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Frequency weighted H_2 optimization of multi-mode input shaper

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Abstract

An optimization based design of a robust multimode input shaper is presented with embedded weighting of the frequency response. By prefixing the delay length in the shaper, the optimization design task is formulated as a standard quadratic programming problem. The shaper robustness in terms of zeroed or upper-bounded residual vibration function for the modes to be precompensated is imposed via the equality and inequality constraints. As the main result, additional degree of freedom in the shaper design is introduced via parametrization of the quadratic form cost matrix and utilized to modulate the amplitude frequency response over a given frequency range. In particular, this allows to embed the high-frequency roll-off to the shaper. The proposed shaper design is demonstrated and experimentally validated on a problem of positioning a flexible manipulator arm with multiple oscillatory modes.

[1] Aaa

Key words: Input shaping, multiple modes, time delay, H_2 optimal design

1 Declaration

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2 Introduction

Input shaping is a well known technique to precompensate oscillatory modes of flexible systems. After the pioneering work by Smith [25], the boom of the technique has come in 1990s by the introduction and thorough analysis of zero-vibration (ZV) shaper, robust zero-vibration-derivative (ZVD) [18, 23] and extra-insensitive (EI) shaper [24]. The attention has also been paid to multi-mode input shapers [11], [21] and discrete-time input shapers, e.g. [26], [1], [2]. For an extensive review on input shaping, see [20].

Next to the input shaping, e.g. Trapezoidal, S-curve functions can be used to smooth rapid changes in the reference or input signals of flexible systems, and thus to decrease the vibrations [14]. As shown in [22], the input shaping is a considerably faster and more efficient technique for reducing vibrations compared to the command smoothing. The properties of input shapers and smoothers have been merged recently by introducing the distributed delay input shapers [30], [29], [16]. Next to smoothing the shaping command at the accommodation stage, their advantage is also at the retarded spectrum of zeros when implemented in the inverse form [10].

For the input shaper design, a prevailing approach is based on minimizing (zeroing) the *residual vibration function* [18], [24]. The shaper design can also be performed in the spectral domain by assigning the dominant zeros, [19], [29]. Taking into consideration various design requirements, the design task can be formulated as an

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optimization problem. In [13] the shaper is considered in the standard FIR form and the design is formulated and solved as l_1-norm constrained quasi-convex optimization problem. A multi-level optimization approach for creating discrete time shapers was proposed in [17].

When both the delays and the gains within the input shaper are to be optimized, it forms a nonlinear optimization problem, which is difficult to solve. Once the time delays in the shaper are prefixed, the optimization task is considerably simplified and can be formulated as linear programming problem, where the equality and inequality constraints are imposed by the functional and structural properties of the shaper [28]. In [27] predictive prefilters based on the model predictive control (MPC) framework are embedded within the optimal design. In [4], a generalized FIR filter with shaping capability is proposed and optimized via linear or quadratic programming problem. Compared to lumped delay shapers, the shaper impulse response has no singularities and as such, it filters discontinuities from a command input. The method was subsequently extended to adaptive input shaping by discrete FIR filter in [2] and [5]. In [3] a fundamental set of shaper solutions was derived that give exact vibration cancellation for a finite number of modes and with low-pass properties with a specified degree of high-frequency attenuation. Recently, an enhanced attention was paid to employing the optimization methods in design of distributed delay shapers. In [29], a method based on the constrained linear least squares optimization was proposed. Consequently, the design task was addressed as a quadratic multi-objective problem balancing the robustness and the response time in [16]. The design of a shaper with piece-wise distribution of the delay was also targeted in [12] where the stability constraint on the spectral abscissa was considered.

In this brief paper, we revisit and extend the quadratic optimal design of a shaper in the form considered in [28]. Structurally, it also complies with the form of a discrete FIR filter considered in [4] and it is also linked to the discretized distributed delay shaper proposed in [16]. Next to generalizing the design to multi-mode shaper case and complementing the structural properties via defining the additional constraints, the main contribution is in formulating the design in the sense of frequency weighted H_2 optimality.

