COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Description of the quasi-periodic response caused by combined harmonic and random excitation

J. Náprstek, C. Fischer

Institute of Theoretical and Applied Mechanics of the Czech Academy of Sciences, Prosecká 76, 190 00 Prague, Czech Republic

Abstract

The generalized van der Pol equation exposed to combined harmonic and random excitation can exhibit a quasi-periodic response. The existence of this particular type of response depends on the detuning between the driving and resonance frequencies. The response is stationary for a ”small” or ”large” value of detuning. The contribution specifies in detail the detuning interval in which the quasi-periodic response occurs.

1. Introduction

When dealing with the frequency characteristics of real dynamic systems, such as those that appear in the dynamics of civil engineering structures, one encounter the phenomenon of the so-called frequency lock-in. It consists in the fact that for certain values of design parameters the change of the response does not correspond to the change of the frequency of the external excitation. For example, when examining the properties of a flow-induced movement of a body, the frequency of vibrations of the body caused by the separation of vortices is approximately proportional to the flow speed. However, once the flow velocity approaches the critical velocity, when the vortex separation frequency is close to the natural frequency of the body, the increase stops. The frequency of vibrations of the body remains constant in the non-zero vicinity of the resonant frequency, i.e. for a certain interval of values of the flow speed. If the flow velocity exceeds a certain limit with the next increase, the linear relationship between vibration frequency and flow velocity is restored.

This effect stems from the non-linear nature of the physical phenomenon. It is often described using the generalized van der Pol equation. In a state very close to resonance, the solution of this equation corresponds to a stable limit cycle. Large amplitudes of vibrations in this state affect the frequency of the vortex shedding, so that the excitation frequency is in fact fixed at the natural frequency of the structure.

The oscillation of the body in resonance is stationary and approximately harmonic. When the difference between the excitation and resonant frequencies increases, the vibrations of the body cease to be stationary and a quasi-periodic response occurs. It has two main components: the natural oscillation corresponding to the respective natural frequency, and the stationary forced oscillation. Their combination then cause the beating effect. The periods and amplitudes of the beating depend on the parameters of the system and on the difference between the natural and excitation frequencies. If the excitation frequency shifts from the natural frequency, i.e. the detuning increases, the period of beats shortens, because the influence of the auto-oscillating component of the response decreases. When this component disappears, the response will be stationary again. Its frequency will correspond to the frequency of forced oscillation.

The assumption of a random additive component, which can represent turbulence effects in the flow, introduces uncertainty into the problem. Experimental studies show that the random component can be considered Gaussian, the spectral density corresponds to von Kármán's velocity spectrum [1]. Theoretical investigation of a combined deterministic and random excitation in the resonance case has been published by the authors [5]. The paper which permits a positive value of detuning is currently under preparation. This work presents several results of a numerical examinations of the mathematical model in the state close to resonance, when the additive random component contributes to forming of beats, and to a general non-stationary character of the response.

2. Mathematical model

The stochastic single-degree-of-freedom van der Pol oscillator with strongly nonlinear damping part, which is a very suitable model of the case described in Introduction, can be written as

$$\ddot{u} - (\eta - \nu u^2)\dot{u} + \omega_0^2 u \;=\; P\omega^2 \cos\omega t + h \cdot \xi(t), \tag{1}$$

where

$u = u(t)$	displacement $[\mathrm{m}]$,	$v = v(t)$	velocity $[\mathrm{m.s^{-1}}]$,
η, ν	parameters of the damping $[\mathrm{s^{-1}},\ \mathrm{s^{-1}m^{-2}}]$,	ω_0	eigen-frequency of the adjoint linear SDOF system,
ω	excitation frequency of the vortex shedding $[\mathrm{s^{-1}}]$,	$P\omega^2$	amplitude of the harmonic excitation force $[\mathrm{m.s^{-2}}]$,
h	multiplicative constant $[\mathrm{m.s^{-2}}]$,	$\xi(t)$	broadband weakly stationary Gaussian random process [1].

Eq. (1) characterizes the nonlinear vibration of an SDOF system modelling the reduced flutter as one of post-critical response types of an aeroelastic system. In general, this equation describes state when the total linear damping component drops below zero due to aeroelastic effects and only nonlinear effects stabilize.

