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Vocal tract acoustic modelling using FEM with specific element

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1. Introduction

The acoustics of the vocal tract are involved in the creation of vowel and the timbre of the voice. Bioacoustics and the creation of the human voice is an intensively researched field. Acoustic analyses of vocal tracts investigate the influence of geometry on the character of the voice. This is based on work [1]. According to this work, the vocal cords generate pressure or velocity pulses (source voice) that are independent of the phonated voice. The vocal tract is a dynamic system that modulates the frequency spectrum of the source voice. The transmission function of the vocal tract between the vocal cords and the mouth determines the vowel and timbre of the voice.

Vocal tract analyses are most often performed using the finite element method (FEM) or experimentally. Other methods are less used. The vocal tract is geometrically complicated. Its shape is characteristic. It is a channel of variable cross-section. Frequencies below 4kHz are important for the human voice. At these frequencies, mainly longitudinal waveforms and the simplest transverse waveforms are existed. A conventional FEM model must have a fine mesh to describe complex geometry. Such a model is suitable for analyses in a wide spectrum of frequencies. It is unnecessarily complex for the frequencies of the human voice. A coarser model would not describe the geometry well. In this work, a new element for FEM is developed. This element is suitable for modelling vocal tracts. It has a minimum of degrees of freedom and yet it can describe the geometry of the vocal tract well. This will make it possible to build numerically more efficient models that are similarly accurate as classic FEM models.

2. Formulation acoustic for FEM

The propagation of acoustic waves in the environment is described by the wave equation. A wave equation of the form with losses in the medium is assumed

$$\frac{\partial^2 p'}{\partial t^2} + \frac{r}{\rho_0}\frac{\partial p'}{\partial t} = c_0^2 \Delta p'. \tag{1}$$

The FEM uses a weak formulation of the partial differential equation problem [2]. Multiply the equation by the test function and integrate over the solved volume

$$\int_V \left(\delta p' \frac{\partial^2 p'}{\partial t^2} + \delta p' \frac{r}{\rho_0}\frac{\partial p'}{\partial t}\right) dV = \int_V (\delta p' c_0^2 \Delta p') dV\,, \tag{2}$$

$$\int_V \left(\delta p' \frac{\partial^2 p'}{\partial t^2} + \delta p' \frac{r}{\rho_0}\frac{\partial p'}{\partial t} + c_0^2 (\nabla \delta p')^T (\nabla p')\right) dV = \int_{\partial V} (n^T c_0^2 \delta p' \nabla p') dS\,. \tag{3}$$

From the theory of potential flow, it is possible to determine the dependence between acoustic pressure and acoustic velocity

$$\rho_0 \frac{dv'}{dt} = -\nabla p'. \tag{4}$$

Then it applies

$$\int_V \left(\delta p' \frac{\partial^2 p'}{\partial t^2} + \delta p' \frac{r}{\rho_0} \frac{\partial p'}{\partial t} + c_0^2 (\nabla \delta p')^T (\nabla p')\right) dV = -\int_{\partial V} \left(n^T c_0^2 \delta p' \rho_0 \frac{dv'}{dt}\right) dS\,. \tag{5}$$

Acoustic velocity is eliminated by using acoustic impedance

$$n^T v' = \frac{p'}{Z}, \tag{6}$$

$$\int_V \left(\delta p' \frac{\partial^2 p'}{\partial t^2} + \delta p' \frac{r}{\rho_0} \frac{\partial p'}{\partial t} + c_0^2 (\nabla \delta p')^T (\nabla p')\right) dV = -\int_{\partial V} \left(c_0^2 \delta p' \frac{\rho_0}{Z} \frac{dp'}{dt}\right) dS\,. \tag{7}$$

The integration is over the entire volume (surface). Since integration is additive, it is possible to convert it to integration over individual elements and use shape functions

$$p'(t,x,y,z) = N_e(x,y,z) P_e(t)\,, \tag{8}$$

$$\delta p'(t,x,y,z) = \delta P_e^T(t) N_e^T(x,y,z)\,. \tag{9}$$

The equation will take the form

$$\delta \hat{P}_e^T \left(\int_{V_e} \left(-N_e^T N_e \ddot{P}_e + N_e^T \frac{r}{\rho_0} N_e \dot{P}_e + c_0^2 (\nabla N_e)^T (\nabla N_e) P_e\right) dV + \int_{\partial V_e} \left(N_e^T c_0^2 \rho_0 \frac{1}{Z} N_e \dot{P}_e\right) dS\right) = 0\,. \tag{10}$$