3 Input shaper structure and optimal design

Consider a feed-forward interconnection of an input shaper $H(s)$ and the flexible system $G(s)$ with the oscillatory modes to be compensated given by damping and natural frequencies $\xi_i, \omega_i, i = 1..m$. Analogously to [28] the input shaper is considered in the form

$$y(t) = \sum_{i=1}^{n} A_i u(t - t_i), \tag{1}$$

where u, y are the input and the output of the shaper, respectively, and t_i are the delays satisfying $0 = t_1 < t_2 < ... < t_n$ and $t_{i+1} - t_i = T_s, i = 2..n$. In the discrete time, the shaper can be represented by a transfer function

$$H(z) = \frac{y(z)}{u(z)} = \sum_{i=1}^{n} A_i z^{-t_i/T_s}. \tag{2}$$

A common design strategy of the input shaper is via the residual vibration function

$$V(\omega, \xi) = \mathrm{e}^{-\xi\omega t_n} \sqrt{C(\omega,\xi)^2 + S(\omega,\xi)^2} \tag{3}$$

where

$$C(\omega, \xi) = \sum_{i=1}^{n} A_i \mathrm{e}^{\xi\omega t_i} \cos(\omega\sqrt{1-\xi^2} t_i) \triangleq \mathbf{c}(\omega,\xi)\mathbf{h}, \tag{4}$$

$$S(\omega, \xi) = \sum_{i=1}^{n} A_i \mathrm{e}^{\xi\omega t_i} \sin(\omega\sqrt{1-\xi^2} t_i) \triangleq \mathbf{s}(\omega,\xi)\mathbf{h}, \tag{5}$$

determining the scaled amplitude of the residual vibration at time $t = t_n$ [18], [23], where

$$\mathbf{h} = [A_1, A_2, ..., A_n]^T \tag{6}$$

is the vector of parameters, and the elements of the row matrices $\mathbf{c}(\omega, \xi) \in \Re^{1\times n}$ and $\mathbf{s}(\omega, \xi) \in \Re^{1\times n}$ are given as $c_i = \mathrm{e}^{\xi\omega t_i}\cos(\omega\sqrt{1-\xi^2} t_i)$ and $s_i = \mathrm{e}^{\xi\omega t_i}\sin(\omega\sqrt{1-\xi^2} t_i)$.

The design of the input shaper can be defined as a constrained optimization problem

$$\min_{\mathbf{h}} \; f(\mathbf{h}) \text{ subject to} \begin{cases} \mathbf{C_{eq}h} = \mathbf{d_{eq}}, \\ \mathbf{Ch} \leq \mathbf{d}, \end{cases} \tag{7}$$

where $f(\mathbf{h})$ is the objective function. The equality and inequality constraints arise from the structural and design properties of the shaper. If $f(\mathbf{h})$ is a linear function in $\mathbf{h}$, as considered e.g. in [28], see also [7], the problem can be solved by *linear programming*. Such an objective function formulation however has a limited impact on the shaper properties, which are mostly imposed by the constraints. Formulating the objective function as

$$f(\mathbf{h}) = \mathbf{h}^T\mathbf{Q}\mathbf{h}, \tag{8}$$

where $\mathbf{Q} \in \Re^{n\times n}$ is a cost matrix, provides a considerably larger potential to achieve a shaper with enhanced quantitative properties. By the simplest option with $\mathbf{Q} = \mathbf{I}$ considered in [4], a shaper with a minimum quadratic gain is obtained. In [15] and [16], for a distributed delay shaper, the objective function

$$f(\mathbf{h}) = \frac{1}{N_\omega N_\xi}\sum_{k=1}^{N_\omega}\sum_{l=1}^{N_\xi} V(\bar{\omega}_k, \bar{\xi}_l)^2 = \mathbf{h}^T\mathbf{Q}\mathbf{h}, \quad (9)$$

carries directly the robustness measure over the region $[\bar{\omega}_{min}, \bar{\omega}_{max}] \times [\bar{\xi}_{min}, \bar{\xi}_{max}]$ covered by $N_\omega \times N_\xi$ grid points. However, as demonstrated below, the requirement on pass-band bounding of the residual vibration function can also be performed via inequality constraints.

