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Complex transonic phenomena in critical flow Venturi nozzle

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1. Introduction

With the recent growth of hydrogen technologies, high precision measurement of flow rates at low pressure ratios is necessary. A widely used methodology for this type of measurement is the ISO 9300 standard [2]. It is in the interest of high quality measurement to recognize the nature of all the complex phenomena that occur in the flow through the critical flow Venturi nozzle (CFVN).

The axisymmetric geometry of the nozzle consists of a convergent part, which is defined as a section of a torus, and a divergent part that is created by a cone. The throat of the CFVN can be of two types – cylindrical and toroidal. The geometries are illustrated in Fig. 1.

The flow is accelerated from low subsonic velocities at the inlet of the nozzle to a supersonic flow. As the transonic conditions are reached, the flow is aerodynamically choked and the flow rate is directed solely by thermophysical properties of the gas, by stagnation conditions and by the geometry of the nozzle.

This work focuses on phenomena, which originate in compressibility of the fluid, namely the sonic line (its shape and position) and local supersonic compression in transonic expansion.

Ideal gas with properties of air (specific gas constant $r = 287.05\,\mathrm{J/(kg\,K)}$ and ratio of specific heats $\kappa = 1.4$) is considered in this work.

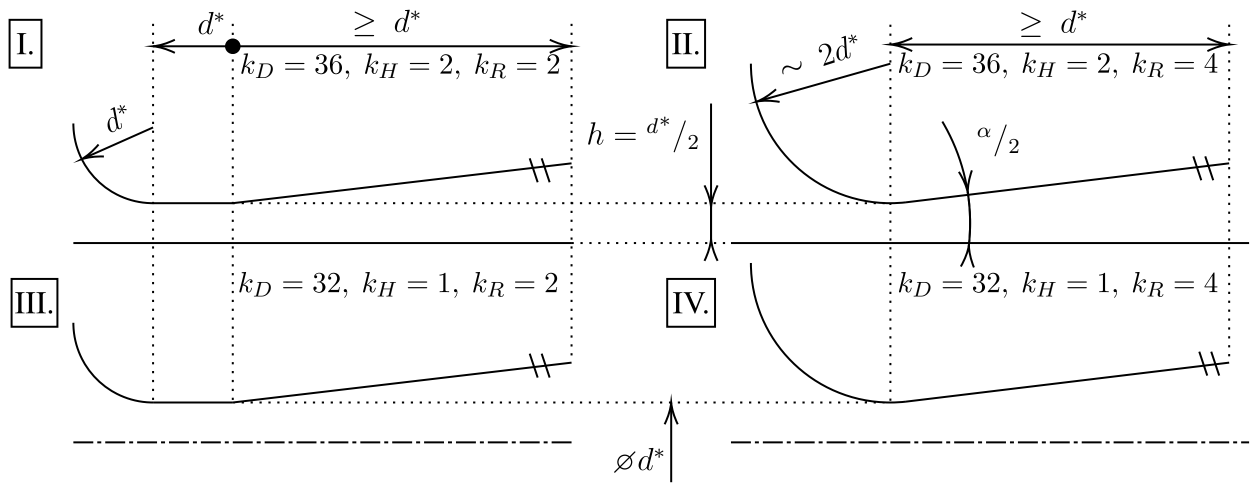

Fig. 1. Types of CFVN

2. Theoretical analysis

Thanks to the nature of transonic flow significant phenomena can be predicted based solely on theoretical background. Analytical description was developed mostly for two dimensional problems, therefore part of the theoretical analysis is performed for idealized flat asymmetrical nozzle (types I. and II. in Fig. 1[1]).

For inviscid flow, the shape and position of the sonic line can be predicted. Position of the sonic line corresponds to the minimum cross-section (throat) of the nozzle. Approximate shape of the sonic line can be determined based on analysis proposed by Shapiro in [4] for both, flat and axisymmetric, types of the nozzle. An assumption made in [4] is that in the vicinity of the sonic line, the curvature of the wall is continuous, which, in studied case, is not met (especially for types I. and III.). Still, this analysis can serve as a very good first step approximation.

