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Subcritical behaviour and stability of a rigid rotor supported by undulated journal bearings

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Journal bearings are widely used as a support for high-speed or heavy rotating machines. Among their main advantages are simple construction, low wear, damping capacity and ability to withstand shock loads. However, the journal bearing can be a source of self-excited subsynchronous vibration of the rotating parts. This behaviour is also called fluid-induced instability. The criterion and method for system stability analysis were studied in [3]. The undesirable behaviour can be suppressed by global geometry modifications leading to lemon bore bearings, multi-lobe bearings, offset-halve bearings and tilting pad journal bearings. If different bearing types are insufficient to improve the system stability, the local changes in the geometry by adding grooves, pressure dams, or, nowadays popular, textures are performed. The textures (undulation) can improve the bearing load capacity due to the fluctuation of the developed hydrodynamic pressure. The hydrodynamic pressure is governed by the Reynolds equation. In the case of an indented bearing shell, the solution of the Reynolds equation is tricky. The finite difference [3, 6] and the finite volume methods are the most used numerical methods for solving this equation. These methods are suitable for plain journal bearing, but they become time-costs inefficient for undulated bearing shells. For the textured bearing, the numerical solution requires a significantly higher number of considered nodes in the computational mesh to obtain valid results, see [6]. This contribution presents a computational approach employing the homogenisation method [4, 5] applied to the problem of hydrodynamic lubrication in the journal bearing with textures, where the homogenisation procedure is performed utilising the SfePy tool [1] – simple finite elements in Python. The SfePy tool is further linked with the continuation toolbox MATCONT [2].

First, a parameterization of the bearing model coupled with the rotor is introduced, Fig. 1. The rotor position can be described by coordinates $\boldsymbol{\Xi} = (\Xi_1, \Xi_2)$ in the Cartesian system xz, or by the polar coordinates $\boldsymbol{\xi} = (\rho, \alpha)$ with $\rho > 0$ denoting the eccentricity, and $\alpha \in]-\pi, +\pi[$ denoting the angular deviation from the reference position. Clearly

$$\boldsymbol{\Xi} = \rho(\cos\alpha, \sin\alpha)\ , \quad \rho = |\boldsymbol{\Xi}|\ . \tag{1}$$

Let $\boldsymbol{x} = (x_1, x_2) \in \Omega \subset \mathbb{R}^2$ describe the position in the plane of the rectified bearing gap. Consider $\Omega =]-s, s[\times]0, L[$, where $s = \pi R$ is the half-circumference given by the bearing shell radius R, and L is the bearing width. Due to the bearing eccentricity, the bearing median gap height h_0 is given by

$$\begin{aligned} h_0(\boldsymbol{x}) &= h^0 - \rho\cos(\theta - \alpha)\ , \quad \theta \in]-\pi, \pi]\ , \\ x_1 &= R\theta = s\theta/\pi\ , \quad x_2 \in]0, L[\ , \end{aligned} \tag{2}$$

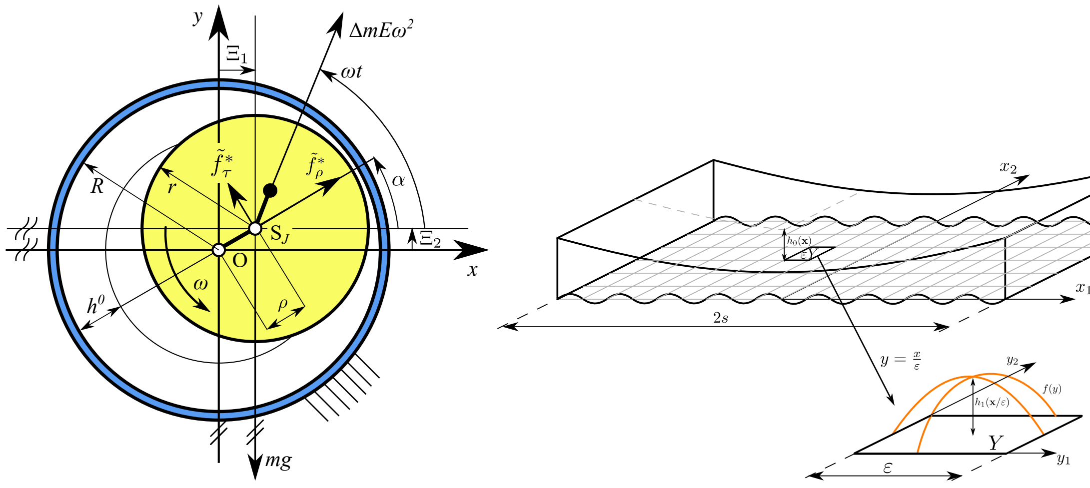

Fig. 1. A scheme of the considered rotor-bearing system with denoted forces (left) and the bearing surface area with undulation (left)

where $h^0 = R - r$ is a nominal bearing gap, given by the bearing shell and rotor radii.

