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Evaluation of the delta-sigma modulator coefficients by genetic algorithm

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Abstract:
The article deals with real aspects of the Delta-Sigma modulator (DSM) coefficients design by genetic algorithm (GA). There are described conventional methods of the coefficients design and limitations of the ideal value reaching due to qualities of the real circuits. The restriction must be considered in a criteria function used by GA. Next, the article describes GA settings for optimal DSM parameters searching.

INTRODUCTION

Two ways to real modulator's coefficients design are currently used. The simplest method is coefficient calculation based on the modulator's transfer functions. The coefficients designed by this method ensure intrinsic modulator stability; nevertheless potential of the noise shaping is not exploited as can be.

Null and pole modelling of the modulator transfer functions is more sophisticated method. The method provides better exploitation of the noise shaping against calculation based on transfer functions. However, the noise shaping potential is also not use fully. The reason is known - modulator stability especially in case of high order modulators. The crucial restriction of the real modulator coefficients design is full voltage swing in circuit requirement accompanied by attenuation of the integrator and summator saturation.

Consequently, the optimization of the modulator noise shaping is possible by the experimentation with zeros and poles placing or by pure computing force. The GA with appropriate criteria function is also perspective method. The criteria function has to consider modulator stability, signal to noise ratio (*SNR*) and voltages in modulator circuitry.

CONVENTIAL METHODS OF THE COEFFICIENTS DESING

Generalized DSM consists of difference amplifier, integrator, timer, quantizer and digital to analog converter (DAC). There is usually anti-aliasing filter in front of the modulator and low-pass digital filter behind it. The second order DSM CIDIDF [1] topology is shown in Fig. 1. The coefficients c_n of the DSM are equated one. It is possible because of other coefficient ratios.

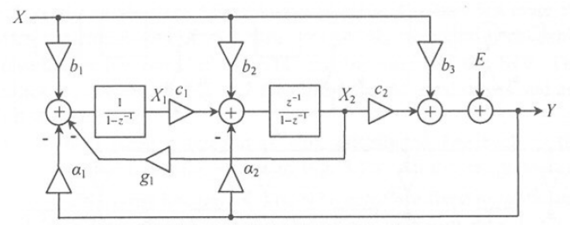

Fig. 1: The second order DSM CIDIDF topology

COEFFICIENTS DESIGN BY ZEROS AND POLES MODELING

There are zeros and poles placing of the second order DSM in the Fig. 2. Changeover of the coefficient a_1 changes root placing, while changeover of the coefficient a_2 has minimal influence. If $a_1 < 1$ (transfer function lays in the unit circle) the modulator is stable. The Fig. 2b shows transfer function of coefficient $a_1 = 0.5$. The transfer function poles are placed in z = 1 and with increasing gain of the k_i go to real axis along the unit circle. One of the poles stays inside the unit circle and second goes into z = -∞.

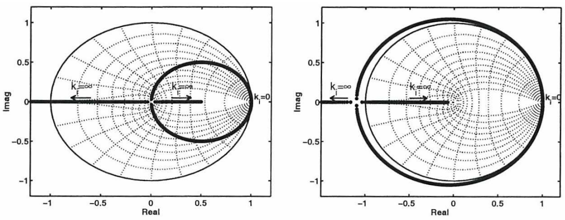

Fig. 2: Zeros end poles placing of the second order DSM **a)** stable state ($a_1 = 0.5$) **b)** instable state ($a_1 = 1.05$).

If $a_1 > 1$, the system is instable. One pole is always out of the unit circle for every value of the k_i. Signal rises up and output of the quantizer reach maximal level. Gain drawdown and poles come back into z = 1, but never come back into the unit circle.

- increasing value of the coefficients a_i improve SNR,
- If the coefficients are increased too much and stability border is reached $a_1 = 1$, the gain markedly decrease and signal amplitude

inside convertor limitedly rise up. If $a_1 > 1$ signal rises unlimitedly and converter is instable.