The relations given in [4] are adjusted based on geometry of the nozzle, so that the resulting relations are dependent only on the heat capacity ratio (κ) and type of the nozzle. The relation is converted to dimensionless, so that it is independent of critical dimension d^* ($r^+ = {}^{2r}/_{d^*},\ x^+ = {}^{2x}/_{d^*}$). The curve describing the sonic line is given by relation (1). Constants $k_i,\ i \in \{D, R, H\}$ (describing geometry and dimensionality of each problem) in relation (1) are given in Fig. 1. In Fig. 2, shapes of sonic line obtained numerically for inviscid flow and by the approximate calculation are compared. The discrepancy may be due to combination of errors of both methods.

$$r^+ = \left[(1 + k_R) - \sqrt{k_R^2 - (k_H k)^2}\right] \sqrt{\frac{k - x^+}{k(1 + k_H)}}, \qquad k = \sqrt{\frac{\kappa + 1}{k_D k_R}} \tag{1}$$

As the inlet of the nozzle is defined by section of a torus with quite a small radius, sonic line is significantly curved. The lower the Mach number, the higher is the angle of streamline and characteristic ($\mu = \pm \arcsin M^{-1}$). These two facts allow the waves (characteristics) to interact with sonic line and be reflected by it. Expansion waves are reflected by the sonic line as compression waves, these cause local increase of the pressure and therefore slow down the transonic expansion. LSCiTE is documented in [5]. Using an interferogram obtained by inviscid numerical simulation, the LSCiTE is illustrated in Fig. 3[2].

In [3], expected frequency of the oscillations of the pressure field is given. Dimensionless

[1]To simplify any comparison of results, critical height of the flat nozzle is set to ${}^{d^*}/_2$.
[2]Constant of the interferogram is 0.011858 kg/m^3.

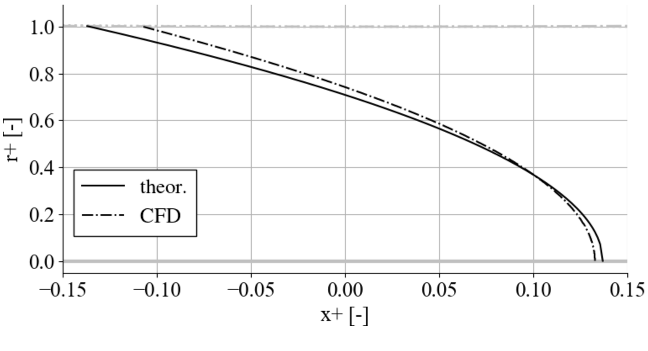

Fig. 2. Comparison of sonic line shapes (type IV.)

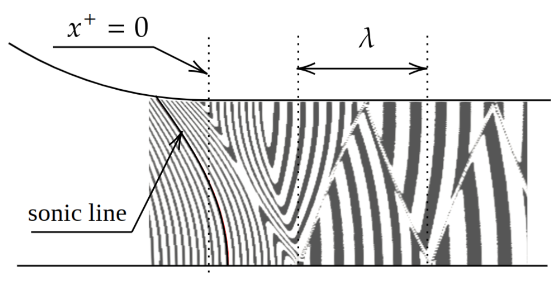

Fig. 3. Numerical interferogram with visible LSCiTE (inviscid, type I.)

wavelength of the oscillations is derived in the following form:

$$\lambda^+ = \sqrt{\frac{\kappa+1}{2}\frac{12}{13}}. \tag{2}$$

The relation is provided only for the nozzle type I., even though similar but more complex behavior can be expected in all other types. Numerically obtained wavelength $\lambda^+ \approx 0.8$, which differs by ≈ 20 % from the theoretically predicted value. This is not surprising since the theoretical relationship is based on the very same description of the sonic line as was shown in previous part of this work and which did not match the numerical results perfectly.