3.1 Equality and inequality constraints

The first equality constraint in (7) is given by the structural requirement on the unity gain of the shaper (1)

$$\mathbf{e}\mathbf{h} = 1, \quad (10)$$

where $\mathbf{e} \in \Re^{1\times n}, e_i = 1, i = 1..n$. The other standard requirement on the shaper properties is the zero residual vibrations, i.e. $V(\omega_k, \xi_k) = 0$ for all or selected oscillatory modes, given by ω_k, ξ_k of the system, $k = 1..p, p \leq m$. This leads to p couples of equality constraints

$$\mathbf{c}(\omega_k, \xi_k)\mathbf{h} = 0, \ \mathbf{s}(\omega_k, \xi_k)\mathbf{h} = 0. \quad (11)$$

In order to enhance the robustness of the vibration suppression at the given mode, the requirements

$$\frac{\partial^l V(\omega_k, \xi_k)}{\partial \omega_k^l}{=}0, \frac{\partial^l V(\omega_k, \xi_k)}{\partial \xi_k^l}{=}0 \quad (12)$$

may be imposed for the first l derivatives.

An alternative way of enforcing the robustness presented here forms the *first technical result* of the paper. A specified level of robustness can be enforced by formulating the inequality constraints

$$V(\bar{\omega}_i, \bar{\xi}_k) \leq V^{max}; \quad (13)$$

for $i = 1..N_\omega, k = 1..N_\xi$. By this inequality, the residual vibration is kept below a predefined value V^{max} over the region $[\bar{\omega}_{min}, \bar{\omega}_{max}] \times [\bar{\xi}_{min}, \bar{\xi}_{max}]$ formed around the neighborhood of the mode ω_i, ξ_k to be compensated. Substituting the boundary values $C(\bar{\omega}_i, \bar{\xi}_k) = \pm\frac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}}$ and $S(\bar{\omega}_i, \bar{\xi}_k) = \pm\frac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}}$ to (3), which yields $V(\bar{\omega}_i, \bar{\xi}_k) = V^{max}$, leads us to the following proposition:

Proposition 1 *The inequality* (13) *can be imposed by*

$$|C(\bar{\omega}_i, \bar{\xi}_k)| \leq \frac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}} \cap |S(\bar{\omega}_i, \bar{\xi}_k)| \leq \frac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}}. \quad (14)$$

Forming the inequalities (14) to comply with (7), four inequalities are obtained for every grid point $\bar{\omega}_i, \bar{\xi}_k$

$$\begin{aligned} \pm\mathbf{c}(\bar{\omega}_i, \bar{\xi}_k)\mathbf{h} &\leq \tfrac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}}, \\ \pm\mathbf{s}(\bar{\omega}_i, \bar{\xi}_k)\mathbf{h} &\leq \tfrac{V^{max}e^{t_n\bar{\xi}_k\bar{\omega}_i}}{\sqrt{2}}. \end{aligned} \quad (15)$$

Bounding $V(\bar{\omega}_i, \bar{\xi}_k)$ this way in order to achieve the enhanced robustness provides a viable design opportunity to utilize the objective function $f(\mathbf{h})$ to impose additional characteristics to the shaper and thus improve its performance, as will be proposed next. Before that, let us mention that the other standard inequality constraint is the non-negativity of the shaper impulse response

$$-\mathbf{h} \leq \mathbf{0}, \quad (16)$$

implying the non-decreasing character of the step response.

4 Main result - H_2 optimal shaper synthesis with frequency weighting

A natural effort for a shaping filter designer should be to limit the high-frequency content of the shaped commands. This comes from a number of practical engineering insights, in particular i) there is a danger of excitation of an unmodelled high-frequency dynamics, typically higher bending modes, actuator and sensor nonlinearities, ii) smooth setpoint commands are easier to be tracked by feedback controllers, a risk of actuator saturation is reduced, energy consumption and mechanical wear of the equipment is decreased. This leads to a conclusion that the amplitude frequency response of an ideal shaping-smoothing filter should resemble a notch shape in the vicinity of plant resonance frequencies to be compensated and a *sufficient high-frequency roll-off is desirable* in order to provide the smoothing functionality.

