COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Form-finding of tensegrity structures based on defined topology and following semi-automatic creation of corresponding Simscape models

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Solving a problem of form-finding of a free-standing tensegrity structure means to determine spatial configuration of the structure based on defined topology and to determine members prestress forces. In most cases, it is also required that the final structure configuration meets the criteria of so called super-stability [3]. There are various suitable approaches for tensegrity form-finding that lead to stable configuration, see [1]. In this work, the Adaptive force density method (AFDM) presented by [2] is used. This method is further modified so that it is possible to assign altitude to arbitrary number of structure nodes.

In the first design stage, feasible force density vector $\mathbf{q}$ is searched for. The vector $\mathbf{q}$ consists of force densities $q_i = \frac{F_i}{l_i}$ of all members, F_i is an internal force in i-th member and l_i is its length. These force densities have to satisfy the necessary condition for super-stability and the non-degeneracy condition for free-standing structures. In d-dimensional space, they can be combined into one requirement: force density matrix $\mathbf{E} = \mathbf{E}(\mathbf{q}, \mathbf{C})$ [3], where $\mathbf{q}$ is a vector of force densities and $\mathbf{C}$ is a connectivity matrix defining topology, must have $d + 1$ zero eigenvalues and all other eigenvalues positive. Because the target is to get an explicit value of some eigenvalues, eigenvalue analysis and spectral decomposition of matrix $\mathbf{E}$ is applied. According to the condition mentioned above, some of detected eigenvalues λ_i are modified so a new set of eigenvalues λ_i^{mod} is created, eigenvectors $\mathbf{v}_i$ remain unchanged. Based on these sets λ_i^{mod} and $\mathbf{v}_i$, a new force density matrix $\overline{\mathbf{E}}$ can be found. When eigenvalues $\overline{\lambda}_i$ of the matrix $\overline{\mathbf{E}}$ are discovered, they probably will not be the same as pre-assigned set of eigenvalues λ_i^{mod} from the last step. But if these steps are repeated, difference between sets λ_i^{mod} and $\overline{\lambda}_i$ will fall to low values after a sufficient number of iterations. So if the difference falls below a specified tolerance, the new force density matrix $\overline{\mathbf{E}}$ is declared as the force density matrix $\mathbf{E}$ that meets the specified requirements.

In the second design stage, self-equilibrated configuration of the tensegrity is determined. By solving the system

$$\begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \mathbf{H}\mathbf{X} = \mathbf{0}, \tag{1}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are coordinates of all nodes in each direction, a super-stable configuration without possibility of controlling altitude of nodes can be discovered.

If specific altitude of some nodes is required, additional equations are added

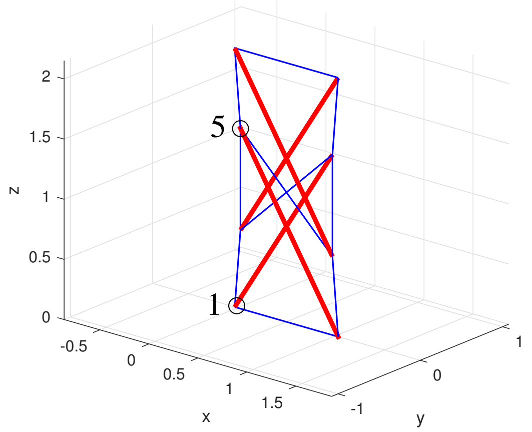
$$\mathbf{A}\mathbf{z} = \mathbf{b}, \tag{2}$$

where matrix $\mathbf{A}$ (containing only zeros and ones) informs which nodes have pre-assigned altitude and vector $\mathbf{b}$ specify altitude of selected nodes. By combining systems (1) and (2), it can be written that

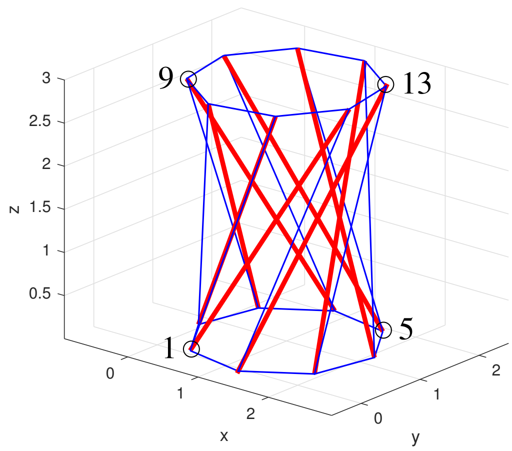
$$\begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \\ \mathbf{0} & \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \widehat{\mathbf{H}}\mathbf{X} = \widehat{\mathbf{b}}. \tag{3}$$

This is the final system of equations that is numerically solved and the solution provides self-equilibrated configuration of the tensegrity.

