Implementation of Constant Phase Elements Using Low-Q Band-Pass and Band-Reject Filtering Sections

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Abstract – **This paper briefly describes possibility to use band-pass and notch filters with low quality factors for straightforward synthesis of the so-called constant phase elements. These filters connected in cascade can form desired alternation of transfer function zeroes and poles. Using proposed approach one-quarter, half and three-quarter fractional-order integrator dedicated for audio and speech applications has been derived and verified by using numerical analysis. Discovered method can be generalized to arbitrary frequency range, allowed phase error and almost any biquad topology.**

Keywords-analog filter; biquadratic section; circuit synthesis; constant phase element; transfer function; zeroes and poles

I. Introduction

Recently various signal processing applications of fractional-order (FO) devices attract increasing interest among analog design engineering community. Circuits described by FO differential equations can be superior in generation of the harmonic signals [1-2], provide a better control in feedback loops via FO PID regulators [3], develop new types of the filtering structures [4] or enrich capabilities of the existing topologies [5-7].

In practice these FO elements are considered to be either two-terminals (fractal capacitors) or two-ports (fractional integrators). Let suppose non-integer value of system order in the interval $\alpha \in (0, 1)$. First case is characterized by impedance $Z(s)=Z_0 \cdot s^{-\alpha}$ with module decreasing with slope 20α if plotted in the log-log frequency scale and constant phase shift between driving source (current) and response (voltage) $-90\alpha°$ in theoretically infinite frequency range. Similarly FO integrator has ideal transfer function in the Laplace transform $K(s)=V_{out}(s)/V_{in}(s)=K_0 \cdot s^{-\alpha}$, i.e. module in dB is decreasing with slope 20α and phase shift should remain constant and fixed at $-90\alpha°$ value. Note that fractional capacitor forms a bridge between resistor having $\alpha=0$ and conventional capacitor with $\alpha=1$.

Due to the natural and unique properties mentioned above these devices are usually denoted in literature as constant phase elements (CPE). Unfortunately CPEs with associated behavior close enough to the ideal one are not commercially available so far. Despite of huge efforts with different materials the manufacturing problems are still serious and satisfactory solution probably will not be available in the near future.

II. Decomposition of Transfer Function

Due to serious fabrication difficulties CPEs must be precisely approximated in predefined operational frequency range by a more complicated analog system with mathematical order much higher that desired FO. It seems that the best approach how to construct two-terminal device with FO dynamics is to adopt passive RC ladder structure described in [8]. Formulas given here can lead us to reasonable values of resistors and capacitors. Of course phase frequency response is still rippled around ideal value; for user-defined frequency range design process undergoes compromise between allowed phase error and circuit complexity.

For real practical applications it is believed that phase error less than 0.5° is necessary. Assuming that operational frequency band for CPE approximation begins with 10 Hz and ends with 10 kHz this precision can be achieved by using the voltage transfer function with eight zeroes and the same number of poles. Using concept of cascade of the bilinear filters eight sections are necessary; each having at least one active element in order to prevent interactions between individual sections. Thus final transfer function can be written as

$$K(s) = K_0 \prod_{k=1}^{8} \frac{s+\omega_{nk}}{s+\omega_{pk}} \qquad \omega_{n1} < \omega_{p1} < \omega_{n2} < \ldots \quad (1)$$

and leads to phase frequency response in degrees

$$\varphi(\omega) = \frac{180}{\pi} \sum_{k=1}^{8} \left(\arctan\frac{\omega}{\omega_{nk}} - \arctan\frac{\omega}{\omega_{pk}} \right). \quad (2)$$

This kind of two-port CPE synthesis is ineffective and should be replaced by a cascade of two band-pass (BP) and the same number of band-reject (BR) filters, i.e. voltage transfer function decomposed into sequence

$$K(s) = \prod_{k=1}^{4} K_k \frac{s^2 + \frac{\omega_{Nk}}{Q_{Nk}} s + \omega_{Nk}^2}{s^2 + \frac{\omega_{Pk}}{Q_{Pk}} s + \omega_{Pk}^2}, \quad (3)$$

where $Q_{Nk}<0.5$ and $Q_{Pk}<0.5$ simultaneously such that roots of nominator and denominator of (3) are real and different numbers. Obviously a combination of BP and BR filter can make a desired formation of four transfer zeroes and poles which alternate on the real axis of the left half-plane of the complex plane.

