AN INNOVATIVE METHOD FOR MEASURING YOUNG'S MODULUS OF FLEXIBLE MULTI-LAYERED MATERIALS (TENSILE RING METHOD)

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1. Introduction

In recent years, flexible multi-layered materials with very high performance are used to establish cost-effective processing with regard to long-term performance and reliability. Therefore, Young's modulus of these materials is very important to predict large deformation.

In this study, an innovative mechanical testing method (*Circular Ring Method*) is provided for measuring Young's modulus of each layer in a flexible multi-layered material. By just measuring the vertical or the horizontal displacement of the ring, Young's modulus of each layer can be easily obtained for thin multi-layered materials.

Measurements were carried out on a two-layered wire (Cu: an electrodeposited material + SWPA: a spring steel material).

The method is based on a nonlinear large deformation theory. Exact analytical solutions are obtained in terms of elliptic integrals. Besides the *Circular Ring Method* for a flexible multi-layered material studied here, the *Circular Ring Method* [1], [2], the *Axial Compression Method* [3] for a flexible single-layered material and the *Cantilever Method* [4] for a flexible multi-layered material have already been developed, based on the nonlinear large deformation theory.

2. Fundamental theory

A typical illustration of a deflection shape is given in Fig.1 for a ring, subjected to opposite tensile forces at two points. Denoting the whole arc length of a circular ring by $4L$ and Taking into the boundary conditions $\zeta_{max}\,(=s_{max}/L)=1$,$\eta_{max}=\delta/L$, and, $\xi_{max}=\lambda/L$, the maximum non-dimensional arc length ζ_{AB}, the maximum non-dimensional vertical displacement η_{AB} and the maximum non-dimensional horizontal displacement ξ_{AB} are obtained as follows.

$$\zeta_{AB}=1=\frac{F(1/k\ ,\ Z_B)-F(1/k\ ,\ Z_A)}{k\sqrt{\gamma}} \tag{1}$$

$$\eta_{AB}=\delta/L=\frac{\left[\begin{array}{l}(2k-1/k)\left\{\begin{array}{l}F(1/k\ ,\ Z_B)\\-F(1/k\ ,\ Z_A)\end{array}\right\}\\-2k\left\{\begin{array}{l}E(1/k\ ,\ Z_B)\\-E(1/k\ ,\ Z_A)\end{array}\right\}\end{array}\right]}{\sqrt{\gamma}} \tag{2}$$

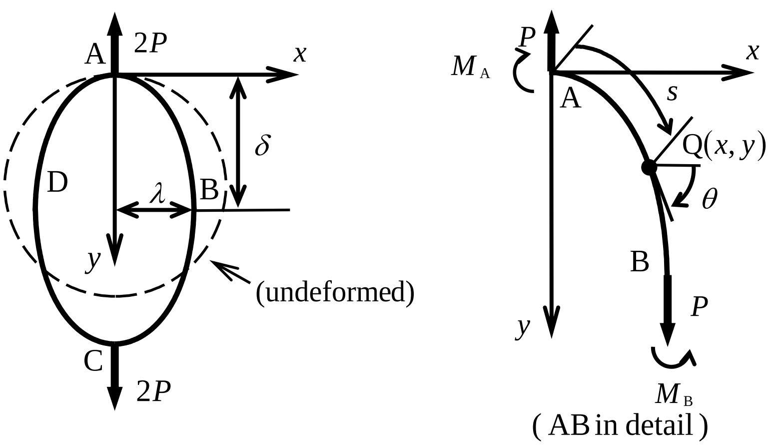

Fig. 1. Schematic illustration of circular ring subjected to opposite tensile forces

$$\xi_{AB} = \frac{\lambda}{L} = \frac{2k\left(\cos\varphi_A - \cos\varphi_B\right)}{\sqrt{\gamma}} \quad (3)$$

where, $k = \sqrt{\left\{2\lambda + (\alpha + 1/\rho_0)^2/(4\gamma)\right\}}$

$$\phi_A = \mathrm{Sin}^{-1}\left[\sqrt{\left\{1/(2k^2)\right\}}\right]$$

$$\phi_B = \mathrm{Sin}^{-1}(1/k)$$

$$Z_A = \pi/4, \quad Z_B = \pi/2$$

$$\gamma = PL^2 \Big/ \sum_{i=1}^{n}(E_i I_i), \quad \alpha = M_A L \Big/ \sum_{i=1}^{n}(E_i I_i)$$

I_i : the second moment of area.

The functions $F(1/k, Z_{A,B})$, $E(1/k, Z_{A,B})$ appeared in Eqs.(1), (2) and (3)are elliptic integrals of the first and second kinds, respectively. Using fundamental Eqs.(1)-(3) it is possible to calculate each Young's modulus E_i from the following Eq.(4).

$$\sum_{i=1}^{n}(E_i I_i) = \frac{PL^2}{\gamma} \quad (4)$$

One quantity γ (: the non-dimensional load) is required to calculate Young's modulus E_i from Eq. (4). The value of γ is obtained from a chart (Nomograph) of γ -δ relation (δ: the vertical displacement) [Method 1] or γ -λ relation (λ: the horizontal displacement) [Method 2].

3. Experimental investigation

Several experiments were carried out using a two-layered wire [a Copper layer: Cu (0.011mm thick, 500mm long) + a spring steel wire: SWPA (0.38mm diameter, 500mm long)]. Young's moduli of Cu and SWPA obtained by applying Method 2 [Method 1 is omitted here.] are shown in Fig. 2 and 3. The measured values remain nearly constant for a tensile load and the standard deviation (S.D.) is small although the method has a little scattered values.

Acknowledgements

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References

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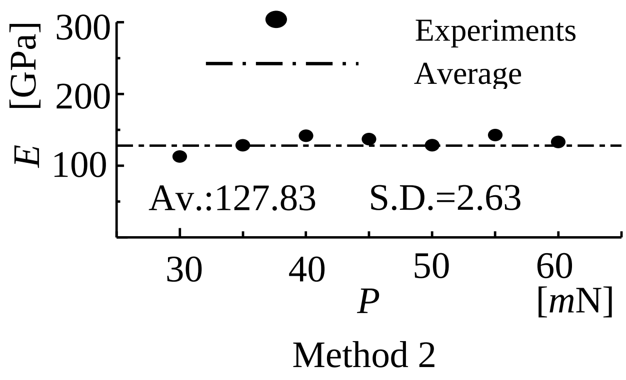

Fig. 2. Young's modulus for an electrodeposited material (Cu) [Method 1 is omitted.]

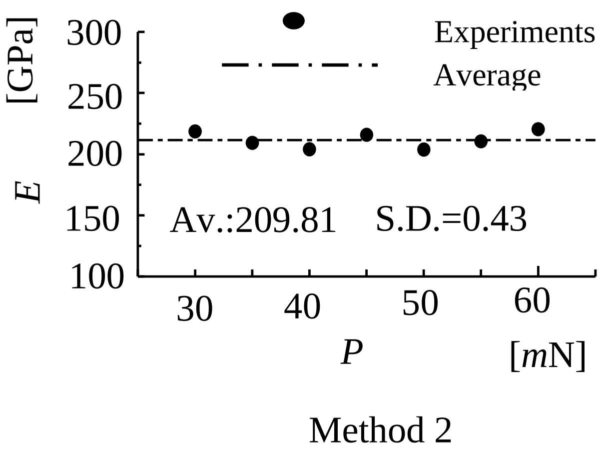

Fig. 3. Young's modulus for a steel material (SWPA) [Method 1 is omitted.]

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