3. Numerical analysis

The numerical analysis was performed using LU-SGS solver [1] implemented in OpenFOAM library. Standard boundary conditions were applied, with total pressure and total temperature set to 10^5 Pa and 303.15 K, respectively. Supersonic outlet is assumed, and therefore homogeneous Neumann BCs are prescribed for all variables. Ideal gas with properties of air is assumed. For the viscous cases the Langtry-Menter 4-equation transitional SST turbulence model was applied with boundary conditions meeting the requirements of the model. Hexahedral mesh – with refinement at the wall for viscous cases – was used. The refinement ensures that the dimensionless size of the cell at the wall is close to one ($y^+ \approx 1$) and the growth in normal direction to the wall is gradual. The value of y^+ is verified after computation, in all cases $y^+ < 2.5$. The higher values are obtained only near the inlet of the nozzle.

Results of computations with inviscid fluid can be easily compared with theoretical assumptions but need to be precised by including viscous effects. The greatest influence of viscosity can be observed in the nozzle with cylindrical throat (III. in Fig. 1), as the evenly developing boundary layer in the throat creates the minimum effective cross-section at the end of the throat. That is why LSCiTE is not observed, see Fig. 4 with comparison of results obtained with an assumption of viscous and inviscid fluid. In type IV. LSCiTE appears even in the viscous cases[3] (Fig. 5). Oscillations of the pressure field should therefore be taken in account. These perturbations can speed up the transition to turbulence which is an effect that should not be neglected as it works directly against the favorable pressure gradient, which suppresses the development of turbulent boundary layer. Moreover, as the ISO 9300 standard requires static pressure measurement behind the nozzle, it is in the best interest of precise experiment to ensure that the tap is not affected by the pressure oscillations.

[3]In the computed case ${}^{\alpha}/_{2} = 6°$ which is maximum allowed ${}^{\alpha}/_{2}$ by the standard.

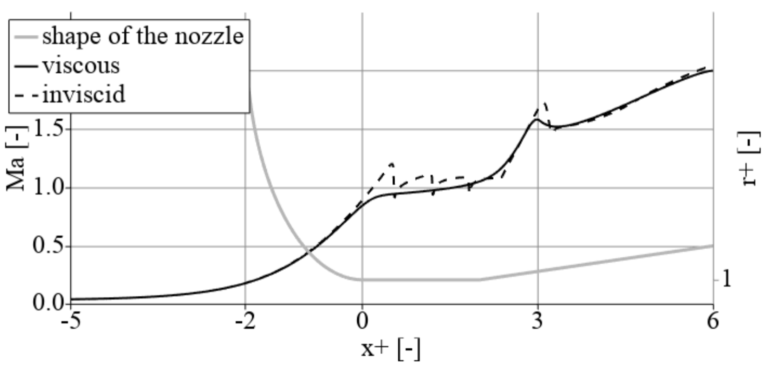

Fig. 4. Development of the Mach number along CFVN axis (type III.)

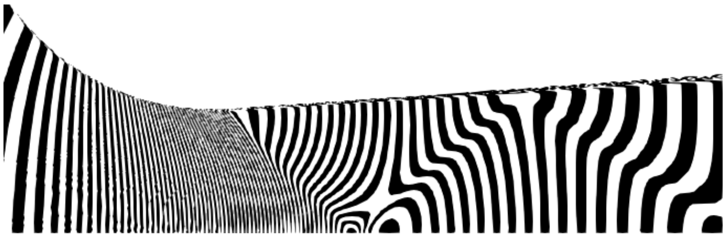

Fig. 5. Numerical artificial interferogram of meridian slice with visible LSCiTE (viscous, type IV.)

4. Conclusions

Theoretical and numerical analysis of flow field in critical flow Venturi nozzle was performed. It was shown that for proper measurement, presence of more dimensional gas dynamics effects should be taken into account. Also, it is important to point out the strong sensitivity of transonics to geometry and physical parameters changes.

For a validation of numerical computations, a set of experiments focused on more dimensional gas dynamics effects in CFVN would be necessary. Plenty of work is still left untouched in the field of (as precise as possible) turbulence modeling. Based on empirical criteria for relaminarization it can be easily shown that in major part of the nozzle, turbulence in boundary layer might be suppressed by the presence of strong favorable pressure gradient. Interference of this effect with LSCiTE should be studied.

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