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III. CPE Dedicated for Audio Applications

Approximation coefficients for audio-domain CPE as well as proposed synthesis method were verified by numerical calculations in Mathcad.

In the case of quarter integrator first BP in cascade has following parameters ω_{N1}=65.16rad/s, Q_{N1}=0.098, ω_{P1}=98.03rad/s and Q_{P1}=0.383 while main parameters of BR are ω_{N2}=66.8rad/s, Q_{N2}=0.383, ω_{P2}=98.11rad/s and Q_{P2}=0.101. Second BP filter has ω_{N3}=29.7krad/s, Q_{N3}=0.101, ω_{P3}=43.29krad/s and Q_{P3}=0.383 while BR has to be implemented by considering ω_{N4}=29.6krad/s, Q_{N4}=0.383, ω_{P4}=46.98krad/s and Q_{P4}=0.093.

In order to obtain half capacitor first BP in cascade should be characterized by parameters ω_{N1}=39.94rad/s, Q_{N1}=0.088, ω_{P1}=98.11rad/s and Q_{P1}=0.383 while main parameters of BR filter are ω_{N2}=45.6rad/s, Q_{N2}=0.382, ω_{P2}=98.107rad/s and Q_{P2}=0.101. Second BP filter has parameter ω_{N3}=20.32krad/s, Q_{N3}=0.1, ω_{P3}=43.32krad/s and Q_{P3}=0.384 while BR should be implemented by using main parameters ω_{N4}=20.24krad/s, Q_{N4}=0.383, ω_{P4}=53.56krad/s and Q_{N4}=0.082.

Three-quarter capacitor can be modeled by using first BP in cascade with parameters ω_{N1}=20.79rad/s, Q_{N1}=0.067, ω_{P1}=98.1rad/s and Q_{P1}=0.383 while main parameters of BR filtering section are ω_{N2}=31.24rad/s, Q_{N2}=0.383, ω_{P2}=98.14rad/s and Q_{P2}=0.101. Second BP filter in cascade has parameters ω_{N3}=13.86krad/s, Q_{N3}=0.101, ω_{P3}=43.29krad/s and Q_{P3}=0.383 while BR should be implemented by using parameters of transfer zeroes ω_{N4}=13.83krad/s, Q_{N4}=0.383 and transfer poles ω_{P4}=53.57krad/s and Q_{N4}=0.082.

Above mentioned audio CPE active realizations using BP and BR filters have been numerically verified as shown in Fig. 1 where module frequency response is drawn by using solid line and phase shift is represented by dotted lines. Note that approximation is perfect providing phase error less than 0.2°.

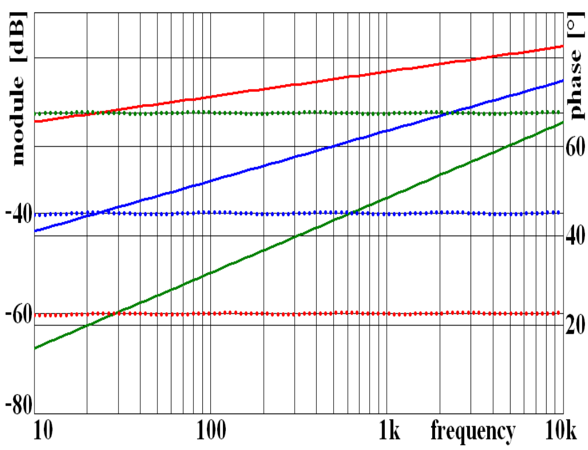

Figure 1. Numerical confirmation of designed CPE valid in audio frequency range: α=1/4 (red), α=1/2 (blue) and α=3/4 (green).

IV. Different Circuit Implementations

Unlikely common CPE two-port realizations which employs a cascade of the bilinear filtering sections [9] much simpler concepts are suggested in this paper. Two examples are provided in upcoming subsections.

However the list of all possible implementations is much longer containing biquadratic filtering sections having almost any active element and functional block (both off-the-shelf and hypothetical), mixed-mode or current-mode multifunctional filters, etc.

