Ročník 2012 electroscope Číslo III

EEG database merging for BCI applications

J. Doležal[1], J. Šťastný[1], M. Švadlenka[1]
[1] Biosignal processing laboratory, Department of Circuit Theory, Faculty of Electrotechnical Engineering, Czech Technical University in Prague, Technická 2, 166 27 Praha 6, Czech Republic
E-mail : dolezja7@fel.cvut.cz; stastnj1@seznam.cz; m.svadlenka@seznam.cz

Anotace:
Příspěvek představuje metodu pro slučování různých EEG záznamů. Sloučením záznamů a stabilitou EEG se zabýváme s ohledem na dlouhodobé používání rozhraní mozek-stroj. Metoda pro slučování záznamů byla otestována na datech z experimentů provedených s časovým odstupem jednoho roku. Pro ověření správnosti metody byla provedena klasifikace pomocí klasifikátoru založeného na skrytých Markovských modelech a použití Laplaceovské filtrace a nezávislých komponent. Výsledky ukazují, že projevy pohybové aktivity v EEG lze detekovat jak v samostatných tak i sloučených záznamech, což dokazuje správnost navržené metody. Předkládaná metoda je nezbytný krok pro vyhodnocení středně a dlouhodobých změn v budoucích experimentech se systémem pro zpracování EEG v reálném čase vyvinutém naší skupinou. Analýza dat a dosažené výsledky klasifikace ukazují, že odezvy EEG na pohybovou aktivitu jsou stabilní.

Annotation:
This paper presents a method for merging of different EEG recordings. We deal with merging of recordings and EEG stability with respect to long-term Brain-Computer Interface usage. Recording sessions from experiments separated by a one year period are used to test the method. Classification results using a Hidden Markov Model based classifier and both Laplacian filtering and Independent Component Analysis are presented to validate the merge. The results indicate that movement-related EEG responses can be detected in both stand-alone and merged sessions which prove viability of the proposed method. The presented method is a necessary step to investigate short-term and long-term changes with future experiments using a real time EEG processing system developed by our group. Both data analysis and classification indicate that the movement-related EEG responses are stable.

INTRODUCTION

Our group has been dealing with research in the field of movement-related EEG recognition towards developing of a Brain-Computer Interface (BCI). BCI is a system that bypasses traditional brain output pathways – peripheral nerves and muscles [1]. The output commands are taken directly from the brain function manifestation, nerves and muscles are bypassed or supplemented with the BCI. The system designed in this manner can be used with completely paralyzed patients [2],[3] or to support rehabilitation after stroke or brain injury.
Our previous work [4] showed that off-line single trial classification of extension and flexion movements of right index finger is possible. The EEG database we used in [4] was originally recorded for a physiological research [5],[6] and has some drawbacks from the BCI experiments point of view. Therefore we recorded a new database more suitable for BCI experiments [7]. As long term stability of BCI systems is rarely dealt-with we repeated the recording with the same experimental subjects to produce thus obtaining EEG database composed of two sessions separated by a year period. We want to find out if the experiment is repeatable, how do the brain responses differ, and whether the recording sessions can be used together in one BCI experiment. Use of separate recording sessions introduces problems in BCI applications therefore we had to develop a method to merge the sessions at first.
This paper presents a method for EEG database merging along with the list of problems related to the merging. We have defined a measure of experiment reproducibility between different recording sessions and evaluated movement detection score using EEG spatial and subspace filtering in order to asses performance of the proposed merging method.
The presented method for merging EEG sessions and it's validation presents a necessary step to investigate both mid- and long-term EEG-changes in future real time experiments using our developed system [8],[9].

EEG STABILITY

Volitional movements have specific responses in EEG; a distinctive temporal behavior of an EEG short time spectrum can be seen in Figure 1:

- Event-Related Desynchronization (ERD): starts usually about 1 second prior to the movement onset and displays as a decrease of power [5]. ERD is usually localized to the C3/CP3 and C4/CP4 scalp area.
- Event-Related Synchronization (ERS): central rhythms display desynchronization prior and during the movement and a rebound in the form of a phasic synchronization after the movement. ERS represents a post movement increase of

power in the band; the phenomenon is located about 1 second after the movement onset [6].

These events are present with both movement execution and imagery.

