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Kinematic and constitutive equations in warping torsion of FGMs beams with spatially varying material properties

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1. Introduction

The most well-known theories of warping torsion beams with constant stiffness, (TTT), were elaborated by Vlasov [6], and Benscoter [1]. As is known, their difference lies in the determination of a new unknown quantity, which characterizes the axial deformation of the cross-section caused by its twisting $u_x(x,y,z)$. For Vlasov, this quantity is the relative torsion angle $\psi'(x)$, which is a dependent quantity. In this case, $u_x(x,y,z) = \omega(y,z)\psi'(x)$, This dependence, considered as a lack of this theory, was addressed by Benscoter by introducing an independent function $F(x)$: $u_x(x,y,z) = \omega(y,z)F(x)$. The warping ordinate function $\omega(y,z)$ depends only on the cross-sectional geometry of the beam. Significant contribution to the theory of warping torsion of thin-walled beams of constant stiffness are articles published by Rubin, e.g. [5], where the analogy between the II. order bending theory of beams and the warping torsion was used. The calculation of maximum normal and shear stresses is performed using known formulas based on TTT. However, the above procedures may not be used for FGM beams with spatial variability of material properties, because not only the primary quantities but also the normal and shear stresses and warping ordinates function depend on the variability. This dependence influences maximum stresses not only in their size but also in their place of action [2-4]. In proposed contribution, the new kinematic and constitutive equations for calculation of warping torsion deformation and stresses in the FGM beams with spatial variability of material properties will be formulated. These equations will include both the effects of material properties variability and the material-dependent warping ordinate function and its gradients. These equations can be used for warping torsion analysis of FGM beams with both open and closed cross sections. The significant influence of spatial variability of material properties on normal and shear stresses by warping torsion will be documented by numerical simulations of FGM thin-walled beams.

2. Kinematic and constitutive equations in warping torsion of FGM beams

Kinematics of a warping free cross-section of FGM beam with spatially varying material properties is shown in Fig. 1. In the cross-section, following quantities have to be known: the bimoment $M_\omega(x)$, the primary $M_{Tp}(x)$ and secondary $M_{Ts}(x)$ torsion moments, the relative twist angle $\psi'(x)$, the part of the bicurvature $\psi'_M(x)$ caused by the bimoment, the effective bimoment stiffness $EI_\omega(x)$ and the primary tosional stiffness $GI_T(x)$ and the effective secondary stiffness $GI_{Ts}(x)$. Further, $E(x,y,z)$ is the spatial distribution of the elasticity modulus and $G(x,y,z)$ is the spatial distribution of the shear modulus in the real FGM beam.

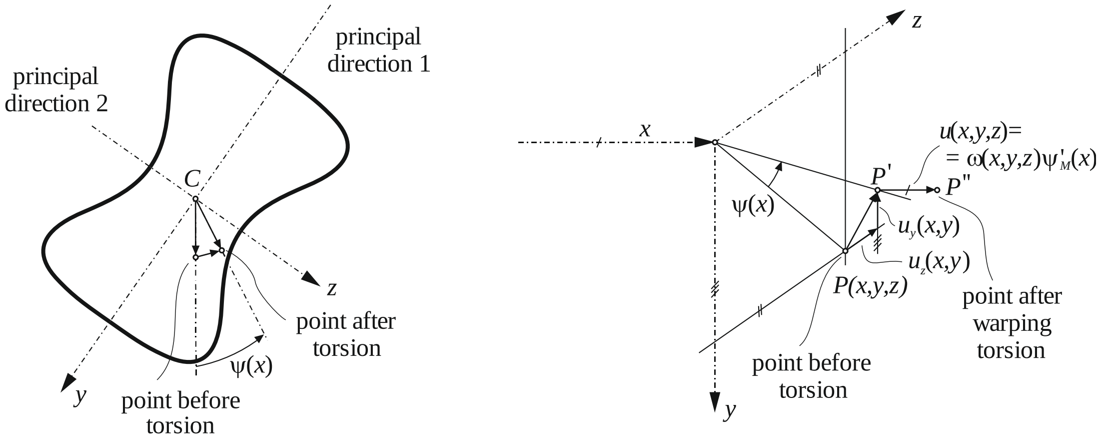

Fig. 1. Kinematics of arbitrary point in the warping free cross-section

After expressing the components of the displacement vector of the selected cross-section, the components of the normal stresses, $\sigma(x,y,z)$, and the shear stresses, $\tau(x,y,z)$, using Hooke's law, we get the resulting relations for their calculation of normal and shear stresses:

