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Compressor cascade total pressure loss correlation modelling at design points using artificial neural networks

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1. Introduction

The analysis of the flows by computational fluid dynamics becomes useful design and optimization method during recent years. Despite the advances in the computational power but it could be still very demanding. Therefore empirical models are commonly used as a main tool for design and prediction of basic performance of axial compressor cascades [1]. The empirical correlations are derived from experimental data obtained from two-dimensional measurements. Unfortunately, sufficient amount of data is available only in cases of well-known airfoils as e.g. NACA 65-series or C.4 profiles. Thus, there is en effort to find a similar relation which will serve in the same manner for another family of the airfoils.

Classical profiles as NACA 65-series and C.4 circular-arc are suitable in case of low Mach number corresponding to subsonic flows. Double-circular arc (DCA) and multi-circular arc (MCA) profiles perform well when the flow is accelerated to high subsonic, transonic even to low supersonic velocities [1]. Controlled diffusion (CD) airfoils are designed and optimized specifically for subsonic and transonic cascade applications, thus they can provide better performance than DCA or MCA profiles. The shape construction employs the concept of shaping the blade beyond the point of peak suction of the surface velocity such that the diffusion rate and associated suction boundary layer results in minimum loss for the airfoil section [6] resulting in relatively tight range of acceptable incidence angles [1].

In some complex engineering applications, e.g., nuclear reactor cooling by an axial compressor as a part of the secondary system, it is necessary to ensure reliable operation of the device when off-design conditions occur. Based on desired pressure distribution on the blade surface, camber line of the profile together with thickness distribution are established as described in [4]. A new airfoil family should outperforms NACA 65-series and it should offer performance comparable with the CD airfoils. Furthermore, the range of acceptable incidences should be much wider.

Flow analysis by means of computational fluid dynamics (CFD) could be still very demanding, thus empirical correlations are commonly used as a tool for design and prediction of axial compressor cascade performance. This contribution aims to searching correlation model for design points of the new airfoils family in order to accelerate the design of compressor cascade using artificial neural network (ANN). In contrast to standard deep neural network, the proposed neural network is built using higher order neural units.

2. Objective statement

The basic objective of the empirical modelling process is to predict the fluid turning and total pressure loss for a compressor cascade. From experimental cascade data for NACA 65-series and C.4 circular-arc blades, Lieblein in [5] developed an empirical correlation for a pressure loss PL as a function of the equivalent diffusion factor D_{eq}

$$PL = \frac{\omega \cos \beta_2}{2\sigma} \left(\frac{W_1}{W_2} \right)^2 = 0.004 \left[1 + 3.1 \left(D_{eq} - 1 \right)^2 + 0.4 \left(D_{eq} - 1 \right)^8 \right], \tag{1}$$

where

$$D_{eq} = \left(\frac{W_{max}}{W_1} \right) \frac{W_1}{W_2} = \left(1.12 + 0.61 \frac{\cos^2 \beta_1}{\sigma} \left(\tan \beta_1 - \tan \beta_2 \right) \right) \frac{W_1}{W_2}. \tag{2}$$

As it can be seen in equations above, the dependence between total pressure loss, cascade solidity σ and parameters of the flow is strongly non-linear that is a suitable task for ANN.

3. Methodology

From a mathematical point of view, processing of the information within neuron is consisted of two separated mathematical operations [2]. The first, synaptic operation contains weights of the synapse which represents storage of knowledge and thus the memory for previous knowledge. The second is somatic operation which provides various mathematical operations such as thresholding, non-linear activation, aggregation, etc. Neural output of the unit $\tilde{y}$ is then scalar as it is indicated in Fig. 1 (left) and expressed by the following equation

$$\tilde{y} = \sigma(s). \tag{3}$$

Let us assume N-th order neural unit, then product of synaptic operation can be written as [3]

$$s = w_0 x_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i}^{n} w_{ij} x_i x_j + \cdots + \sum_{i_1=1}^{n} \cdots \sum_{i_N = i_{N-1}}^{n} w_{i_1 i2 \ldots i_n} x_{i_1} x_{i_2} \ldots x_{i_n}, \tag{4}$$

where $x_0 = 1$ denotes threshold and n stands for length of input feature vector.