Consider the shaper (1) design task to be formulated as the *quadratic programming problem* (7)-(8). As the starting point, let us outline properties of the specific choice $\mathbf{Q} = \mathbf{I}_{n\times n}$ considered in [4]. The cost function has then a direct physical meaning of a squared H_2 norm defined as a total energy of the shaper impulse function

$$f(\mathbf{h}) = \mathbf{h}^T\mathbf{h} = \sum_{\forall k}|A_k|^2 = ||H(z)||_2^2. \quad (17)$$

A corresponding frequency domain interpretation immediately follows from the Parseval's theorem

$$||H(z)||_2^2 = \frac{1}{2\pi}\int_{-\pi/T_s}^{\pi/Ts} |H(e^{j\omega T_s})|^2 \mathrm{d}\omega. \tag{18}$$

Therefore, the minimization of the quadratic cost leads to the *reduction of the average filter gain* over the whole frequency range. This is in agreement with the aforementioned smoothing functionality.

4.1 Frequency weighted H_2 minimization

The choice $\mathbf{Q} = \mathbf{I}$ often leads to a viable solution. However, there are no additional degrees of freedom allowing the designer to influence the shaper dynamics. A *frequency-dependent weighting* is introduced to overcome this limitation. Consider a serial connection of the shaping filter and a properly selected weighting function

$$F(z) = H(z)W(z), \tag{19}$$

where $W(z)$ is a discrete FIR weighting filter. The overall impulse function of the serial connection (19) is given by the convolution operator

$$\begin{aligned} h_F(k) &= \mathcal{Z}^{-1}\{F(z)\} = (h * h_W)(k), \\ h_W(k) &= \mathcal{Z}^{-1}\{W(z)\},\ dim(\mathbf{h_W}) \triangleq w. \end{aligned} \tag{20}$$

The criterion (17) can be replaced by its weighted version in the form of

$$f_W = ||H(z)W(z)||_2^2 = \mathbf{h_F}^T\mathbf{h_F} = (h * h_W)^T(h * h_W). \tag{21}$$

The discrete convolution which forms the vector $\mathbf{h_F}$ in (21) can be expressed as a matrix product

$$\mathbf{h_F} = \mathbf{W}\mathbf{h}, \tag{22}$$

where $\mathbf{W} \in \mathbb{R}^{f\times n}$ is a convolution matrix constructed from the impulse function of the weighting filter $W(z)$

$$\mathbf{W} \triangleq \begin{bmatrix} h_W(0) & \dots & 0 & 0 & 0 \\ h_W(1) & h_W(0) & \dots & 0 & 0 \\ h_W(2) & h_W(1) & h_W(0) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & h_W(w-1) & h_W(w-2) & h_W(w-3) \\ 0 & 0 & \dots & h_W(w-1) & h_W(w-2) \\ 0 & 0 & 0 & \dots & h_W(w-1) \end{bmatrix}. \tag{23}$$

Substitution of (22) to (21) leads to the quadratic cost

$$f_W = \mathbf{h_F}^T\mathbf{h_F} = \mathbf{h}^T\mathbf{W}^T\mathbf{W}\mathbf{h} \triangleq \mathbf{h}^T\mathbf{Q}\mathbf{h}, \tag{24}$$

which reveals a suitable choice of the weighting matrix $\mathbf{Q} = \mathbf{W^T W}$.

In order to enforce the required smoothing functionality of the shaper by imposing a specified high-frequency roll-off, a *high-pass characteristics* has to be embedded in the weighting filter $W(z)$ dynamics. Numerous methods known from the field of digital signal processing can be used to form a suitable weight $W(z)$ (IIR approximation, windowing, direct optimal design, etc.), [6].

An important and unique result of the proposed shaper design is that *the length of the weight $W(z)$ does not affect the length of the resulting optimal filter*. It only serves for the introduction of the frequency-dependent scaling to the criterion function.