- If the multi-bit quantizer is used, the quantizer becomes more linear and DSM is more stable.

The wider range of the coefficient values can be used in switched capacitors circuitries. Unfortunately, it leads to high capacitive load of the opamps and then to higher power consumption. As noted previously, the voltage swing in inner nodes is crucial during real design of the DSM.

CALCULATION FROM TRANSFER FUNCTIONS

It comes out from implementation of the input signal distribution during coefficients design. The signal distribution can be analog as shown Fig. 1 or digital as shown Fig. 3.

Unit gain of the STF

It's relatively strict criteria which support modulator stability, especially low order modulators.

$$STF(z) = 1 - NTF(z) \qquad (1)$$

and for high order modulators:

$$NTF(z) = (1 - z^{-1})^2 \qquad (2)$$

Unit gain STF is desired because

- relaxing input anti-aliasing filter requirements,
- reduces out-band spectral components in front of the quantizer, that can overload it.

Analog implementation

Desired transfer functions have to have form

$$NTF(z) = (1 - z^{-1})^2 \qquad (3)$$

and

$$STF(z) = 1 \qquad (4)$$

Implementation of the analog feed forward (AFF) is complicated by analog summation requirement. It is usually solved by:

- active summation. There is operational amplifier in the summator that leads to increased power consumption.
- passive switched capacitors. This low power consumption solution can be used especially if number of quantizer bits is low.

Digital implementation

Desired transfer functions have to have form

$$NTF(z) = (1 - z^{-1})^2 \qquad (5)$$

and

$$STF(z) = 1 - NTF(z) \qquad (6)$$

The implementation requires an additional quantizer (Fig. 3) against AFF. The quantizer introduces additional quantization noise into signal path; nevertheless feed forward with gain *R* can effectively eliminate it.

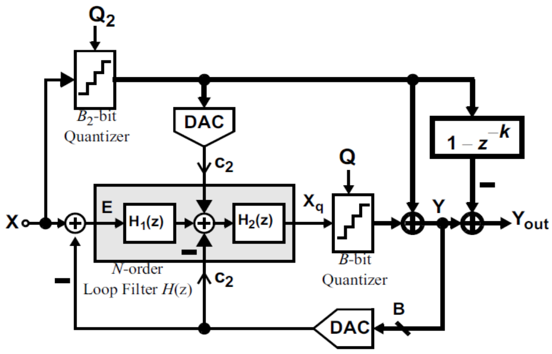

Fig. 3: Digital feed forward SDM with unit gain

The quantization noise of the quantizer Q_2 has not influence on the modulator STF. Difference between DSM input and output is led into the output of the loop back filter integrator. The loop back filter is designed as output modulator:

$$Y = X + (1 - z^{-1})^N \cdot Q + (1 - z^{-K}) \cdot Q_2 \qquad (7)$$

where N is order of the loop filter and K is design parameter [1]. Next step has to be suppression of the addition quantization noise Q_2 of the modulator output (X + Q_2). Output of the SDM is then:

$$Y_{OUT} = STF \cdot X + NTF \cdot Q \qquad (8)$$

where STF = z^{-K} and NTF = $(1-z^{-1})^N$

Loop filter process only shaped quantization noise.

$$E = -NTF \cdot Q - (1 - z^{-K}) \cdot Q_2 \qquad (9)$$

Input of the quantizer include only quantization noise

$$X_q = (NTF - 1) \cdot Q - z^{-K} \cdot Q_2 \qquad (10)$$

The voltage swing of the integrator in front of the quantizer internal nodes is reduced. The design for K=1 provides total suppression of the quantization noise leakage. This setting minimizes sensitivity of the DSM to the DC gain of the opamp. Value of the parameter K is designed K=1 or K=2 [6], [7].