The rotor dynamic is driven by

$$\boldsymbol{M}\ddot{\Xi} = \tilde{\boldsymbol{f}}(\Xi, \alpha, p) + \boldsymbol{g}(\omega, \phi) \quad \text{with } \phi(t) = \omega t \;, \tag{3}$$

where $\boldsymbol{M} = m\mathbf{I}$ is the mass matrix, vector $\tilde{\boldsymbol{f}}$ is given by pressure p by virtue of the Reynolds equation, and $\boldsymbol{g}$ is the rotor unbalance. Recall that the rotor angle position α and eccentricity ρ are related to the Cartesian rotor position (1). The force $\tilde{\boldsymbol{f}}^* = (\tilde{f}^*_\rho, \tilde{f}^*_\tau)$ is defined in the $\rho - \tau$ (radial, tangential) coordinate system as follows

$$\begin{aligned} \tilde{f}^*_\rho(p, \alpha, t) &= \int_0^L \int_\theta p(t, x_1(\theta), x_2) \cos(\theta - \alpha) R \mathrm{d}\theta \mathrm{d}x_2 \;, \\ \tilde{f}^*_\tau(p, \alpha, t) &= \int_0^L \int_\theta p(t, x_1(\theta), x_2) \sin(\theta - \alpha) R \mathrm{d}\theta \mathrm{d}x_2 \;, \end{aligned} \tag{4}$$

where $\theta =]-\pi, 0[$ so that, in the Cartesian system (Ξ_1, Ξ_2),

$$\tilde{\boldsymbol{f}} = \boldsymbol{R}(\alpha)\tilde{\boldsymbol{f}}^* \;, \text{ where } \begin{array}{l} \tilde{f}_1 = \tilde{f}^*_\rho \cos\alpha - \tilde{f}^*_\tau \sin\alpha \;, \\ \tilde{f}_2 = \tilde{f}^*_\rho \sin\alpha + \tilde{f}^*_\tau \cos\alpha \;, \end{array} \text{ thus, } \boldsymbol{R}(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} . \tag{5}$$

Here, the perfectly balanced rotor supported by an undulated journal bearing is taken into account. The dynamic behaviour of the system described generally by (3) is investigated using two methods: numerical integration in the time domain and numerical continuation of equilibrium solution with respect to a chosen bifurcation parameter (nondimensional rotor speed $\overline{\omega}$). Both methods need to compute the pressure field in the bearing gap to determine the hydrodynamic forces which depend on the actual position and velocities of the journal center. Firstly, for a given bearing undulated geometry, the homogenized coefficients need to be calculated. Then, these homogenised coefficients are used to determine the pressure field in the bearing gap in the homogenised model of the Reynolds equation. For illustration, the pressure distribution in the bearing gap in plain journal bearing and undulated journal bearing is shown and compared in Fig. 2.

The rotor dynamic model (3) is transformed into nondimensional form and nondimensional rotor speed and rotor eccentricity are defined as $\overline{\omega} = \omega\sqrt{h_0/g}$ and $\overline{\rho} = \rho/h_0$, where $\rho =$

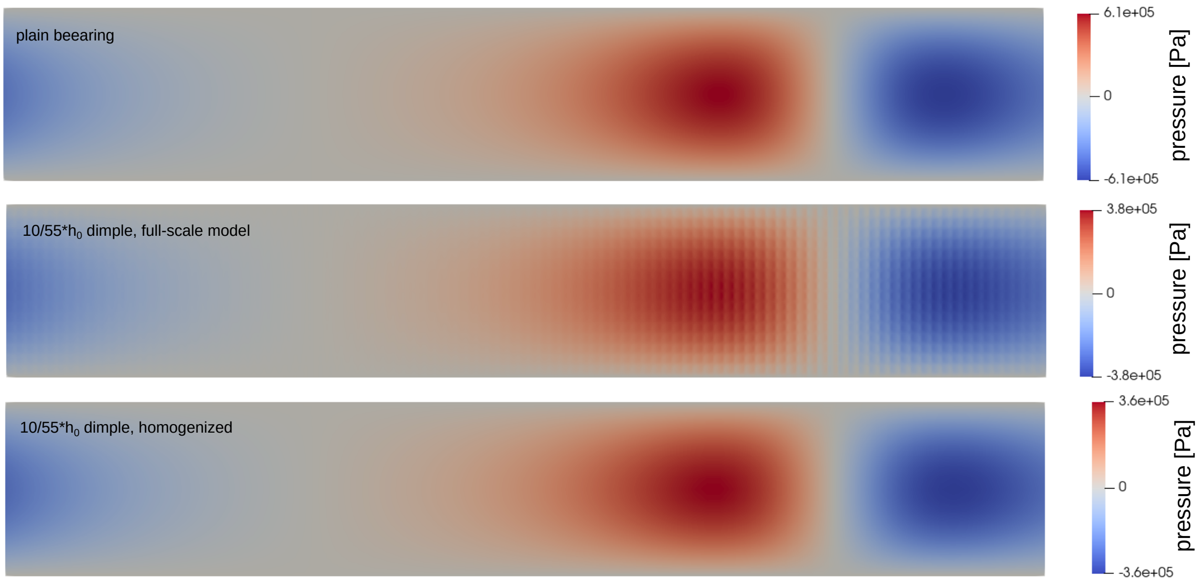

Fig. 2. Comparison of pressure distribution in the bearing gap for $\overline{\omega} = 1.5$, $\rho = 0.5h_0$ and $\alpha = 1.396$ rad; for plain bearing (top), undulated bearing with dimple height equalled to $10/55h_0$ – full-scale model (middle), homogenized model (bottom)