3. Deterministic stationary case

The possible stationary solution to Eq. (1) in the vicinity of the resonance can be characterized using the harmonic balance approach. The procedure described in [3] assumes the solution in a harmonic form

$$u = U\cos(\omega t + \varphi)\,, \tag{2}$$

where the stationary amplitude U is given by

$$U^2\left(4\Delta^2 + \left(\eta - \frac{\nu}{4}U^2\right)^2\right) = \omega^2 P^2\,. \tag{3}$$

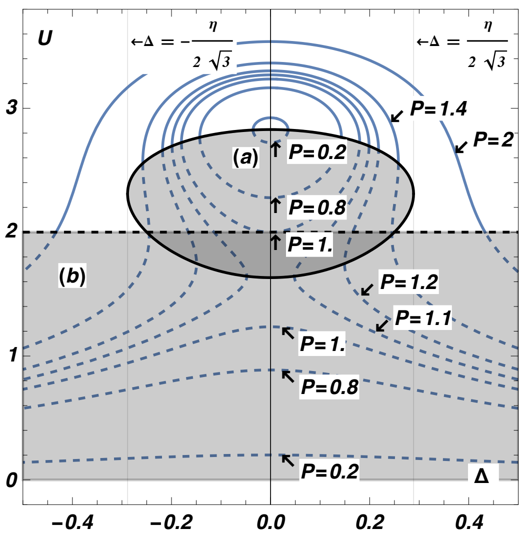

Fig. 1. Stationary amplitudes and instability areas

Stability of the admissible solutions can be assessed using two Routh-Hurwitz conditions

$$\text{(a)}\quad 64\Delta^2 + \left(4\eta - 3\nu U^2\right)\left(4\eta - \nu U^2\right) \geq 0\,, \qquad \text{(b)}\quad 2\eta - \nu U^2 \geq 0, \tag{4}$$

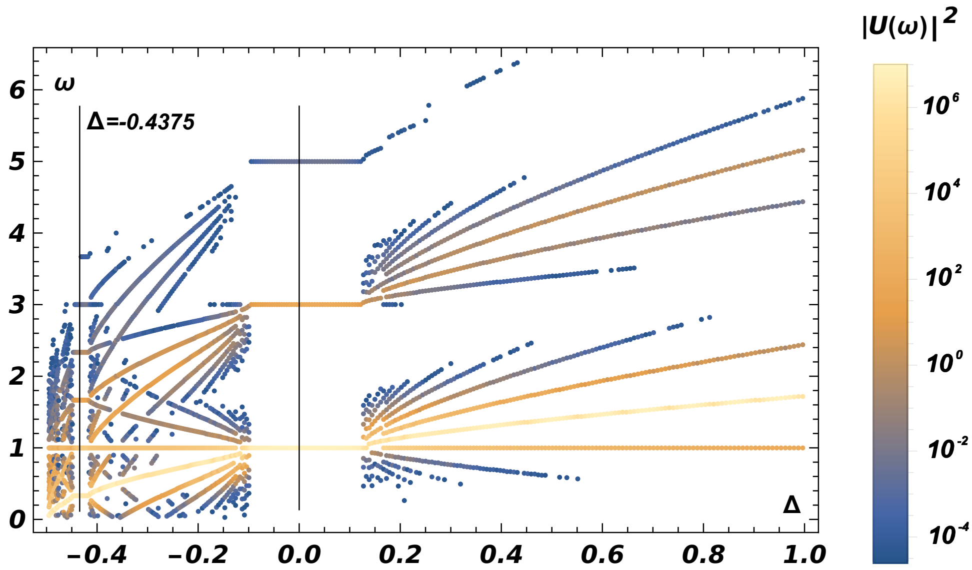

Fig. 2. Frequency characteristics of the response of Eq. (1). Abscissa: detuning Δ. Ordinate: angular frequency axis of the response. Colours: absolute values of dominant Fourier coefficients

where $\Delta = \frac{\omega_0^2-\omega^2}{2\omega} \approx \omega_0 - \omega$ and η is assumed positive. Fig. 1 shows the amplitudes U for various excitation levels $P = 0.2, \ldots, 2$ depending on detuning values Δ. The greyed areas show where the conditions (4) are violated. Values used: $\eta = 1, \nu = 1/2, \omega = 1$.

4. Deterministic non-stationary case

When the detuning exceeds limits given by the stability conditions (4), the stabilised response given by Eq. (1) forms non-stationary quasi-periodic time histories. The frequency characteristics of the response are visible in Fig. 2. All simulations were performed for a prescribed value of the excitation frequency and amplitude $\omega = 1, P = 1$. Thus, the varying detuning Δ on the horizontal axis in Fig. 2 represents in fact the system eigenfrequency ω_0. The dominant peaks of the periodogram for each value of ω_0 are plotted vertically. This way the ordinate represents the Fourier frequency of the response. The color intensity corresponds to absolute values of the dominant Fourier coefficients in a logarithmic scale. The stationary lock-in interval appears for $-0.1 \lessapprox \Delta \lessapprox 0.12$, although two super-harmonic components ($\omega = 3\omega_0$, $5\omega_0$) are also clearly visible. Also the presence of a sub-harmonic resonance interval for $\Delta \approx -7/16$ (i.e., for $\omega_0^2 = 1/8$) indicates a complex behaviour of the nonlinear response. For theoretical explanation of such effects see [4]. It is worth noting that the curves corresponding to the individual peaks are linear when plotted as dependent on ω_0.