A. *CPE via audio equalizers*

First possible circuitry implementation of CPE can be considered as analog audio equalizer; composed by series resonant tanks with lossy synthetic inductors as shown in Fig. 2. Integrated circuit AD844 is used as the only active devices. It is second generation current conveyor followed by a voltage buffer. Thus voltage transfer function can be expressed as

$$K(s)=\frac{V_{out}(s)}{V_{in}(s)}=\prod_{k=1}^{4}\frac{L_{k2}}{L_{k1}}\cdot\frac{s^2+\frac{R_{k2}}{L_{k2}}s+\frac{1}{L_{k2}C_k}}{s^2+\frac{R_{k1}}{L_{k1}}s+\frac{1}{L_{k1}C_k}}, \quad (4)$$

where both resonant sub-circuits utilize the same value of capacitor. In order to keep real values of transfer function zeroes and poles following inequalities need to be satisfied

$$R_{k1}>\sqrt{\frac{4L_{k1}}{C_k}} \quad \wedge \quad R_{k2}>\sqrt{\frac{4L_{k2}}{C_k}}, \quad (5)$$

where i=1, 2, 3, 4. Basic biquad parameters can be calculated by following formulas derived directly from comparison of (3) and (4) as

$$L_{k2}=\frac{1}{\omega_{Nk}^2 C_k} \qquad R_{k2}=\frac{1}{\omega_{Nk}Q_{Nk}C_k}, \quad (6)$$
$$L_{k1}=\frac{1}{\omega_{Pk}^2 C_k} \qquad R_{k1}=\frac{1}{\omega_{Pk}Q_{Pk}C_k}$$

It is obvious that location of transfer zeroes and poles can be adjusted independently.

Quarter integrator can be achieved by using values for the first section C_1=1mF, L_{12}=236mH, R_{12}=157Ω, L_{11}=104mH, R_{11}=27Ω, second filter with parameters C_2=1mF, L_{22}=224mH, R_{22}=39Ω, L_{21}=104mH, R_{21}=101Ω, third section having C_3=10nF, L_{32}=113mH, R_{32}=33kΩ, L_{31}=53mH, R_{31}=6kΩ and finally two-port with following passive components values C_4=10nF, L_{42}=114mH, R_{42}=8821Ω, L_{41}=45mH and R_{41}=22.9kΩ.

Half integrator can be constructed by considering values for the first two-port C_1=1mF, L_{12}=627mH, R_{12}=284.5Ω, L_{11}=104mH, R_{11}=26.6Ω, second filter with C_2=1mF, L_{22}=481mH, R_{22}=57.4Ω, L_{21}=104mH, R_{21}=100.9Ω, third filter with parameters C_3=10nF, L_{32}=242mH, R_{32}=49.21kΩ, L_{31}=53mH, R_{31}=6kΩ, and final two-port having following components C_4=10nF, L_{42}=244mH, R_{42}=12.9kΩ, L_{41}=35mH and R_{41}=22.8kΩ.

Three quarter integrator can be obtained by using values of first filter C_1=1mF, L_{12}=2.3H, R_{12}=717.57Ω, L_{11}=104mH, R_{11}=26.6Ω, parameters of second two-port C_2=1mF, L_{22}=1.027H, R_{22}=83.7Ω, L_{21}=104mH, R_{21}=100.9Ω, third section with C_3=10nF, L_{32}=521mH, R_{32}=71.44kΩ, L_{31}=53mH, R_{31}=6031Ω and final filter having C_4=10nF, L_{42}=523mH, R_{42}=18.9kΩ, L_{41}=21mH and R_{41}=22780Ω.

Note that this realization does not allow external electronic control of CPE approximation (CPE order α cannot be changed) unless self-inductance of synthetic inductors is achieved. Contributions of the individual blocks in cascade to the module and phase frequency response are given in Fig. 3. Also note that inductors have quite large values of self-inductances. However this does not represent a big problem because synthetic equivalents of the loss inductors are supposed here. Many circuit structures can do the trick; for example well-known Prescott's inductor.