Lemma 2 *A global optimal solution of the formulated frequency-weighted H_2 minimization problem exists provided that a nonempty set of feasible shaping filters exists for the given design constraints, a chosen filter length and a nonzero impulse function of the weighting filter.*

PROOF. The convolution matrix $\mathbf{W}$ in (23) has a full column rank from the principle of construction for any non-zero weighting vector $\mathbf{h_w}$. Therefore, the matrix $\mathbf{Q} = \mathbf{W^T W}$ is always positive definite and there is a unique global minimum of the cost function $f_W = \mathbf{h}^T\mathbf{Q}\mathbf{h}$, provided that a feasible solution exists with respect to the formulated equality and inequality constraints. □

5 Experimental validation

The proposed frequency weighted H_2 optimization design of the shaper was applied to a control of flexible manipulator arm shown in Fig. 1. It consists of a servo drive and a flexible mechanical arm. It is a distributed parameter system with multiple resonance modes allowing to emulate various practical motion control problems. PID controller tuned by robust partial pole-placement method [9] is used in the feedback position compensator. In order to determine the oscillatory modes, an amplitude frequency response was identified as shown in Fig. 2. As can be seen, there are multiple resonances caused by lateral and torsional bending modes of the arm and a compliance of the motor-arm coupling. By further experiments, the first two dominant oscillatory modes were determined as

$$\omega = \{77, 609\}\ rad/s, \xi = \{0.09, 0.004\}. \tag{25}$$

The higher modes are considered as unmodelled high-frequency dynamics in this scenario. The flexible modes cannot be sufficiently damped by the fixed-structure

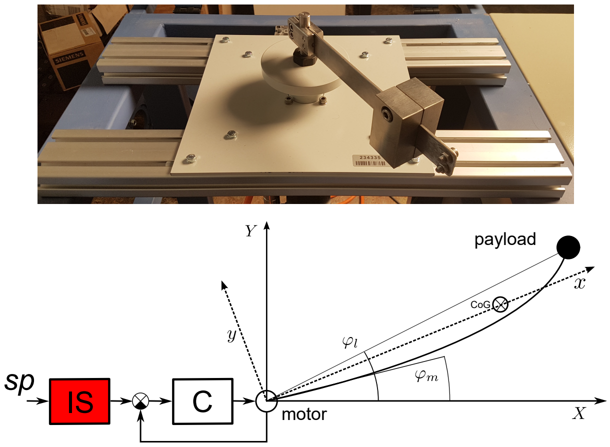

Fig. 1. *Mechanical setup used for the experiments*, schematics of the 2DoF position controller structure, C - feedback position compensator, IS - *input shaping* filter

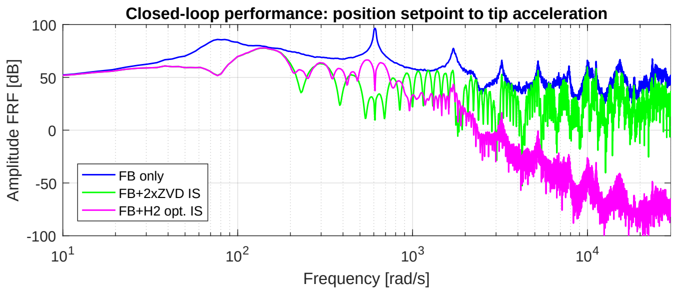

Fig. 2. Measured frequency responses: i) Feedback system (FB) without shaper with nine resonant frequencies (blue), ii) FB with convolved ZVD shaper (green), iii) FB with H_2 optimal shaper (magenta)

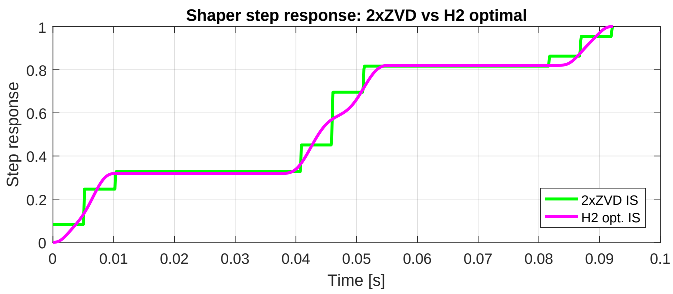

Fig. 3. Step responses of input shapers: i) Convolved ZVD shaper (green), ii) Discrete H_2 optimal shaper (magenta).