$$\sigma(x,y,z) = E(x,y,z)\left[\frac{\partial\omega(x,y,z)}{\partial x}\left(\frac{M_{Tp}(x)}{GI_T(x)} - \frac{M_{Ts}(x)}{GI_{Ts}(x)}\right) - \omega(x,y,z)\frac{M_\omega(x)}{EI_\omega(x)}\right], \quad (1)$$

$$\tau_{xy}(x,y,z) = G(x,y,z)\gamma_{xy}(x,y,z)$$
$$= G(x,y,z)\left(\left(\frac{\partial\omega(x,y,z)}{\partial y} - z\right)\frac{M_{Tp}(x)}{GI_T(x)} - \frac{\partial\omega(x,y,z)}{\partial y}\frac{M_{Ts}(x)}{GI_{Ts}(x)}\right), \quad (2)$$

$$\tau_{xz}(x,y,z) = G(x,y,z)\gamma_{xy}(x,y,z)$$
$$= G(x,y,z)\left(\left(\frac{\partial\omega(x,y,z)}{\partial z} + y\right)\frac{M_{Tp}(x)}{GI_T(x)} - \frac{\partial\omega(x,y,z)}{\partial z}\frac{M_{Ts}(x)}{GI_{Ts}(x)}\right). \quad (3)$$

The warping ordinate function is denoted by $\omega(x,y,z)$. The displacements of the point $P(x,y.z)$ are:

$u_x(x,y,z) = \omega(x,y,z)\psi_M'(x)$ and $u_y(x,z) = -z\psi(x)$, $u_z(x,y) = y\psi(x)$,

from which the normal and shear strains can be obtained. Finally, by use of the Hooke's law the expressions for the normal and shear stresses (1-3) have been established.

3. Conclusions

In the author's contribution, by using the proposed warping torsion FGM WT beam finite element [3, 4], the results from the non-uniform torsional analysis of thin-walled cross-section FGM beams with spatially varying material properties are presented. These results agree very well with the ones obtained by a very fine mesh of the 3D solid FE. New equations for calculation of the normal and shear stresses caused by warping torsion of the FGM beams with spatially varying stiffness are presented. The deformation effect of the secondary torsion moment and the part of the bicurvature caused by the bimoment is accounted. The warping ordinates function and its gradients, which may depend on the spatially varying material properties, are established and implemented in the calculation of normal and shear stresses. Results of numerical experiments and their verification by very fine mesh of the solid finite elements will be presented in the conference presentation It is originally shown that a strong continuous change in material properties causes significant bimoment normal stresses not only at clamped cross-section but also in the field of the beam, and also in the internal points of the cross-section. The variability of material properties in the cross-section of a twisted beam can cause the rise of the maximum stresses at other points of the cross-section than is assumed by the classical theory of warping torsion of thin-walled shafts. For this reason, its use in such cases is inappropriate. Proposed warping torsion beam finite element is very effective - the FGM beam with a longitudinal polynomial variation of the effective material properties can be modelled with only one FGM WT beam finite element. The presented equations (1-3) allow calculation of the normal and shear stress at any point of the cross-section of the beam loaded by warping torsion.

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References

[1] Benscoter, S., A theory of torsion bending for multicel beams, Journal of Applied Mechanics (1954) 25-34.
[2] Kugler, S., Fotiu, P.A., Murin, J., A novel GBT-formulation for thin-walled FGM-beams-structures on reference beam problem, Composite Structures 257 (2021) 113-158.
[3] Murin, J., Kugler, S., Hrabovský, J., Kutis, V., Paulech, J., Aminbaghai, M., Influence of spatially varying material properties on the bimoment normal and shear stresses by warping torsion of FGM beams, Composite Structures 256 (2021) 113043.
[4] Murin, J., Kugler, S., Hrabovský, J., Kutiš, V., Paulech, J., Aminbaghai, M., Warping torsion of FGM beams with spatially varying material properties, Composite Structures 291 (2022) 115592.
[5] Rubin, H., Torsional warping theory including the secondary torsion-moment deformation-effect for beams with constant cross-section, *Stahlbau* 74 (2005) 826-842.
[6] Vlasov, V., Thin-walled elastic beams, Israel program for scientific translations, 1961.