Abstract – Autonomy in vehicles is a rapidly expanding technology that is of interest in many major car companies. Autonomous driving enables safer journeys by removing human error in driving, as well as reducing driving time and fuel consumption by optimizing the engine and brake actuation. While certain autonomous functions in cars have been in use for over a decade, such as lane keeping and automatic parking these features are subject to specific scenarios. A fully automated vehicle needs to take into account the unpredictable and complex environments that cars drive in. The idea of a self-driving car in a city environment needs specific requirements such as reliable path following, accurate sensor data and safe drive. The scope of this paper is to create a simple path following solution which enables an autonomous car to accurately traverse along any given path.

Keywords - adaptive control; adaptive controller; continuous; digital; model reference; adaptive control

I. INTRODUCTION

A control system is a device that regulates or controls the dynamics systems or process. Adaptive control is one of the widely used control strategies to design advanced control systems which can change properties according to changed conditions. Model Reference Adaptive Control (MRAC) is a direct adaptive strategy with some adjustable controller parameters and an adjusting mechanism to adjust them. In this work the adaptive control is used for autonomous vehicle trajectory following with minimal error (see Fig. 1).

II. ADAPTIVE CONTROL

On contrary to the well known and simple structured fixed gain, e.g. PID controllers, adaptive controllers are very effective to handle the unknown parameter variations and environmental changes. An adaptive controller usually consists of two loops, an outer loop or normal feedback loop and an inner loop or parameter adjustment loop.

The principle of MRAC is shown on Fig. 2. The output (position) of reference model (RM) is $y_{m}(t)$ and $y(t)$ is the output (position) of a vehicle or an autonomous car (ACA). The error between the output of RM and ACA is $e(t)$. The adaptive algorithm evaluates $y(t)$ and $e(t)$ and generates a vector of parameters $\theta$ for the controller. Adaptive control is based on MIT rules developed in 1960 by the researchers of Massachusetts Institute of Technology (MIT) and used to design the autopilot system for aircrafts [1–5].

The cost function (based on MIT rules [6–8]) is defined as
where \( e \) is the output error, while \( \theta \) is an adjustable vector which must be adjusted to minimize \( J \). Let’s assume 2nd order linear MODEL \( F_m(s) \) and VEHICLE \( F(s) \) dynamics

\[
F_m(s) = \frac{1}{s^2 + 1.41s + 1} \quad (2)
\]

\[
F(s) = \frac{0.5}{s^2 + 2s + 1} \quad (3)
\]

Step responses of the MODEL and VEHICLE are shown in Fig. 3.

Figure 3. Step responses of MODEL and VEHICLE

The block diagram of continuous adaptive system is displayed in Fig. 4. The adaptive algorithm is designed according to following equations (4) – (8)

\[
x_1(t) = p_0 + y_m^2 \quad (4)
\]

where \( p_0 \) is small positive number (\( p_0 = 0.01 \)).

\[
x_2(t) = y_m (y - y_m) \quad (5)
\]

\[
x_c(t) = k_c \frac{x_c(t)}{x_1(t)} = k_c \frac{y_m (y - y_m)}{p_0 + y_m^2} \quad (6)
\]

where \( k_c \) is a gain of the controller (must be negative). The output of the adaptive algorithm is \( x_c \), see eq. (7).

\[
x_c(t) = \int x_c(t) dt = k_c \int \frac{y_m (y - y_m)}{p_0 + y_m^2} dt \quad (7)
\]

The control signal \( U(t) \) (trajectory) modified by \( x_c \) is given in eq. (8)

\[
u(t) = U(t) x_c(t) = U(t) \cdot k_c \int \frac{y_m (y - y_m)}{p_0 + y_m^2} dt \quad (8)
\]

In Fig. 5 is presented the prescribed trajectory and MODEL trajectory

Figure 5. The prescribed trajectory (dash-dot, black) and MODEL trajectory (solid, red)

In Fig. 6 is the presented prescribed trajectory, MODEL trajectory (solid, red), VEHICLE trajectory for gain \( k_c = 1.5 \) (dash, blue) and VEHICLE trajectory for gain \( k_c = 0.5 \) (dash, brown)

Figure 6. Prescribed trajectory (dash-dot, black), MODEL trajectory (solid, red), VEHICLE trajectory for gain \( k_c = 1.5 \) (dash, blue) and VEHICLE trajectory for gain \( k_c = 0.5 \) (dash, brown)

In Fig. 6 is the presented prescribed trajectory, MODEL and VEHICLE trajectory for 2 different gain values of \( k_c \). It is important to note that for higher values of \( k_c \) the system can be unstable.
In Fig. 7 is shown time evolution of parameter $x_d$ also for 2 values of $k_C$. It can be seen that a big overshoot exists for higher gain $k_C$.

So far the continuous adaptive system was presented. In next part the simulation results for the discrete adaptive system simulations are shown.

III. DIGITAL ADAPTIVE CONTROL

In this part the digital version of adaptive control is described. On the contrary to the continuous adaptive control this approach enables simple changing of parameters or structure of the system. Block diagram of MRAC is shown in Fig. 8. The system is similar to the continuous system (see Fig. 4), but two analog/digital (A/D) converters and one digital/analog (D/A) converter is used. For the VEHICLE, the continuous transfer function according to eq. (3) is used but the continuous MODEL was converted into discrete version by means of the bilinear transformation

$$F_m(z) = \frac{0.128 + 0.256 z^{-1} + 0.128 z^{-2}}{1 - 0.767 z^{-1} + 0.279 z^{-2}}$$ (9)

Step response of $F_m(z)$ is displayed in Fig. 9.

In Fig. 10 is shown the prescribed trajectory, MODEL and VEHICLE trajectory for 2 different values of $k_C$. In Fig. 11 is presented the time evolution of parameter $x_d$ also for 2 values of $k_C$. The overshoot can be seen for higher gain $k_C$.

It should be noticed that previous results were derived from the discrete control system, with VEHICLE continuous and all other parts as discrete blocks.
In Fig. 12 the photo of VEHICLE used for tests is presented. The VEHICLE has 2 independent 9V DC motors with rotary encoders, the optical sensors and Bluetooth module for communications. Weight is 0.5 kg.

For testing purposes the car driving simulator was also developed in department of Applied Electronics. In Fig. 13, the hardware is shown. In Fig. 14, the photo of female subject during test is presented. The infrared sensor S1 for head position detection is shown. The 2 detectors (on left and right sides) are used.

IV. CONCLUSION

In this paper the MRAC was described. The theoretical derivations were confirmed by simulations. Two approaches were simulated – continuous and discrete with similar results. Systems, MODEL-VEHICLE, with different dynamics were deliberately tested. The adaptive algorithms were tested for different gains, because gain value is important for stability and speed of parameter estimation. The car driving simulator was also constructed. The system can be used for further development.

REFERENCES