After dividing the equation by the square of the sound speed, it is possible to introduce the matrices defining the element

$$M_e = \frac{1}{c_0^2} \int_{V_e} (N_e^T N_e) dV\,, \tag{11}$$

$$B_e = \frac{1}{c_0^2 \rho_0} \int_{V_e} (r N_e^T N_e) dV + \rho_0 \int_{\partial V_e} \left(N_e^T \frac{1}{Z} N_e\right) dS\,, \tag{12}$$

$$K_e = \int_{V_e} \left((\nabla N_e)^T (\nabla N_e)\right) dV\,. \tag{13}$$

The integral of one element turns into the form

$$\delta P_e^T \left(M_e \ddot{P}_e + B_e \dot{P}_e + K_e P_e\right) = 0\,. \tag{14}$$

Since the integration is additive, the contributions of all elements must be added. This will create a global mass, stiffness and damping matrix

$$\delta P^T \left(M \ddot{P} + B \dot{P} + K P\right) = 0\,. \tag{15}$$

The nodal sound pressure variation is any value satisfying the boundary conditions. Therefore, the form of the equilibrium equations is as follows

$$M \ddot{P} + B \dot{P} + K P = 0\,. \tag{16}$$

3. New element

The purpose of the new element is to describe the complex shape of the cross-section of the geometry with a minimum of nodal points. Significant vibration shapes are longitudinal in nature. Transverse sound pressure gradients should be less significant. That is why we are looking for an element that ideally describes the entire cross-section. The elements are then stacked on top of each other, this makes it possible to change the cross-section along the length. Internal element nodes are not desirable. They increase the size of the model and the computational complexity.

A separate element that would have the described properties was not created. The created element must be used at least 3 times in each section. The element used is shown in Fig. 1. The element is based on a triangular element in the η-ξ plane. It is an isoparametric element in the reference coordinate system [3]. This element is of the sixth order of accuracy. It has 28 nodes, these nodes have been eliminated at the shape function level. Only corner nodes and nodes on coordinates satisfying the equation η+ξ-1=0 are left. Such a planar element is swept out into 3D space. In the third spatial direction, the element is linear. The resulting element is shown in Fig. 1. The nodes at coordinates (0,0,0) and (0,0,1) are located on the midline of the vocal tract. The other nodes form the surface of the vocal tract. In this work, 6 elements are used in one layer. There are 36 nodes in the section on the circuit.

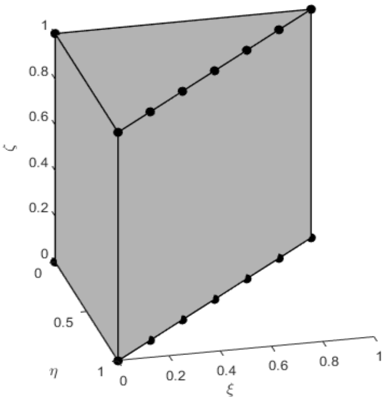

Fig. 1. Used element

Numerical tests showed excellent conditionality of the element even with complex geometric configurations. Fig. 2 shows that for a circular cross-section the worst-case condition is 2.5 and for a strongly non-convex geometry it is 5.5.

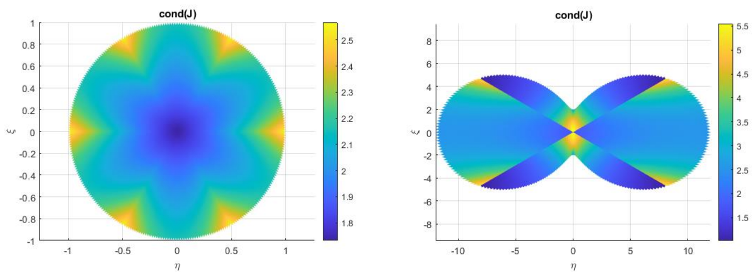

Fig. 2. Conditionality elements creating circle and nonconvex area

4. Vocal tract modelling

Vocal tract geometry was obtained using MRI. This is the geometry for the vowel /a:/.