Since desired outputs are known, machine learning is called as supervised learning which is the task of learning a function that maps input to an output represented with cost function $\vec{\boldsymbol{e}}$. As we could see, the neural output is strongly dependent on the neural memories represented by vector of the weights $\overrightarrow{\boldsymbol{W}}$. Thus, processing of the information should be done in a way which leads neural unit to be learned. Batch Levenberg-Marquardt algorithm for weights updating [2] is employed in this work

$$\overrightarrow{\boldsymbol{W}} = \overrightarrow{\boldsymbol{W}} + \Delta \overrightarrow{\boldsymbol{W}}, \quad \Delta \overrightarrow{\boldsymbol{W}}^T = - \left(\vec{\vec{\boldsymbol{J}}}^T \vec{\vec{\boldsymbol{J}}} + \frac{1}{\mu} \vec{\vec{\boldsymbol{I}}} \right)^{-1} \vec{\vec{\boldsymbol{J}}}^T \vec{\boldsymbol{e}}. \tag{5}$$

Coefficient μ is learning rate, $\vec{\vec{\boldsymbol{I}}}$ is $n_w \times n_w$ identity matrix, n_w number of weights and $\vec{\vec{\boldsymbol{J}}}$ represents $n \times n_w$ Jacobian matrix.

Usually, training data set is divided into three subsets. The first, training set which serves for learning and weights updating. The second is validating set. After each epoch of learning algorithm, error estimation is performed on this subset in order to avoid neural unit overfitting. Training continues until validating error is increasing. Third part is called testing set which measures error after learning is terminated.

In order to obtain training data set for neural network and replace experimental measurement, various numerical simulations with different geometrical setups and inlet boundary conditions were performed. Design incidence angle was found through number of simulations as the flow angle with minimum pressure loss as described in [1].

Designed neural network is consisted of two neurons in the first layer and single neuron in the output layer as it can be seen in Fig. 1 (right). Synaptic operation of all neurons was assumed as quadratic polynomial in the designed ANN. As the activation function $\sigma(\cdot)$, bipolar sigmoid was used in the first layer and linear one in the output layer. Error propagation through the network is performed using multilayer backpropagation algorithm described in [3].

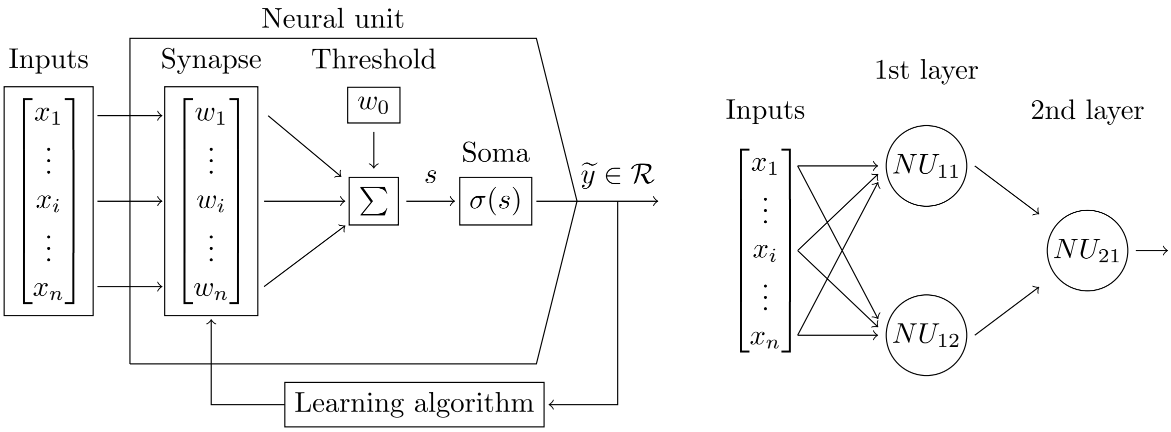

Fig. 1. Neural network: single neural unit (*left*); shallow neural network (*right*)

4. Results

Data set was divided into three aforementioned parts, 80% of samples belongs to training subset and the rest was equally distributed to validating and testing subsets. Learning rate μ in weight updating formula (5) was set to $\mu = 0.4$. Referring to Fig. 2 (left), twenty epochs was sufficient to neural network got learned with testing error 0.0192. Progress of the Lieblein's correlation and the function learned by ANN is shown in Fig. 2 (right).