CRITERIA FUNCTION

At first, it is crucial to determine criteria function for genetic algorithm (GA) use. The criteria function is variable that has to be maximized or minimized. The *SNR* of the DSM was chosen as a criteria function to get optimal values of the coefficients. If the voltage in internal node saturates, the result of criteria function is set to zero. Using Fast Fourier Transformation (FFT) is necessary for the *SNR* calculation. The calculation is quite difficult. To avoid inadequate calculation, input data vector is tested to internal voltages, and if the transfer function curve lays inside the unity circle, before starting FFT calculation.

GENETIC ALGORITHM

It's crucial appropriate setting up parameters when the genetic algorithm is used. Variations of the number of the generations and number of the members of these generations are not so crucial to get tolerable results.

Why only tolerable results? Because the GA is a stochastic process of the solution searching that can not guarantees finding the most optimal result. The GA can works with many possible solutions at the same time, but it still cannot guarantee finding the highest value of solutions with plenty of local extremes. Hence, there is essential to determine the value that the criteria function can reach at the beginning. Then it should be known, if exists one or more solutions in range of searched global extremes, and how much the global extremes are covered by local extremes. It´s very helpful to know the waveform of the criteria function.

At first, the most important issue is appropriate settings of the solution parameters that will be searched. Then determine their range and resolution. The criteria function has to appropriately evaluate these parameters. The resolution of separated parameters is most critical at this time; because of individual subjects ("solutions") in GA generation haven't inadequate bit length.

Appropriate generation's ***elimination method*** can be chosen if the estimation waveform of the criteria function is known. The criteria function is overloaded by number of local extremes in case of optimal coefficients search. Due to the ***tournament method*** was chosen. An advantage of the tournament method inhere in the part of previous generation is moved into the new generation. Simultaneously, the new group of solution is better than eliminated one.

Now, the ***number of subjects (I)*** in generation must be evaluated. Evaluation of the value is difficult because the number of the local extremes is usually not known. Accidently, the number of global extremes is even unknown. It is recommended that the number of ones should be appropriate to avoid undesired elimination of the solution leaded to finding global extreme. Nevertheless it's not necessary set number of members comparable to number of local extremes. The GA can overcome local extreme and continues in solving.

Examination and analysis of extremes positions is crucial for new generation creating. If extremes are situated near one area or are randomly spread through the solution and what gradient they have. All above noted influences settings of the ***hybridization coefficients*** (hc) and ***mutation coefficients*** (mc). The hybridization coefficient has the most influence on new subject searching in other space of the searched solution. The mutation coefficient serves to solution searching near close space of solutions. Suma of hc and mc should be equated 1, but it is not necessary. In some cases the summa of hc and mc should be higher than 1. It is relatively aggressive method of new subject creation that is acceptable only in cases when number of subject of the new generation is too small and is desirable to extend decimated generation.

The ***number of generation (G)*** than is necessary to proper solution searching can be only difficultly determined. It is due to that in majority of cases is number of solution extremely high. It's impossible to process all possible solution space. Two main variant exist what next? The first possibility inheres in algorithm running until the required value is searched. This method should be in many cases very time-consuming and is useful limit time of computing. Second possibility inheres in number of generation setting before start of computing depending on time needed to criteria function evaluation. In both cases is crucial to control values of the searched solution depending on number of generations. If the GA culminated, as can be determined from this waveform, the better solution searching is not expectable. There are searched values of SNR depending on set number of generations in followed figures.

