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Shape optimization of nonlinear Gao beam

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The contribution is devoted to an optimal shape design problem of a nonlinear stepped elastic beam. The nonlinear Gao beam model [2] is motivated by a limitation of the Euler-Bernoulli beam model for relatively small deformations. The governing equation for the Gao beam reads as follows

$$E\mathcal{I}w'''' - E\,\alpha\,(w')^2\,w'' + P\,\mu\,w'' \,=\, f, \quad \text{in } (0, L),$$

where $\quad \mathcal{I} \,=\, \dfrac{2}{3}\,t^3\,b, \quad \alpha \,=\, 3\,t\,b\,(1-\nu^2), \quad \mu \,=\, (1-\nu^2), \quad f \,=\, (1-\nu^2)\,q.$

Here E denotes the elastic modulus of the material, $\mathcal{I}$ is the area moment of inertia of the beam's cross-section, w is the transverse displacement of the beam, ν is the Poisson ratio, P is the axial force acting at the point $x = L$, q is the distributed vertical load, $2t$, b and L represent the height, the width and the length of the beam, respectively. Here the modified integral constant μ is used in the governing equation, see [5]. The existence analysis of the state problem was studied in [4].

The objective of the design problem is to find the thickness distribution minimizing compliance of the beam represented by the functional

$$J(w(t)) = \int_I f w(t)\mathrm{d}x,$$

where $w(t)$ is a solution of the state problem for a given admissible thickness distribution t. The volume of the beam is preserved and fixed during the optimization. Therefore, a set of admissible thicknesses U^{ad} is defined by

$$U^{ad} = \left\{ t \in L^\infty(I) : 0 < t_{min} \le t \le t_{max} \text{ in } I,\ t|_{I_i} = t_i \in P_0(I_i)\,\forall i, \int_I t(x)\mathrm{d}x \,=\, \gamma \right\},$$

where the stepped beam is represented by an interval $I = (0, L)$ composed of r mutually disjoint subintervals $I_1, I_2, \ldots I_r$. Finally, the thickness optimization problem reads

$$\begin{cases} \text{Find } t^* \in U^{ad} \text{ such that} \\ J(w(t^*)) = \min\limits_{t \in U^{ad}} J(w(t)). \end{cases}$$

The concept of shape optimization in the context of the linear Euler-Bernoulli model is well understood, see [3]. The corresponding analysis for the nonlinear Gao beam model is provided in [1] where the existence of a solution to the optimization problem is proved and the convergence analysis of the discretization using the finite element approach is covered.

For illustration, we consider a propped cantilever beam and a simply supported beam composed of 8 elements subject to the constant vertical load $q = -10^6\,\mathrm{Nm}^{-1}$ and no axial force. The material and geometric parameters are defined as follows: $L = 1\,\mathrm{m}, b = 0.1\,\mathrm{m}$, $E = 21 \cdot 10^{10}\,\mathrm{Pa}$, $\nu = 0.3$, $t_{min} = 0.01\,\mathrm{m}$, $t_{max} = 0.08\,\mathrm{m}$, $\gamma = 0.0425\,\mathrm{m}^2$. The initial thickness distribution is constant $t_i = 0.0425\,\mathrm{m}, \forall i$.

The results for the optimal thickness distribution of the Gao beam are shown in Fig. 1. The optimal thickness distribution of the Gao beam plotted with a solid line is compared with the reference constant thickness distribution plotted with a dashed line, left above for the propped cantilever beam and right above for the simply supported beam. Similarly, the deflection of the beam with the optimal thickness distribution is plotted with a solid line and the deflection of the beam with the initial constant thickness distribution is plotted with a dashed line below.

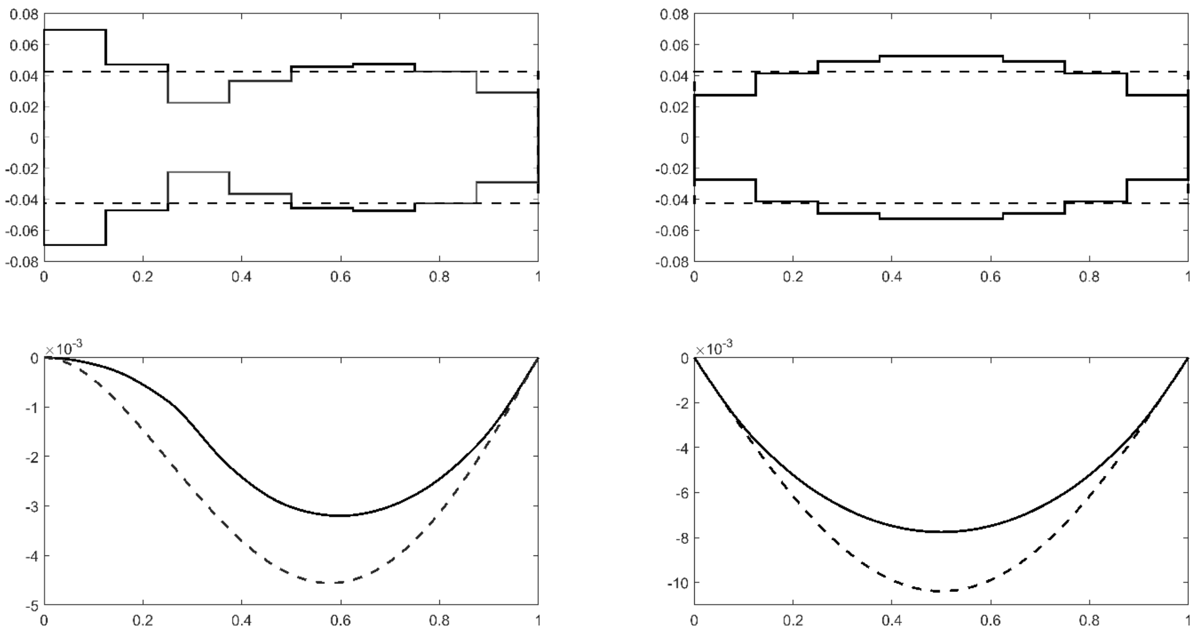

Fig. 1. Optimal thickness distribution and resulting deflection of the nonlinear Gao beam

The value of the compliance cost functional decreases from $2393\,\mathrm{Nm}$ for the reference beam with the constant thickness distribution to $1617\,\mathrm{Nm}$ for the beam with the optimal thickness distribution in the case of the propped cantilever beam and from $6044\,\mathrm{Nm}$ for the reference beam to $4840\,\mathrm{Nm}$ for the optimized beam in the case of the simply supported beam.

References

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