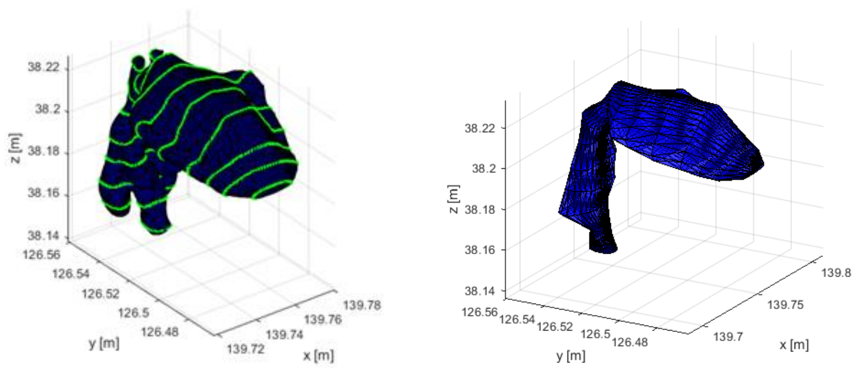

Fig. 3. MRI measurement with cross sections and FEM model

The geometry is shown in Fig. 3. The green lines are the positions of the sections, and the resulting FEM model is on the right. The model was subjected to a modal analysis with velocity boundary condition at the inlet and acoustic impedance at the outlet. The impedance was set according to [4].

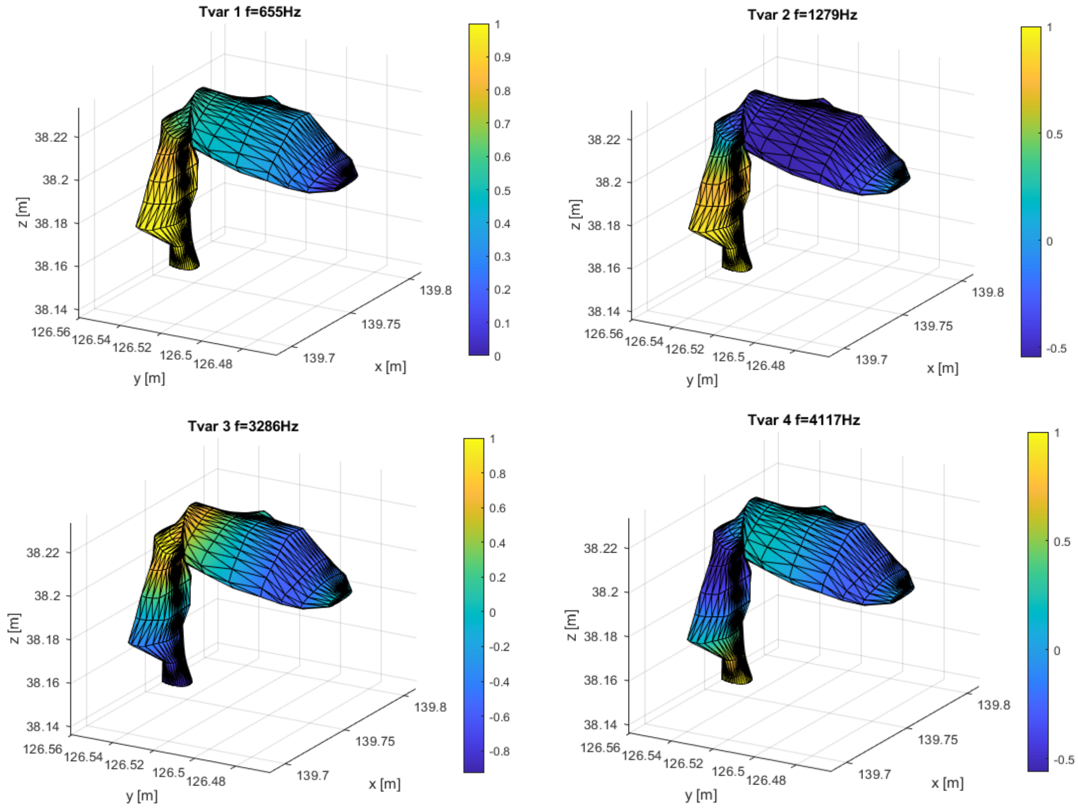

Fig. 4. First 4 eigenmodes

The results of the analyses from Fig. 4 show a good agreement with the generally accepted data, as well as with other better models.

5. Conclusion

The designed element has ideal properties for modelling the acoustics of vocal tracts. The model of the vocal tract composed of 15 layers with 6 elements has only 592dof and in the frequency band 0-6kHz shows a deviation from analytical models of less than 5%. The FEM model with new elements can accelerate the analysis and research of the biomechanics of the human voice.

Acknowledgements

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