Parameters are generated in 8bit resolution:

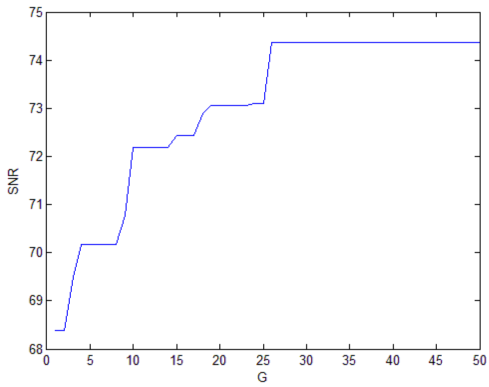

Fig. 4: G50 I40 HC02 MC08 8b (2000)

a1= 0.21875 a2= 0.484375
b1= 0.2421875 b2= 0.6171875 b3= 0.015625
SNR= 74.3616779427

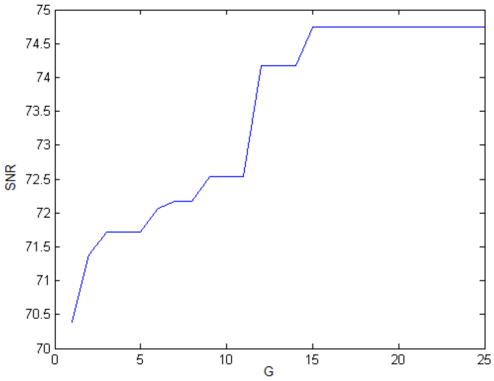

Fig. 5: G25 I80 HC02 MC08 8b (2000)

a1= 0.359375 a2= 0.8046875
b1= 0.3828125 b2= 0.4375 b3= 0.07421875
SNR= 74.7393170883

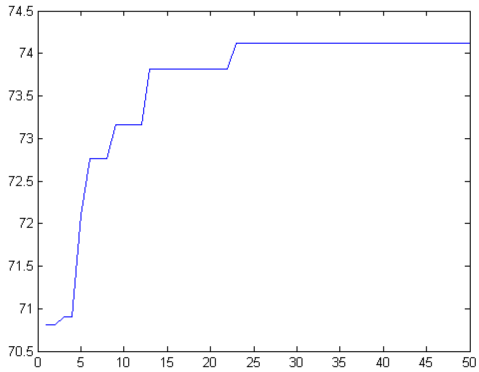

Fig. 6: G50 I80 HC02 MC08 8b (4000)

a1= 0.28125 a2= 0.58203125
b1= 0.31640625 b2= 0.49609375 b3= 0.0546875
SNR= 74.1157581753

Parameters are generated in 10bit resolution:

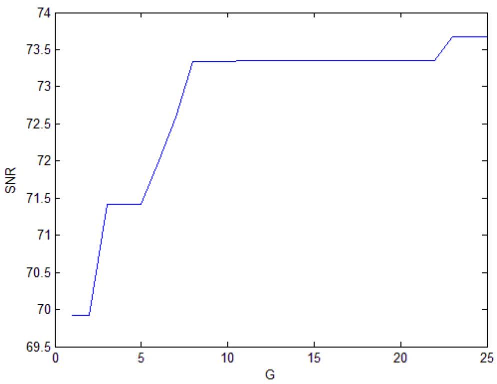

Fig. 7: G25 I80 HC02 MC08 10b (2000)

a1= 0.1484375 a2= 0.3671875
b1= 0.16894531 b2= 0.597656 b3= 0.79394531
SNR= 73.6742137149

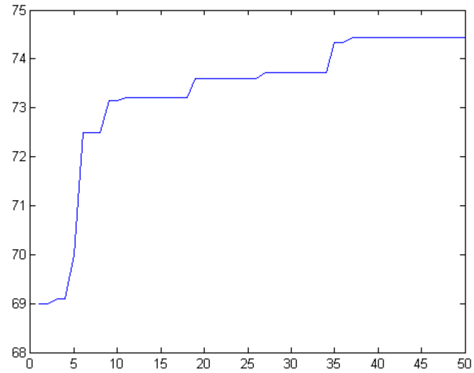

Fig. 8: G50 I80 HC02 MC08 10b (4000)

a1= 0.0576171875 a2= 0.1259765625
b1= 0.0576171875 b2= 0.1005859375
b3= 0.2871093750
SNR= 74.4455346152

