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The fuzzy approach for investigation of the steady-state response of rotors excited by loading effects of uncertain magnitude

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Behavior of rotors depends on their geometric shape and composition, materials they are manufactured of, and on their loading and interactions with the environment and other bodies. All these factors are described by a number of parameters, the values of some of them can be uncertain.

Several approaches making it possible to treat this problem have been developed. Among them there are:

- the worst scenario method,
- the fuzzy number approach,
- the probabilistic approaches.

Application of each of these approaches requires to define uncertainty of the input parameters. The worst scenario method requires to know only the interval of values, which the uncertain quantities can acquire. The probabilistic approach utilizes mostly the Monte Carlo method. It is based on knowledge of the probability density function related to each uncertain quantity and requires corresponding generators of the random numbers.

The fuzzy approach assumes that the uncertain quantities take values from some intervals. Then the degree of membership, the magnitude of which is between 0 and 1, is assigned to all of them. This manipulation is based on experience. Some rules and recommendations how to determine the degree of membership can be found in [4]. The following solution consists in performing the interval analysis for all values of uncertain quantities, the degree of membership of which is equal or greater than the chosen one (the same for all uncertain parameters). This gives intervals of values of resulting parameters. Their degree of membership is equal or greater than the chosen one. The fuzzy number approach makes it possible to consider credibility of the obtained results. More details on fuzzy sets and fuzzy numbers can be found in [1], [3-5], [9].

The fuzzy number approach was applied in different fields of mechanics. Moens and Vandepitte [2] used a fuzzy finite element procedure to analyze the uncertain frequency-response function of damped structures, Vaško et al. for frequency analysis [8], Sága et al. [6] for analysis of vibration of vehicles with uncertain parameters, and Vaško and Sága [7] for damage prediction of mechanical structures. This paper deals with analysis of the steady state response of a rotor supported by nonlinear hydrodynamic bearings.

The investigated rotor is rigid. At both its ends it is supported by the hydrodynamic bearings. The rotor is loaded by its weight, by a force acting perpendicularly to its axis, and is excited by the imbalance. The whole system can be considered as symmetric. Scheme of the rotor is depicted in Fig. 1.

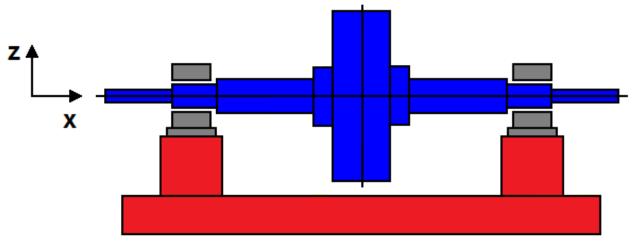

Fig. 1. The studied rotor system

In the computational model the rotor was considered as absolutely rigid and the hydrodynamic bearings were represented by force couplings. The Reynolds equation and computational integration of the obtained pressure distribution were applied to determine the bearing forces. The trigonometric collocation method was used to determine the rotor steady state response.

The mass of the rotor was 426 kg, the length/diameter of the bearing hole were 120/100 mm, the oil viscosity was 0.004 Pas, and the rotor rotated at the speed of 300 rad/s. Eccentricity of the rotor center of mass was assumed to be 30 ± 30 μm and magnitude of the horizontal (acting in the positive direction of axis y) and vertical (acting in the negative direction of axis z) components of the stationary force 2400 ± 72 N and 3600 ± 108 N, respectively.

The task was to find out if maximum displacement of the rotor from the axis connecting the centers of the bearing holes does not exceed 185 μm.

The steady state orbit of the rotor center determined for the most probable values of the uncertain parameters can be seen in Fig. 2. The history of the rotor radial displacement during one rotor revolution is depicted in Fig. 3. The analysis shows that the maximum radial displacement of the rotor center is 181 μm. It implies the allowable value of 185 μm is not exceeded.

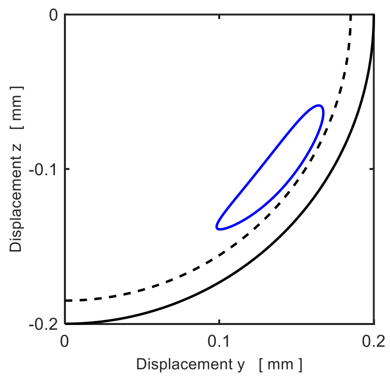

Fig. 2. Orbit of the rotor for the most probable magnitude of loading

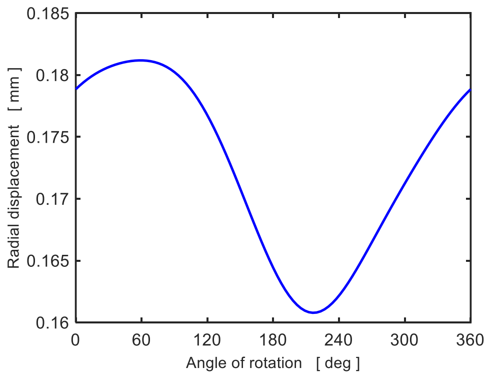

Fig. 3. History of the rotor radial displacement during one rotor revolution

In the next analysis the fuzzy number approach was applied. The membership function (Fig. 4) for the eccentricity is assumed to have a triangular shape (the probability of the lower and higher values goes down). The membership function for the components of the stationary force is estimated to be rectangular (the same probability for all assumed values), which implies the degree of membership of all possible values of the stationary force is 1.

