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Modelling of gear couplings in the framework of multibody systems

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A computational investigation of multibody systems dynamics [2] is widely applied in various practical and research problems. Since the multibody system is composed of rigid or flexible bodies, interconnected with multiple joints, and loaded by different types of forces, all relevant parts must be included in the mathematical model of such a system. In general, the bodies can perform large motions and are driven by drives, that are often controlled by a human operator or by some autonomous control strategy. The gear couplings (GC) are the most common parts of the drive trains for various multibody systems, such as robotic manipulators, wind power plants, vehicles, and other systems with rotating parts. The purpose of the GC is to transfer the rotary motion of a drive (motor, engine, etc.) to another body with desired rotation ratio. This also applies to robotic manipulators based on tensegrity structures, where the length of cables is adjusted using motors. The motor rotation needs to be transferred with proper ratio to a cable winch to achieve stable motion of the whole tensegrity structure and also to achieve good dexterity and controllability. This paper deals with the summary of standard approaches of gear and gear coupling modelling in the framework of multibody system dynamics. The most common gear coupling types are spur, helical, bevel, hypoid and worm.

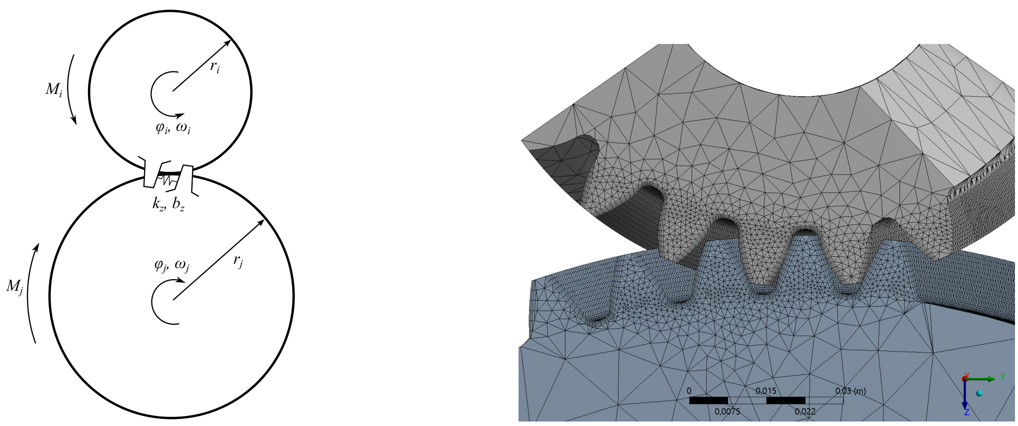

Fig. 1. Simplified scheme of the gear coupling (*left*) and the example of gear stiffness estimation using a software based on the finite element method (*right*)

There are several possible approaches for including the GC in mathematical models of multibody systems, and they can be divided by their complexity. The simplest approach is based on the kinematic constraint formulation in the form of an algebraic equation. Considering the gear coupling from Fig. 1, the algebraic equation that fully constrains the rotation of the gears can be expressed as

$$r_i\varphi_i - r_j\varphi_j = 0, \tag{1}$$

where r_i, r_j are gear pitch circle diameters and φ_i, φ_j are the gear angles of rotation. Since algebraic constraints are commonly used in problems of multibody system dynamics, several methods exist to solve the resulting differential-algebraic equations. On the other hand, this basic approach does not reflect any flexibility of the coupling and also no transverse vibrations of gears are considered, which limits the possible usage of this simple model.

Another approach to describe the GC is the force-based model of the coupling. The basic version couples only the rotational motion of the gears by using appropriately defined moments. Considering the gear coupling from Fig. 1 again, the gear coupling moments M_j and M_i can be expressed as

$$\begin{aligned} F_z &= k_z(r_i\varphi_i - r_j\varphi_j) + b_z(r_i\omega_i - r_j\omega_j), \\ M_i &= F_z \cdot r_i, \\ M_j &= F_z \cdot r_j, \end{aligned} \tag{2}$$

where F_z is the gear force, k_z is the gear stiffness coefficient, and b_z is the tooth damping coefficient. This model allows a flexible transition of the rotational vibrations from one gear to another. The estimation of the gear stiffness can be performed using analytical approaches [3], or the software for structural analysis based on the finite element method can be used; see Fig. 1. This simple force model can be enhanced by varying gear stiffness, kinematic transmission error and teeth backlash that further affects the dynamic behaviour of the whole system [1]. For another improvement of this model, it is possible to include transverse, axial and tilting motion of gears into the expression for teeth deformation, see [1], which makes this approach sufficiently accurate and still computationally effective. This approach can be extended by teeth normal and axial forces together with friction forces, which makes it more detailed.

The most complex force-based method utilizes the general contact of two bodies. In problems of multibody system dynamics, the contact model is most often formulated based on the modified Hertz theory that includes damping in contact. This approach is the most computationally demanding because some algorithm for a contact of solid bodies needs to be employed; thus, its usage in large systems is limited.

The typical software for simulations of multibody systems, such as Adams, Simpack, or Recurdyn, often includes some GC modelling modules, which employ all mentioned methods. Regarding Adams, its force-based approach does not include phenomena such as varying gear stiffness and kinematic transmission error, and its kinematics is based only on the gear rotation angles. Thus, there is still a possibility to extend this kind of modelling tool with the more detailed GC force models, which are needed to be used when the motor-induced vibrations can amplify through the GC to other system parts. This may lead to an increase in noise during mechanism operation. The adequately chosen GC modelling approach helps to analyse possible causes of highly vibrating parts that produce undesired noise.

Acknowledgement

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