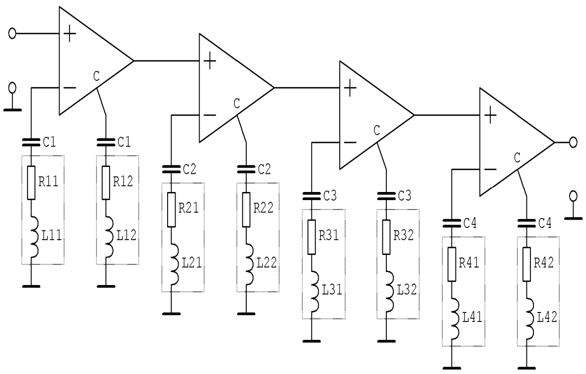

Figure 2. Complete CPE circuit implemented by using general audio equalizer technique.

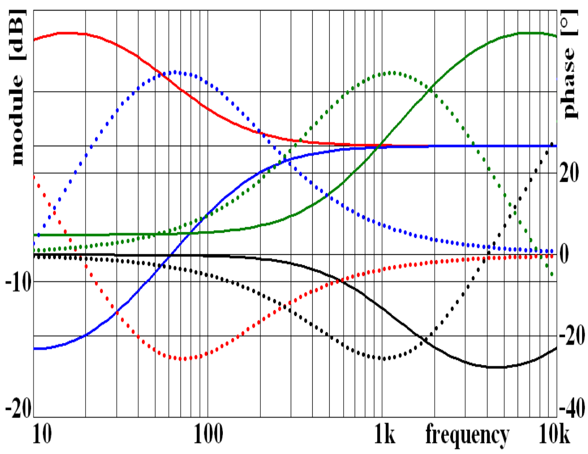

Figure 3. CPE implemented by four-segment audio equalizers: first block (red), second two-port (blue), third block (green) and final biquad (black).

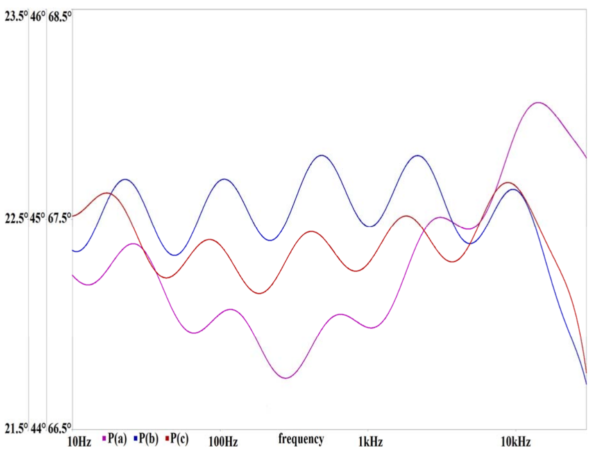

Figure 4. Visualization of phase shift ripple in the operational frequency range from 10Hz up to 30kHz, vertical scale 90α°±1° for values α=1/4, α=1/2 and α=3/4.

B. *FDNR based transfer function synthesis*

Second proposed CPE approximation network is very similar to previous one; both from the viewpoint of function and final complexity. Individual sections utilize a couple of the so-called frequency dependent negative resistors (FDNR) which leads to the voltage transfer function in Laplace transform

$$K(s)=\frac{V_{out}(s)}{V_{in}(s)}=\prod_{k=1}^{4}\frac{s^2+\frac{C_{k1}}{D_k}s+\frac{G_{k1}}{D_k}}{s^2+\frac{C_{k2}}{D_k}s+\frac{G_{k2}}{D_k}}, \qquad (7)$$

where D_k is k-th super-capacitance. Similarly to audio equalizer two-port transfer zeroes and poles can be adjusted independently by capacitors and resistors. Basic design rule for this network concept can be expressed as

$$C_{k1}>\sqrt{4D_kG_{k1}} \quad \wedge \quad C_{k2}>\sqrt{4D_kG_{k2}}\,, \qquad (8)$$

for i=1, 2, 3, 4. Since fraction (7) is analogical to (4) circuit components can be calculated by using similar formulas

$$R_{k1}=\frac{1}{\omega_{Nk}^2D_k} \qquad C_{k1}=\frac{\omega_{Nk}D_k}{Q_{Nk}}$$
$$R_{k2}=\frac{1}{\omega_{Pk}^2D_k} \qquad C_{k2}=\frac{\omega_{Pk}D_k}{Q_{Pk}}\,, \qquad (9)$$

where super-capacitance in each section can be chosen arbitrarily to prevent circuit realizations with extreme values of the passive elements.