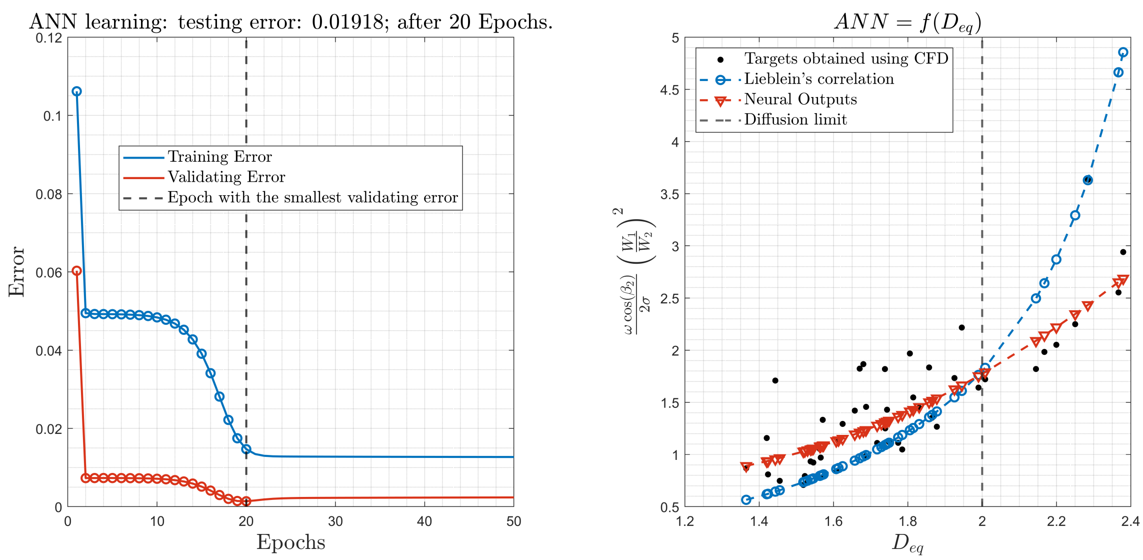

Fig. 2. Results: progress of the learning (*left*); ANN results compared to Lieblein's correlation (*right*)

Deviations performed by artificial neural network and the Lieblein's correlation compared to data obtained by CFD are listed in Table 1, both measured with mean square error (MSE). Approximation using ANN is more than threefold more accurate that Lieblein's correlation model in the whole interval of equivalent diffusion ratio D_{eq}. Although the difference between discussed methods is smaller, it is shown that total pressure loss modelling using ANN offers better approximation than Lieblein correlation in the region under diffusion limit ($D_{eq} < 2$).

Table 1. Mean square error comparison

Interval	Whole interval	$D_{eq} < 2$
MSE: Lieblein's correlation	0.3555	0.1571
MSE: correlation using ANN	0.1096	0.0901

5. Conclusion

An approach for loss correlation was presented in this paper. Based on CFD simulations that was taken as input data set, artificial neural network was learned to predict total pressure loss at design point of axial compressor cascade designed with the new family of the airfoils. Results of the learning are compared against Lieblein's empirical model [5]. Approximation using ANN outperformed available correlation model from the literature as it can be seen in Table 1.

Further work should aim to axial compressor cascade performance predicting at off-design points which will require much larger training data set. Moreover, some geometrical parameters and parameters of the flow probably should be taken into account as inputs to ANN.

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