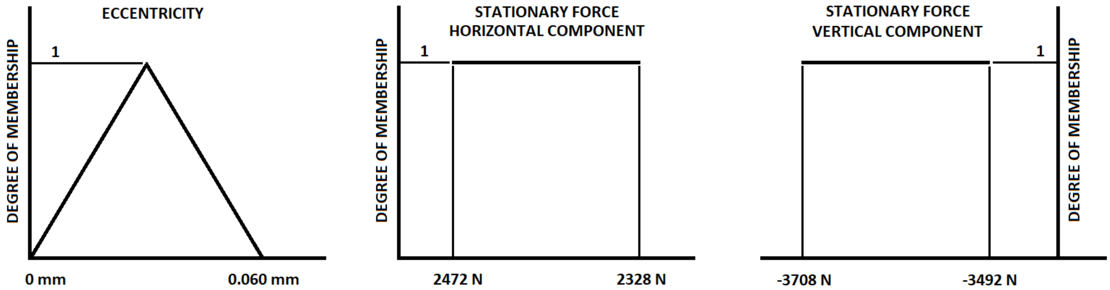

Fig. 4. The membership functions

Six levels of the membership function related to the eccentricity of the rotor center of mass was chosen. Consequently, the corresponding values of the eccentricity were determined. The values related to the uncertain force components were specified by uniform dividing the interval of the assumed magnitudes in 4 parts. The interval analysis was done for all combinations of the values specified for the individual uncertain parameters. Each analysis arrives at the value of the radial displacement of the rotor center. Its degree of membership corresponds to the lowest degree of membership of each component of the uncertain input parameters combination.

The performed analysis shows that membership function of the rotor radial displacement has the shape close to trapezoidal (Fig. 5). Fig. 6 depicts intervals of the rotor radial displacements for several membership levels.

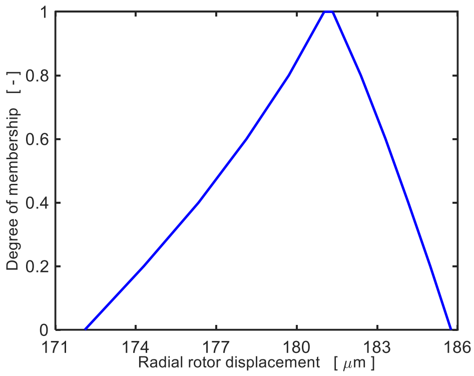

Fig. 5. The membership function of the radial displacement of the rotor center

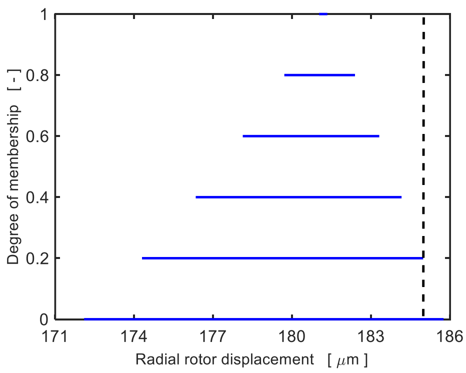

Fig. 6. Determined intervals of the rotor radial displacement for different degrees of membership

The interval related to the degree of membership of 0 corresponds to the worst case. The ratio of lengths of intervals corresponding to the chosen degree of membership and the degree of 0 can be considered as the degree of credibility, with which the results were obtained (Table 1). The allowable value of the rotor radial displacement is not exceeded for the results obtained with the degree of credibility of 78.2 %. This corresponds to the degree of membership 0.2.

Table 1. Degree of membership and credibility

Degree of membership	Degree of credibility
1.0	2.3 %
0.8	19.7 %
0.6	38.0 %
0.4	57.3 %
0.2	78.2 %
0.0	100.0 %

The approach for analyzing the systems with uncertain parameters based on application of the fuzzy numbers has some advantages. It does not require knowledge of the probability density function of uncertain parameters and corresponding random numbers generators. If the number of uncertain parameters is not large, the number of performed simulations can be considerably lower than those if the Monte Carlo method were applied.

Acknowledgements

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