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Identification of viscoelastic material properties using the analysis of waves propagated in thin rods

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This work deals with the identification of material parameters of a thin prismatic viscoelastic homogeneous rods. The first part of this paper discusses transient waves in a viscoelastic 1D medium and presents the relations describing the acceleration of a rod in the space-time domain for specified boundary conditions and for different types of axial load. Subsequently, experiments performed on rods of various material properties, which verify the correctness of the derived solution, are presented in the next part. The following paragraph describes the procedure of material properties identification, which is implemented in the Matlab environment and uses a built-in optimization function and an algorithm of numerical inverse Laplace transform (NILT). Finally, the results of the optimization are analysed and discussed.

Rheological viscoelastic discrete models, which contain serial or parallel connections of elastic and viscous elements, are most often used to describe stresses and strains in viscoelastic solids (see [3]). The generalized Zener model (GZM) is used in this work, as it can represent simpler models in limit cases (see [2]). The relation describing acceleration of a viscoelastic rod of length l, the behavior of which is characterized by GZM, can advantageously be derived in the Laplace domain. Considering the zero initial conditions and the boundary conditions which describe free-free rod, the Laplace transform $A(x,p)$ of the rod acceleration depending on the longitudinal coordinate x and the variable $p \in \mathbb{C}$ can be written as

$$A(x,p) = \frac{\Sigma_0(p) C_0(p) p \cosh \dfrac{px}{C_0(p)}}{E^*(p) \sinh \dfrac{pl}{C_0(p)}}. \tag{1}$$

The complex function $\Sigma_0(p)$ is the Laplace transform of applied axial loading, $C_0(p)$ characterizes the complex wave velocity and $E^*(p)$ represents the complex modulus.

To verify the relation (1), the measurement of the non-stationary response of thin rods of 4 different materials was performed. These were both typically viscoelastic materials, namely acetal copolymer POM-C and polycarbonate PC1000, and typically elastic materials, such as steel and aluminium. The rods were loaded at one end with $Brüel\ \&\ Kjær\ Miniature\ Impact\ Hammer$ – $Type$ 8204 and the acceleration was measured using $Brüel\ \&\ Kjær\ Miniature\ DeltaTron^{®}\ Accelerometer$ – $Type$ 4519 at the other end. The signals from the impact hammer contained noise which was filtered out by applying a rectangular window on a part with the requested pulse. If the noise influences the pulse itself, the individual points were interpolated and scaled to the original maximum, if necessary.

To obtain the acceleration $a(t)$ from relation (1) in time domain, a Matlab code based on the NILT algorithm was created. This program calculates $A(x, p)$ for given material and geometric properties at required complex points p and for all x in the first phase. In the next step, the NILT algorithm taken from work [1] and based on the fast Fourier transform and the ε-algorithm accelerating the convergence of infinite series is applied. This procedure provides the response in time domain. Due to the high efficiency of the described method, the same core is used also for solving the inverse problem of material parameters identification. This parametric optimization has been solved using the standard Matlab fuction $fmincon$. It works with the target function f_c, which expresses the deviation from the measured acceleration, as follows

$$f_c = \left| \frac{(\mathbf{a}_E - \mathbf{a}_A)\mathbf{W}(\mathbf{a}_E - \mathbf{a}_A)^T}{\mathbf{a}_E \mathbf{W} \mathbf{a}_A^T} \right| . \tag{2}$$

The vectors $\mathbf{a}_A$ and $\mathbf{a}_E$ denote the analytically and experimentally determined acceleration vectors, respectively, and $\mathbf{W}$ is the diagonal weight matrix, which is identity in this case. In order to make the optimization precedure effective, the analysis of the target function f_c was performed. It is clear from Fig. 1, which presents the target function for two-parametric Maxwell model, that f_c is more sensitive to the modulus of elasticity E than to the normal viscosity λ and that it has several local minima in the area of interest. Since the $fmincon$ function is not able to find the global minimum without the known approximate values of the optimal parameters, the $GlobalSearch$ object was used in the program. It founds an approximate location of the global minimum in the specified domain. In addition, the created program is divided into three phases, which will speed up the optimization process. In the first phase, the $GlobalSearch$ object and the problem structure $createOptimProblem$ defining the task are created using the $fmincon$ method. Using the run function it starts an algorithm to find the approximate position of the global minimum while changing all moduli of elasticity contained in the rheological model. In the second phase, this algorithm is repeated for normal viscosities. Finally, the aproximate results are refined by running the $fmincon$ function itself with the starting parameters found in the first two phases.