$\sqrt{\Xi_1^2 + \Xi_2^2}$. When neglecting the unbalance, the dynamic response of the model is formed by an equilibrium which is either stable or unstable. Based on this fact, the stability of the system is determined computationally and briefly described by the following steps:

- Initialisation: determination of parameters of homogenised model of Reynolds equation.
- Numerical integration in the time domain of the model (3) for given initial conditions of rotor position and velocity and for given nondimensional rotor parameters: rotor speed $\overline{\omega}$ and rotor bearing parameter λ. The homogenized Reynolds equation is solved in every time-integration step to determine the hydrodynamic forces.
- Finding steady-state (equilibrium) solution $\hat{\boldsymbol{\Xi}}(\overline{\omega}, \lambda)$ using the numerical integration.
- The continuation of equilibrium solution $\hat{\boldsymbol{\Xi}}(\overline{\omega}, \lambda)$ is performed with respect to parameter $\overline{\omega}$.
- In each continuation step, the stability of the solution is determined based on the properties of eigen-values of the linearized model in the equilibrium point.
- Determination of special bifurcation points. In case of equilibria continuation, Hopf's bifurcation points can be detected. These are points where the stability of the equilibrium solution is lost. Simultaneously, as the equilibrium loses its stability, a stable (or unstable) limit cycle solution can be born at Hopf's point.
- Moreover, Hopf's bifurcation points form so-called Hopf's (stability) curve, see Fig. 3 green curve. This curve can be found using codim-2 continuation of the equilibrium solution starting from Hopf's bifurcation point and the continuation is performed in two parametric space $(\overline{\omega}, \lambda)$. On the Hopf's curve, there can be detected Generalized Hopf's bifurcation point which splits the curve into two parts. The first part is formed by Hopf's points from which a stable super-critical limit cycle solution can arise. The second part is formed by Hopf's points from which an unstable sub-critical limit cycle solution arises. For more details, see Fig. 3

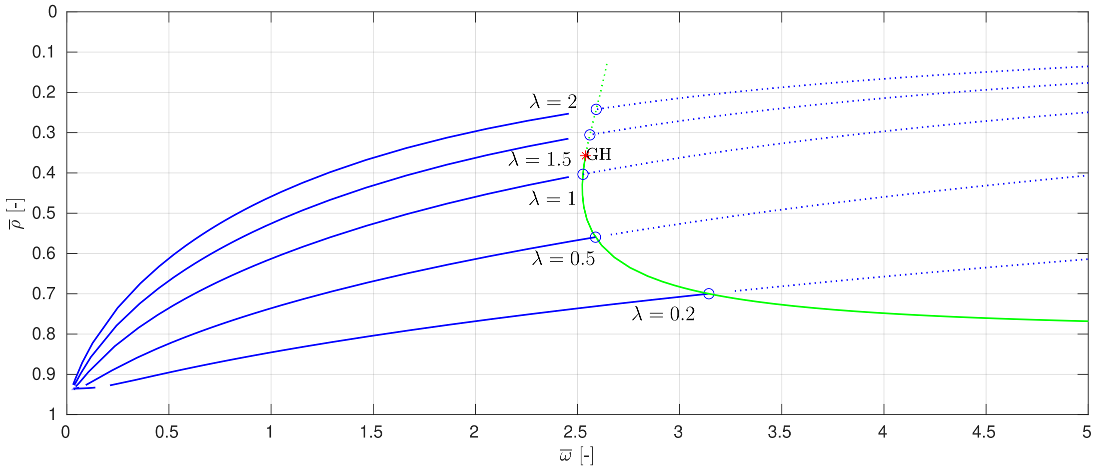

Fig. 3. Stability chart; '—' stable equilibria; '- -' unstable equilibria; ○ Hopf's bifurcation points; ∗ Generalized Hopf's bifurcation point; '—' super-critical Hopf's curve; '- -' sub-critical Hopf's curve

The work presents a novel approach which uses the homogenisation method for analysis of the pressure field in the undulated journal bearing described by the Reynolds equation in tasks of journal bearing dynamics. Based on this, a weakly-coupled dynamic model of a rotor supported by undulated journal bearings is formulated. A corresponding computational model integrates the homogenised model created in the Python-based SfePy tool into the Matlab-based continuation tool MATCONT. This method enables to perform computationally efficient dynamic analyses of weakly-coupled models.

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