Overall, outputs from the modified AFDM are coordinates $\mathbf{x}, \mathbf{y}, \mathbf{z}$ of all nodes and force densities $\mathbf{q}$ in all members.



(a) a 2-D tensegrity



(b) a 3-D tensegrity

Fig. 1. Examples of super-stable tensegrity structures

In Fig. 1, two examples of tensegrity structures can be seen. Their configuration was determined by the modified AFDM. In case of the 2-D tensegrity in Fig. 1a, altitude equal to 0 in the node no. 1 and altitude equal to 1.5 in the node no. 5 was required. In case of the 3-D tensegrity in Fig. 1b, altitude equal to 0 in the nodes no. 1 and 5 and altitude equal to 3 in the nodes no. 9 and 13 was required at the same time. Resulting structures meet all requirements for altitude and super-stability (verified by a simulation).

Based on calculated force densities in structure members and coordinates of nodes, a model in the Simscape software (SIMULINK environment) is generated using MATLAB script. The program is able to compile a 2-D or a 3-D tensegrity model of any class but the tensegrity can only consist of 1-D elements (cables, struts). The core of the MATLAB script creating Simscape models consists of invocations of functions *add_block()* and *add_line()* that adds a specified SIMULINK-like block or connects existing blocks together.

Most important features of the resulting model in Simscape are these facts: struts are modelled as thin rigid cylinders, cables are massless – they represent one-sided viscoelastic forces between two nodes. It is intended to model a tensegrity as a structure lying on a flat surface, so the node with the lowest altitude is connected to the ground by a spherical joint, remaining nodes with the same altitude are connected to the ground by planar constraints, other nodes can move freely.

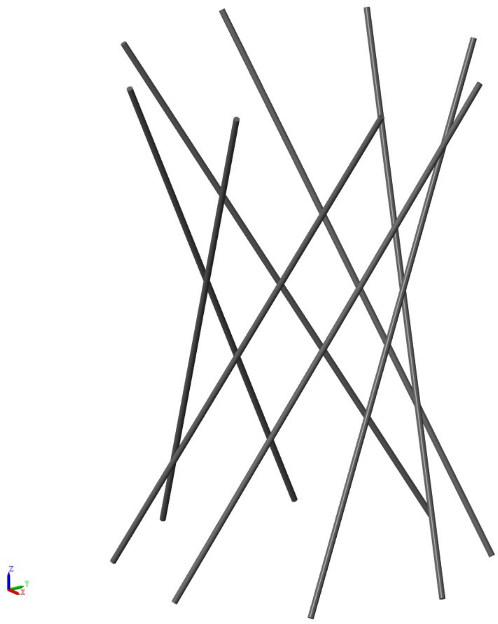


Fig. 2. Example of a Simscape model of a super-stable 3-D tensegrity

In Fig. 2, a visualisation of the Simscape model is depicted, it represents the same 3-D tensegrity as in Fig. 1b. The visualisation does not offer a possibility to show cables but struts can be observed clearly.

The whole methodology allows fast generation of stable spatial tensegrity computational models that can be further analysed from the viewpoint of mechanical properties, such as overall stiffness and modal properties. The resultant tensegrity structures usually reach good strength to weight ratio and they can be used as substructures of special robotic manipulators. On the one hand, described concept of the modified AFDM does not take the weight of members into account, which may be considered as a disadvantage. On the other hand, the structure consists only of thin struts and massless cables so deviation of total structure mass from originally considered massless state is relatively small. The essence of super-stable structures is that they remain stable even when a slight deviation of structure parameters occurs.

Acknowledgements

This research work was supported by project SGS-2019-009 of the Czech Ministry of Education, Youth and Sport, and by the Czech Science Foundation project number 20-21893S.

References

[1] Juan, S. H., Tur, J. M. M., Tensegrity frameworks: Static analysis review, Mechanism and Machine Theory 43 (2008) 859-881.

[2] Zhang, J. Y., Ohsaki, M., Adaptive force density method for form-finding problem of tensegrity structures, International Journal of Solids and Structures 43 (2006) 5568-5673.

[3] Zhang, J. Y., Ohsaki, M., Tensegrity structures: Form, stability, and symmetry, Springer, Tokyo, Japan, 2015.