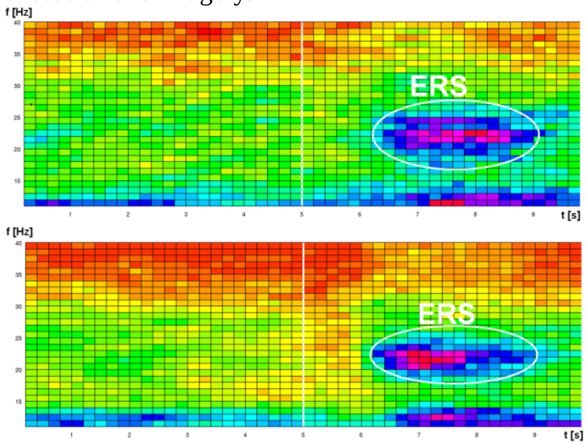

Figure 1: Short-time spectral magnitude EEG time-development (spectrogram) from both recording sessions (1st session is in the upper picture). Finger extension movement was performed at the 5th second (see vertical white line). One can clearly see the marked ERS in both recording sessions therefore merging the sessions makes sense.

Alpha and beta activity has shown stable long-range temporal correlations unique for each of the subjects [10]. The correlation was more stable under closed eyes condition [10], which is also our case. More importantly, the study has shown a linear decreasing trend of alpha and beta powers during a course of one experiment, and suggests that ERD/ERS patterns depends on the power of the EEG signal before the movement [10]. This means that ERS may be difficult to observe in the initial part of the recording since the beta activity power is already high (ceiling effect) and that ERD may be difficult to observe in later part of the recording since the power of alpha/μ-rhythm oscillations is low (floor effect) [10].

It has been shown that cognitive ERD/ERS responses are surprisingly stable in time [11]. Motor imagery induced ERS is also stable, has been suggested for person identification [1] and found suitable for realizing a brain on-off switch [12]. However a different (although similar) EEG databases were analyzed [11] or the responses were analyzed during continuous long-term training of the subject [1]. On the contrary, we aim to evaluate reproducibility of the recording directly by using two sessions of the same database merged together in a single BCI experiment. Therefore we had to address the merge-related problems.

A BCI system had to cope with both short- and long-term changes to classify the movements and thus allow the user to control the device properly. As some intra and inter-session differences are always present it is recommended to perform (re)training of the classifier at start of all online experiments [3], this is sometimes referred as calibration measurement. Short-term changes such as the influence of feedback had to be dealt with by means of continuous adaptation of the system [13]. Long-term changes need to be dealt with if the device is not used continuously. However, the classifier must be able to work even at the very beginning of the experiment (although with lower accuracy) despite all the above mentioned differences in order to provide the feedback facilitating user training later on [9],[13].

The method we propose below aims to enable us to use previously recorded data to train the classifier which can save precious time at the start of BCI experiments.

USED EEG DATABASE

The recordings took place at the laboratory of evoked potentials at the Medical Faculty of Charles University in Hradec Králové. Ten male subjects took part in the experiment with average age of 32 years (σ = 11.8 see Table 1). None of them had a previous experience with such an experiment. We used 64 unipolar scalp Ag/AgCl electrodes placed in standard 10-10 montage system. The ground electrode was mounted on the ears. The real exact positions of the scalp electrodes were measured with the help of the 3D scanner. In addition to the scalp electrodes vertical and horizontal EOGs and EMG electrodes were used [7]. The EEG was recorded with the sampling rate of 1024 Hz.

Each subject sat in a comfortable armchair in a silent and dim shielded cabin with both hands lying on the armrest in such a way so as he might freely perform the required extension or flexion movements. The subject was asked to keep his eyes closed and to avoid other movements than those asked for during the recording. Further, the subject was told to be as much relaxed as possible but not to fall asleep. Before the recording was started, each subject was trained to perform the required movements properly.

One recording session consisted of four blocks. The subject was performing the required self-paced voluntary movements during the first three blocks. The order and time between the movements were left at the subject's free will; no stimulation was used. Four kinds of movements were performed during the recording – brisk extensions and flexions of left or right index finger. As we were not sure if the movements can be distinguished properly based on the EMG traces, we also recorded video of the experiment. Each of the three recording blocks contained about 30 movements; the blocks were separated by 5 minutes of rest. During the fourth block, the resting EEG was recorded. We used this EEG as a referential one for false detection rate estimation later on.

The results of the experimental procedure were four blocks of about 15 minute long EEG per subject.

Second session was recorded after one year period using the same procedure to address the stability of the system. Since the movement related activity is highly individual, same experimental subject as in the first session had to attend the second session. As one subject was not available, only 9 out of 10 subjects took part in the recording of the second session.

Database processing

Since we wanted to perform a single-trial offline analysis and classification, we had to extract the movement-related EEG epochs. As asynchronous recording protocol was used and the movement types were randomly selected by the subject we had to tag the movements in order to provide exact temporal localization.

A linux based Mplayer video player extended by shell scripts was used to tag the movement types and store approximate timings [14]. This was done by pressing keys mapped to the movement types while watching the video. Matlab script then showed the signals at the given time intervals and the beginnings of the EMG traces were marked and used for precise time synchronization. The resting period was tagged automatically by resting tag periodic insertion with 10-second period. This allows to process the database quickly.