controller due to the low resonance ratio of the first mode [8]. Therefore, an input shaper was applied in the 2DoF structure (Fig. 1) to cope with the residual vibrations and to improve the position tracking performance during rapid maneuvers. A conventional input shaping filter was designed first for comparison by convolving two standard single-mode ZVD shapers computed for the first two dominant flexible modes. The overall length of the 2×ZVD shaper imposed by the design is $92.3ms$. For the discrete time implementation, the sampling period $T_s = 0.1ms$ was considered, implying $n = 923$. The

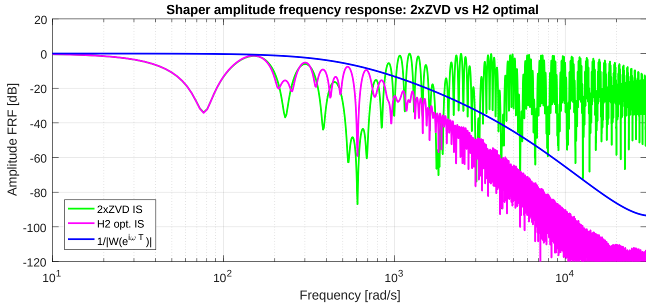

Fig. 4. *Shaping filters comparison*: suboptimal 2xZVD convolved filter (green) vs frequency-weighted H_2 optimal filter (magenta), inverse of weight $W(z)$ magnitude (blue)

step response of the discretized shaper is shown in Fig. 3 consisting of nine step-wise segments.

For the frequency-weighted input shaper (1), the identical length was chosen as for the 2×ZVD shaper, i.e. $n = 923$, considering $T_s = 0.1ms$. Next to the unity gain (10) and impulse response positivity (16), the zero-vibration constraints (11) and first derivative robustness conditions (12) were set for the first two flexible modes to achieve a low frequency behavior comparable to the 2×ZVD shaper. The weighting filter (19) with a high-pass characteristics was introduced to improve the high-frequency roll-off. The weight was computed from a prototype Blackmann-Harris window [6] leading to the impulse function

$$h_W(k) = \{2.1e^{-5}, -0.0076, 0.076, -0.24, 0.35, -0.24, 0.076, -0.0076, 2.1e^{-5}\}. \quad (26)$$

The optimization problem (7) with $f(\mathbf{h}) = \mathbf{h}^T\mathbf{W}^T\mathbf{W}\mathbf{h}$, $\mathbf{W}$ given by (23), was solved using the `quadprog` function of Matlab.

The amplitude frequency responses of both shaping filters are shown for comparison in Fig. 4. A significant high-frequency gain is observed in the case of the 2×ZVD filter which may potentially excite higher bending modes or other unmodelled dynamics. On the contrary, the propopsed H_2 optimal shaper achieves a favorable low-pass characteristics which follows inversely the introduced frequency-dependent weighting function. As can be seen in the step response comparison in Fig. 3, this leads to smooth transients in the time-domain compared to the discontinuous steps, while the overall response time of the shapers is the same. The performance improvement is best observed in the frequency domain. The preshaped closed-loop system was excited by a wide-band pseudo-random signal, the amplitude frequency response was measured and compared to the feedback only configuration in Fig. 2. It can be seen that the conventional 2×ZVD filter fails to attenuate the higher resonances whereas the H_2 shaper provides much better level of damping due to its desirable smoothing characteristics.

6 Conclusions

The paper provides a systematic approach to optimization based design of multimode input shapers. Next to supplementing the constraint conditions in the quadratic optimization task by limiting the residual vibration function amplitude, the frequency-dependent weighting is embedded to the quadratic objective function as the main result. It offers additional degrees of freedom allowing to gain control over the shaper amplitude response. This is used for the formulation of additional frequency-domain requirements aiming at an improved high-frequency roll-off. A unique aspect of the design is that the signal smoothing is achieved without the need to enlarge the action time of the filter, compared to the traditional combination of a shaper and a smoother.

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