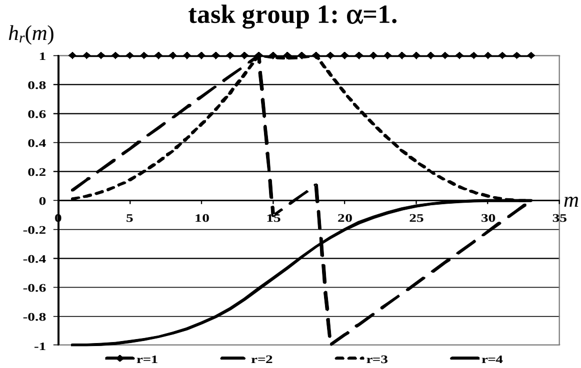

Figure 2. First four constructed kernels for the jointly calculated LLFs (for convenience, we put kernels to the range [-1,1]).

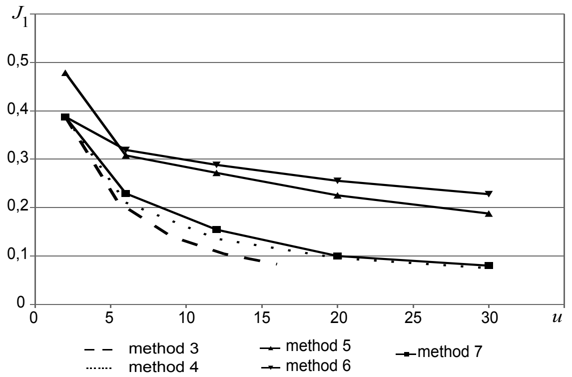

Figure 3. Analysis of the proposed efficient LLFs: comparison of the sets of jointly (method 3) and independently (methods 4-7) calculated LLFs; task group 1: α=1.

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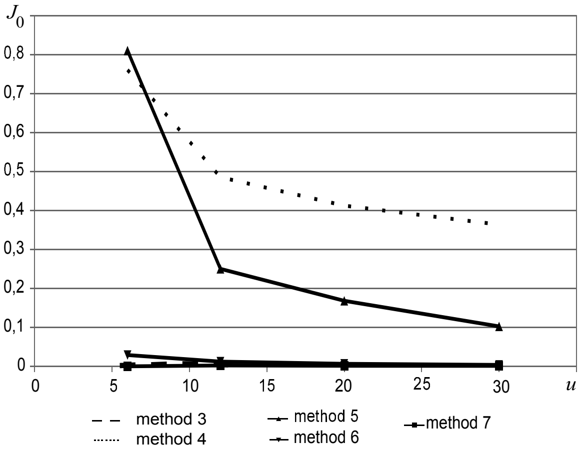

Figure 4. Analysis of the proposed efficient LLFs: comparison of the sets of jointly (method 3) and independently (methods 4-7) calculated LLFs; task group 2: α=0.

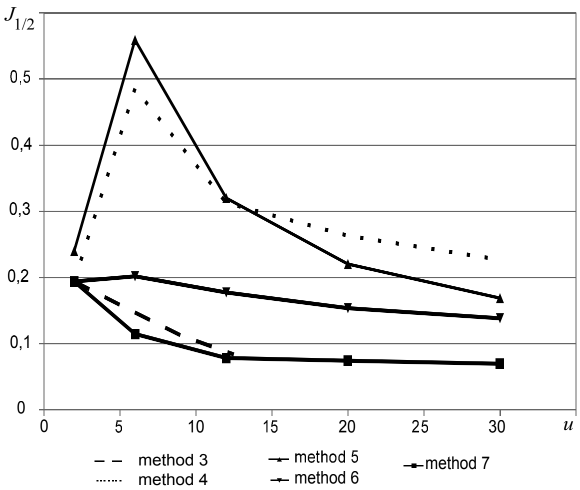

Figure 5. Analysis of the proposed efficient LLFs: comparison of the sets of jointly (method 3) and independently (methods 4-7) calculated LLFs; task group 3: α=1/2.

5. CONCLUSIONS

In this paper two approaches to the construction of a set of linear local features for digital signals are analyzed. It is shown that, depending on the comparison criteria the proposed approaches can have advantages and disadvantages. In the general case, it can be concluded that these approaches are comparable by efficiency value (in terms of parameters pair - quality and computational complexity). This fact allows the developer of a particular signal or image processing system to choose the approach that is convenient and/or familiar to him. Conducted in the paper experiments show, that the proposed approaches have convincing advantages over a typical "best" way to solve the model digital image analysis/representation problem (in terms of parameters pair - quality and computational complexity).

Further research will be related to the following:
- development of alternative ways to introduce efficient linear local features;
- development of numerical methods and algorithms for a quick solution of the particular (and extended particular) problem of constructing an efficient set of jointly calculated LLFs and set of independently calculated efficient LLFs.

6. ACKNOWLEDGMENTS

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