The EEG was decimated to 256 Hz and then extracted into 10-second-long epochs centered at the tags indicating when the movement was performed.

Further we localized artifacts. Artifacts were separated manually by visual inspection of separated movement EEG epochs. Any movement or resting tag was changed into an artifact tag if any artifact was found in the 10 second long epoch centered on the examined event. Also, the EMG traces were checked and outliers were discarded. See Tables 1 and 2 for the number of resulting epochs.

Tab. 1: Number of epochs per subjects and classes: Left Extension (LE), Left Flexion (LF), Right Extension (RE), Right Flexion (RF) and Resting (R). Session 1.

Person no. / Movement type	*LE*	*LF*	*RE*	*RF*	*R*
1	26	26	29	20	51
2	33	31	29	31	56
3	21	18	16	19	66
4	25	33	31	29	67
5	30	25	31	27	60
6	17	13	16	14	73
7	11	22	11	13	77
8	13	23	31	21	65
9	34	35	33	29	67
10	9	41	32	36	57

We wanted to perform a single classification experiment using data from both sessions to evaluate reproducibility of the recording procedure. Therefore the sessions had to be merged together.

Tab. 2: Number of epochs per subjects and classes: Left Extension (LE), Left Flexion (LF), Right Extension (RE), Right Flexion (RF) and Resting (R). Session 2.

Person no. / Movement type	*LE*	*LF*	*RE*	*RF*	*R*
1	35	30	32	26	54
2	53	59	59	63	43
4	54	60	55	60	57
5	30	42	45	49	48
6	46	42	46	40	47
7	4	21	32	28	74
8	43	40	49	53	71
9	44	45	34	39	50
10	32	37	34	33	46

Problem related to the merging

The following issues are met when we are going to merge two EEG sessions separated by a long time period:

- Electrode on-scalp positions can be different between the sessions. The EEG cap placement can not be exactly the same in both sessions. Also, a different EEG cap although of the same type was used with part of the subjects and session combinations.
- The impedance of the electrodes can differ between the sessions due to the quality of skin contacts. The different impedances result in different signal powers. This include bad-contact and noisy electrodes.
- The electrode wirings can be different in the second sessions as the recording laboratory is being used for various experiments. This can happen due to a mere mistake of the operator.
- Different biological and technical artifacts are present in both sessions.

METHODS

The proposed method for merging the recording sessions consists of the following steps:

1. Electrode montages: Comparison of electrode montages validates correctness of measured electrode coordinates using the 3D tracker. Also, proper position of the EEG cap on the subject’s head in both recording sessions is thus checked.
2. Power normalization: The normalization of signal power combats different electrode impedances between the recording sessions.
3. Similarity of spectrograms: Evaluation of spectrograms similarity provides a measure of experiment reproducibility and detects problematic electrodes.

Electrode montages

Positions of the electrodes were measured by a 3D scanner; see Figure 2 with an example of electrode montages from two sessions with the same subject. Mutual distances of the electrodes are computed from the coordinates and used to increase the accuracy of spatial filtering. The electrode montage is not exactly the same with different sessions. However, we have found the changes in electrode distances insignificant as having only a marginal effect on the spectrograms and classification scores. Therefore, we can use the electrode distances of other subject when the electrode coordinates are not complete for the given subject due to the 3D tracker malfunction.

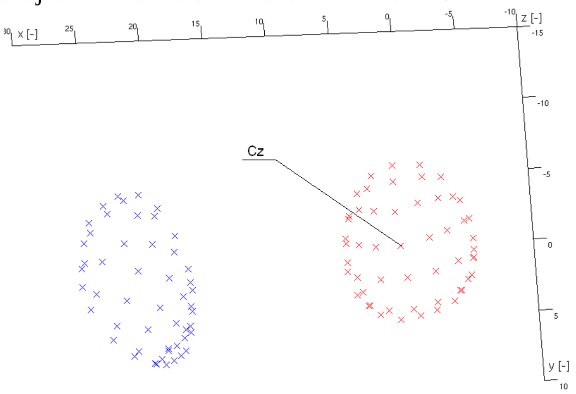

Figure 2: Electrode placement recorded by 3D tracker for both recording sessions.

In this step, the electrode distances are compared with distances of one selected subject with complete electrode montage and if the distances differ by more than 50% the distances of the selected subject are used instead.

Power normalization

Due to the different impedances of electrodes within each session power normalization must be applied before merging both sessions together.

The estimate of EEG signal power must be computed using only the intended signal, the artifacts present in the database must be omitted as they have many times larger amplitude and therefore power than the usable signal. This is made by computing the power estimates only from extracted movement epochs, which were checked for artifacts during database processing.