5. Random excitation

When the response of a system in deterministic case is of a quasi-periodic character, the random response is generally neither stationary nor ergodic. Consequently, it would prevent the application of procedures which are commonly used for evaluation of stochastic parameters along the time coordinate. However, the detailed parameters, e.g. stochastic moments, repeat in a cyclic regime. Consecutive quasi-periods are similar to those observed on synchronously running two or more parallel realizations of the response process. Such processes are called in literature cyclo-stationary processes. For further details on the topic, see [2].

Fig. 3 illustrates behaviour of the quasi-period length for an increasing intensity of the random component. Naturally, the variance of the period length for a low noise intensity is small

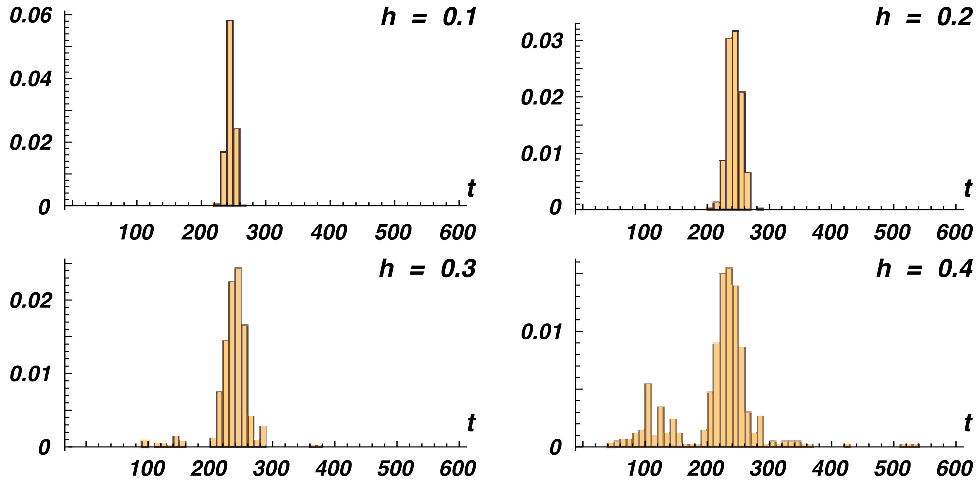

Fig. 3. Histograms of the quasi-period length for an increasing intensity of the random component

and increases proportionally to the noise. It appears, however, that an additional source of uncertainty is a limited possibility to detect the length of the quasi-period. Spurious quasi-periods appear for values of the parameter as low as $h = 0.3$; in the plot for $h = 0.4$ the character of the response changes significantly. A new peak appears in this case for periods of approximately half the original length. If the noise intensity increases further, the original quasi-period will no longer be recognizable in the histogram.

6. Conclusions

The lock-in regime can be significantly complicated when a random noise is considered in addition to a harmonic aeroelastic force. Despite of its complexity, this effect can be modelled using the single-degree-of-freedom van der Pol oscillator with a strongly non-linear damping part. As a beginning of a larger study, the beating quasi-periodic response type in the vicinity of the system eigen-frequency has been studied numerically. It has been shown that the coincidence of both frequencies provides a stationary response. For the response outside the lock-in regime, the complex behaviour has been examined. When random excitation is considered, the shape of the response PDF can qualitatively change due to values of the damping parameters and can exhibit local extremes which can emerge or disappear. Detailed analysis of both excitation components (harmonic, random) is apparently very important.

Acknowledgements

The kind support of Czech Science Foundation project No. 19-21817S and of the RVO 68378297 institutional support are gratefully acknowledged.

References

[1] Dyrbye, C., Hansen, S. O., Wind loads on structures. Wiley and Sons, Chichester, 1997.

[2] Gardner, W. A., Napolitano, A., Paura, L., Cyclostationarity: Half a century of research. Signal Processing 86 (2006) 639-697.

[3] Náprstek, J., Fischer, C., Analysis of the quasiperiodic response of a generalized van der Pol nonlinear system in the resonance zone. Computers & Structures 207 (2018) 59-74.

[4] Náprstek, J., Fischer, C., Super and sub-harmonic synchronization in generalized van der Pol oscillator, Computers & Structures 224 (2019) No. 106103.

[5] Náprstek, J., Fischer, C., Pospíšil, S., Trush, A., Modeling of the quasi-periodic galloping response type under combined harmonic and random excitation, Computers & Structures 247 (2021) No. 106478.