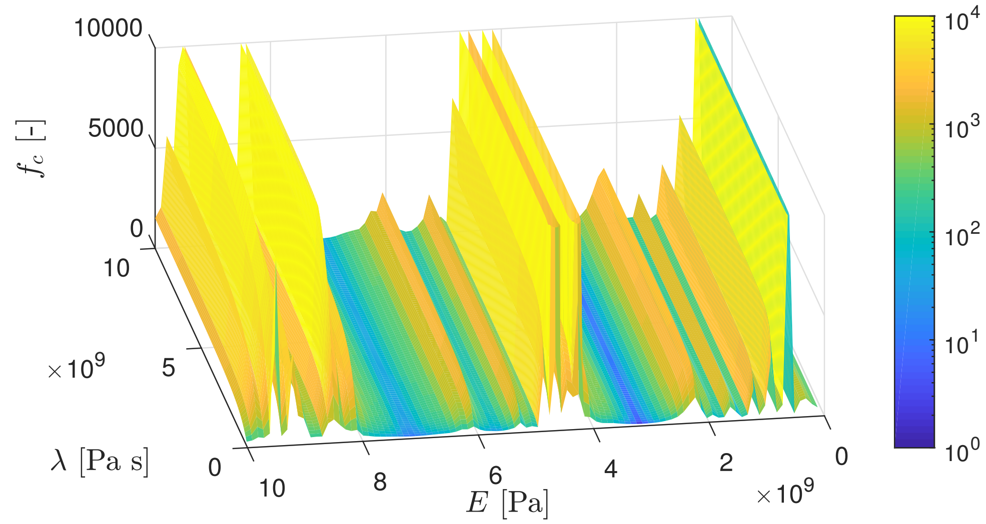
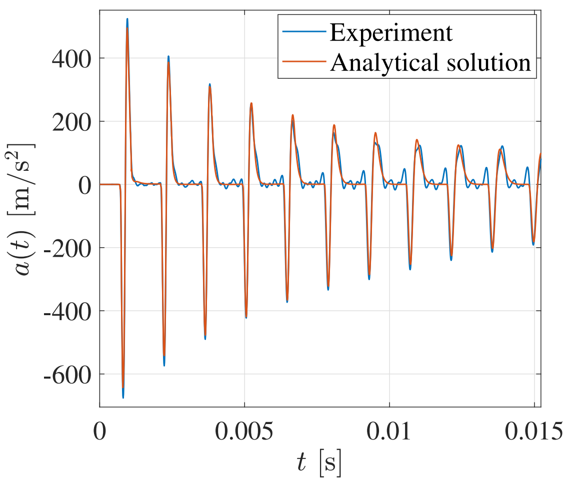


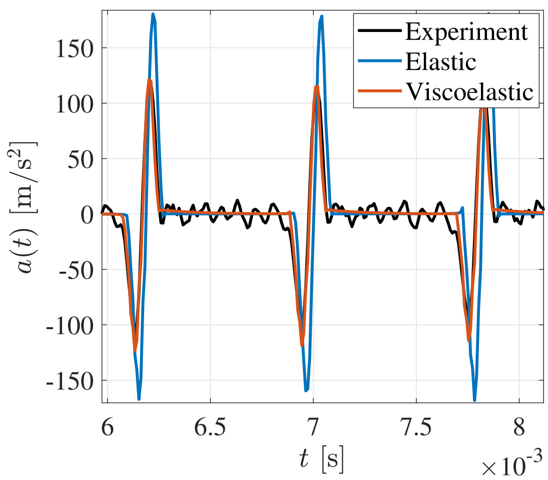
Fig. 1. Target function f_c in the considered domain of optimized parameters

As proved by performed computations, the developed code is very effective and gives reliable results for all the above-mentioned materials with low demands on computational time. Fig. 2a shows the comparison of the acceleration calculated analytically using the optimized material parameters with the measured rod response. It can be seen that the curves have formally the same courses and that the wave speed and the dissipation of energy (i.e., damping) is well preserved in the whole studied time interval. The comparison of the responses for the elastic and viscoelastic model with the real data measured for aluminium is shown in Fig. 2b. It is clear that it is advantageous to include viscosity also for modeling of transient waves in ”pure” elastic materials like aluminium because of non-negligible attenuation observed in the measured response for longer times.

In the first part of this work, the response of a viscoelastic thin rod to transient axial loading is derived for generalized Zener model in the Laplace domain. Then, due to an efficient algorithm for numerical inverse Laplace transform, this procedure is implemented into a program for the identification of material parameters making the use of experimental data. Finally, the correctness and the precision of obtained results were proved by the comparison of measured rods responses and the responses calculated using found optimized parameters.



(a) response of PC1000 rod using viscoelastic GZM model



(b) response of aluminium rod using elastic and viscoelastic GZM model

Fig. 2. Comparison of calculated accelerations with measured data

Acknowledgements

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