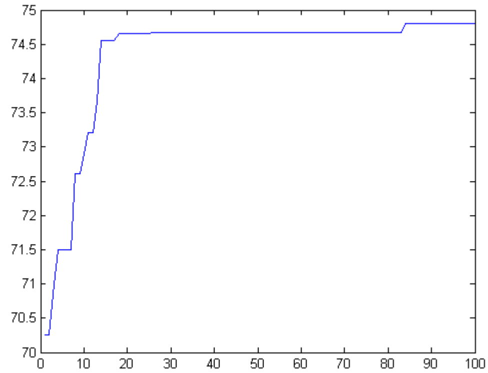

Fig. 9: G100 I80 HC02 MC08 10b (8000)

a1= 0.369140625 a2= 0.8046875
b1= 0.400390625 b2= 0.484375 b3= 0.5146484375
SNR= 74.7975158267

The previous figures present curves of searched *SNR*. It is obvious that optimal GA configuration is in case of GA = 25 I = 80 hc = 0.8 mc = 0.2 and 8bits resolution of the parameters. Even if the best result of SNR wasn't reached under this configuration, solution was found successfully fast before maximal number of calculated generations ware reached. It is expectable that appropriate value of SNR will be reached until 25 generation. This presupposition acknowledges even waveform catches SNR for 50 generations and solution was found before 25^{th}. Additionally under this GA settings only 2000 times calculation of criteria function were needed. Thanks that the searching was relatively fast (one value of criteria function has been calculated for approximately 8 seconds). The GA should not be terminated before the adequate solution has been searched. It's obvious from Fig. 10 and Fig. 11; where the solution was reached about 25^{th} generation. If parameters were generated in 10bit resolution the optimal values of SNR were reached after higher number of GA calculations steps.

COEFFICIENTS SEARCHING RESULT

Different requirements for each realization of the DSM are needed. Coefficients of the CIDIDF topology of the DSM (Fig. 12) were searched by using GA for different maximal voltages in inner nodes of DSM. The result is catch from Fig. 13 where searched parameters are shown.

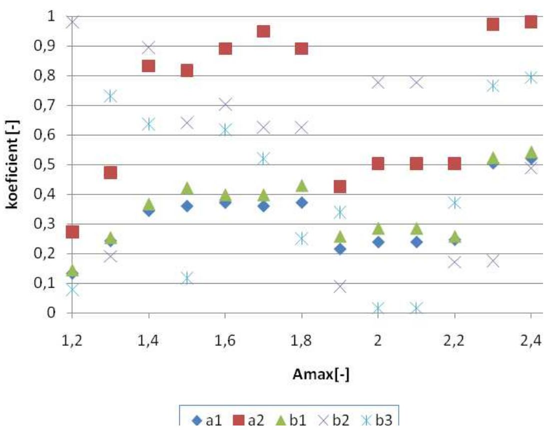

Fig. 13: Values of DSM coefficients for some maximal voltages in inner nodes

Coefficients c1 and c2 were set equal to 1 and g1 was equal to zero. Resulted *SNR* was around 74.2 – 74.6 dB. Theoretical value of *SNR* is 79.2 dB. It's quite ambiguous to determine degree of the optimal coefficients unsuccessfulness because "measurement" error is strongly dependent on input signal frequency and number of FFT spectral bins. The difference up to 2.5 dB can be observed as a simulations result depending on number of spectral bins (from 2 on 13 to 17) for the same DSM.

Tab. 1: Modulator parameters

Parameter	***Value***
Bandwidth	22,05 kHz
OSR[1]	64
Bins of FFT	16 384
Fin	2,93kHz
Ain	0,5

CONCLUSION

The article describes conventional methods of DSM coefficients design and take thinks about limitations of real modulator design that limits conventional methods. Alternative to conventional methods should be genetic algorithm. The article deals with appropriate settings of GA algorithm used to evaluate DSM coefficients. The best reached solution is presented as well.

[1] Over-Sampling Ratio

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