Using this conception quarter integrator can be realized by considering first filtering stage $D_1=10^{-8}$, R_{11}=23.55kΩ, C_{11}=6.65μF, R_{12}=10.41kΩ, C_{12}=2.56μF, second stage with $D_2=10^{-8}$, R_{21}=22.4kΩ, C_{21}=1.744μF, R_{22}=10.39kΩ, C_{22}=9.714μF, third two-port having $D_3=10^{-12}$, R_{31}=1134Ω, C_{31}=294nF, R_{32}=534Ω, C_{32}=113nF and final section with parameters $D_4=10^{-12}$, R_{41}=1141Ω, C_{41}=77.3nF, R_{42}=453Ω, C_{42}=50.5nF.

Half integrator dedicated for audio applications can be achieved by using first section with $D_1=10^{-8}$, R_{11}=62.7kΩ, C_{11}=4.54μF, R_{12}=10.39kΩ, C_{12}=2.56μF, second section with parameters $D_2=10^{-8}$, R_{21}=48.1kΩ, C_{21}=1.193μF, R_{22}=10.39kΩ, C_{22}=9.714μF, third two-port with values $D_3=10^{-12}$, R_{31}=2422Ω, C_{31}=203nF, R_{32}=533Ω, C_{32}=113nF and final stage with $D_4=10^{-12}$, R_{41}=2441Ω, C_{41}=52.9nF, R_{42}=349Ω, C_{42}=653nF.

Three quarter integrator can be reached for first stage composed by the following values $D_1=10^{-8}$, R_{11}=231kΩ, C_{11}=3.1μF, R_{12}=10.39kΩ, C_{12}=2.56μF, second filter with $D_2=10^{-8}$, R_{21}=103kΩ, C_{21}=817nF, R_{22}=10.38kΩ, C_{22}=9.7μF, third stage having $D_3=10^{-12}$, R_{31}=5206Ω, C_{31}=137nF, R_{32}=534Ω, C_{32}=113nF and final two-port with parameters $D_4=10^{-12}$, R_{41}=5228Ω, C_{41}=36nF, R_{42}=206Ω, C_{42}=1.106μF.

Proposed FDNR-based CPE audio-range network with above calculated values of elements have been verified by means of Orcad Pspice circuit simulator. Corresponding results are shown in Fig. 6 and proves that total phase error is lower than 1° in the audio frequency range. However a small deviation from the optimal values mentioned in parameter list causes increased phase ripple and this error is cumulative. The same situation holds for the passive elements of equalizer-based CPE approximation network.

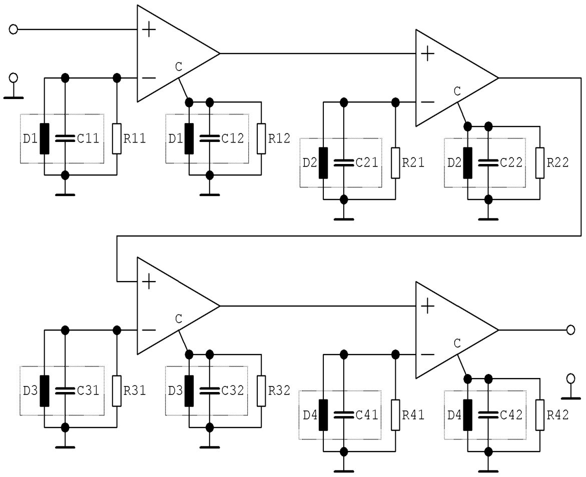

Figure 5. Complete realization of FDNR-based cascaded biquadratic filters towards CPE approximation.

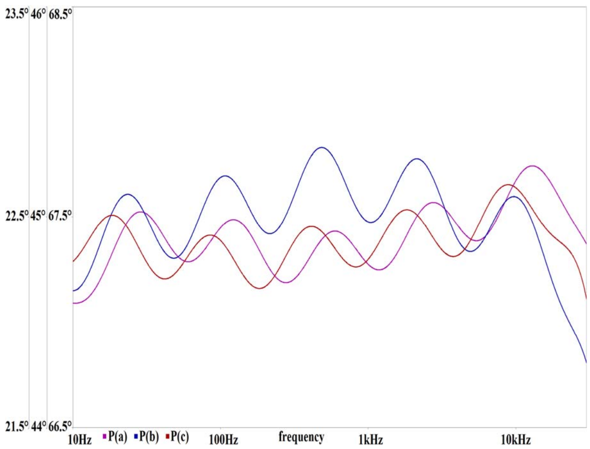

Figure 6. Phase frequency responses of designed FO two-ports with ripple in the frequency range from 10Hz up to 30kHz, vertical scale 90α°±1° for values α=1/4, α=1/2 and α=3/4.

C. *OTA-based biquad network solutions*

Another possible realization of CPE is to utilize four fully reconfigurable biquadratic filtering sections such as those discussed in [10] or, even better, in [11]. Electronic control of transfer function zeroes and poles make these circuits handy for full system-on-chip implementation. However additional voltage buffers are required between the individual sections. Authors believe that other topologies of the biquadratic filters without this drawback can be derived. Thus this is a wide place for future research.

D. *CCII-based CPE approximation*

Recently synthesis of the analog biquadratic filters employing second generation current conveyors (CCII) and various (unfortunately mostly hypothetical) active building blocks derived from CCII is a favorite topic of many research papers and engineering studies. From the viewpoint of possible application in CPE approximation two-ports a class of universal MISO filters should be focused.

One such promising example working in current-mode is given in [12] where two CCII and single multiple-output current amplifier creates desired transfer function. Second filter structure with big potential for CPE synthesis is working in voltage-mode and is introduced in [13]. The last but not least two-port structure which can be directly used for CPE synthesis can be found in [14]. Two-port section given here uses two current controlled current conveyor trans-admittance amplifiers and provides simple thus easily exploitable current transfer function.

From the viewpoint of practical applications of synthesized CPE it is good to know how it behaves outside operational frequency range. This question can be answered by means of polar plot of the complex transfer function (3), see Fig. 7. As expected for the same number of zeroes and poles phase frequency response begins and ends with zero phase shift.

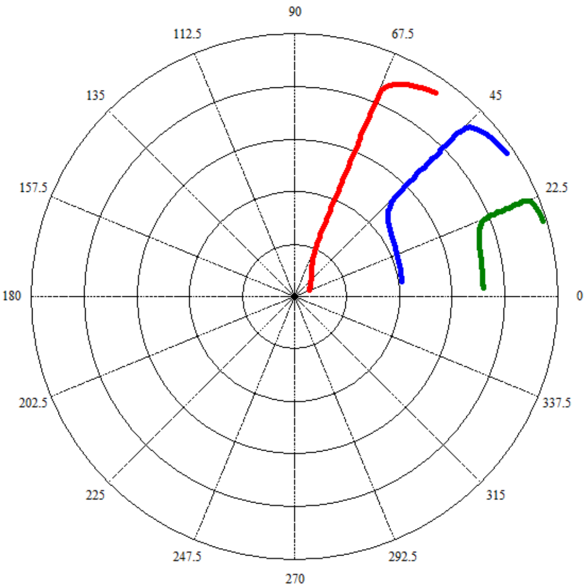

Figure 7. Polar plot of the individual CPE approximations in the frequency range from 100mHz up to 100kHz: green for α=1/4, blue for value α=1/2 and red represents α=3/4.

V. CONCLUSION

This paper enriches current knowledge in the area of CPE synthesis in the form of lumped analog circuit. Proposed method should be preferred over conception based on cascade of the bilinear filtering sections since final network is simpler, with less active elements and without harsh requirements. Used biquadratic filters can be realized by using some of the well-known topologies having only single active element and, if chosen wisely, without need of impedance separation. There is a strong reason to believe that the voltage-mode realizations will be preferred over current-mode circuits. However fully passive realizations of BP and BR filters, for example those containing only resistors and capacitors such as Wien two-port and T bridges, cannot be directly utilized here due to unwanted interactions between the individual sections. However in case of additional voltage buffers there will be no problems as well. Note that conventional Wien-type two-port should be modified in order to convert single transfer zero located at the origin of the complex plane into a pair of two different real zeroes.

Reason for finding alternative implementations of CPE can be also found in high frequency band where BP and BR filtering structures are commonly used. Thus CPE construction based on the microstrip lines can be interesting topic for further studies.

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