The signal was filtered by a 5-40 Hz band pass filter prior to computation of the estimates. The frequency band of the used preprocessing filter (FIR, 256th order, designed using frequency sampling method) was selected as it contains the movement-related responses which are utilized by our classification system later on. The selection of the frequency band is thus given by the application. The only important thing is to use the same frequency band for computation of the all the estimates, independent components, and features fed to the classifier.

We refer to the inverse value of the power estimate as normalization coefficient, i.e. after multiplying with the intended signal a unit power is obtained.

The whole signal from each electrode was then normalized using the computed normalization coefficients. The normalization coefficients can be used as an indicative measurement as shown in Figures 3 and 4. Blocks recorded within one session are considered to be the same (i.e. no differences are presumed). One can see that the first block (red line) significantly differs from the others in Figure 3. This is in full compliance with findings of study [10]: the power at posterior electrodes is significantly higher in the first block (due to alpha and beta activity [10]), while the power at anterior electrodes is increasing during all four blocks.

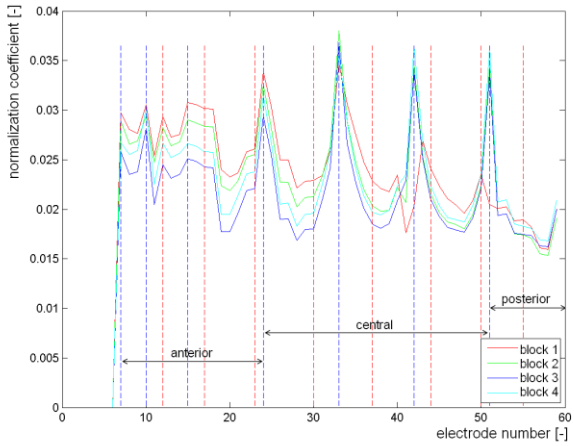

Figure 3: Comparison of power normalization coefficients estimated for all electrodes and all the recording blocks of one session. One can see that the first block (red line) differs from others.

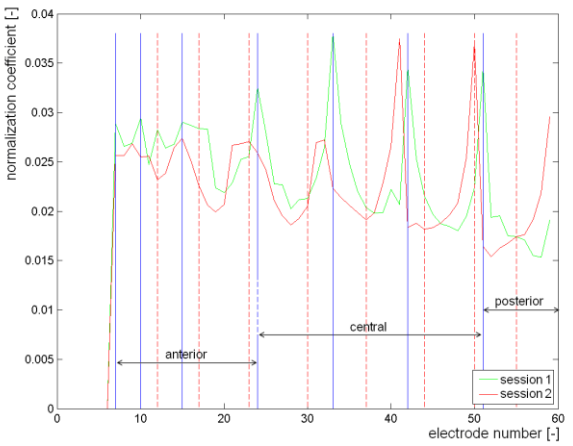

Figure 4: Comparison of power normalization coefficients estimated for all electrodes from second block of recording sessions separated by a year period.

The changes at the very beginning of the recording are also due to the fact that the experimental subject is calming down and getting used to the task as well as the conductive gel penetrates the outer layer of the skin thus impedances of the electrodes are stabilizing.

The comparison of power can be used to tell when the EEG is stable enough to start with the experiment based on the decreasing continuous power estimate difference at the beginning of the recording experiments. We plan to use this with our developed real-time processing system [8],[9] as an additional objective measure to the currently used subjective EEG visual inspection.

An example comparison of second recording block between sessions separated by a year period is shown in Figure 4. One can see that significant differences are present therefore sessions separated by a long period can not be used together in classification experiment without power normalization.

Similarity of spectrograms

Comparison of total signal powers between electrodes and blocks does not tell much about the movement-related changes. As the movement-related changes are visible in the spectrograms (see Figure 1 and 6 for examples) we evaluate their similarity. We compute the spectrograms in the same way as features fed to the classifier (see section Classification for details) as we prepare the database for movement classification. A visual comparison for all the electrodes would be very time consuming therefore we have developed an automatic method. We compare the spectrograms using the following formula,

$$Y_{jk} = \sum_{m} \sum_{n} \frac{\left| S_{1j}[m,n] - S_{2k}[m,n] \right|}{S_{1j}[m,n] + S_{2k}[m,n]} \qquad (1)$$

where S_{1j} is the spectrogram from the first recording session and S_{2j} from the second; j is the electrode number from the first session and k from the second; m and n are the numbers of rows (frequency axis) and columns (time axis) of each spectrogram. The matrix of spectrogram differences for all the electrodes between both sessions and one subject is visualized in Figure 5.

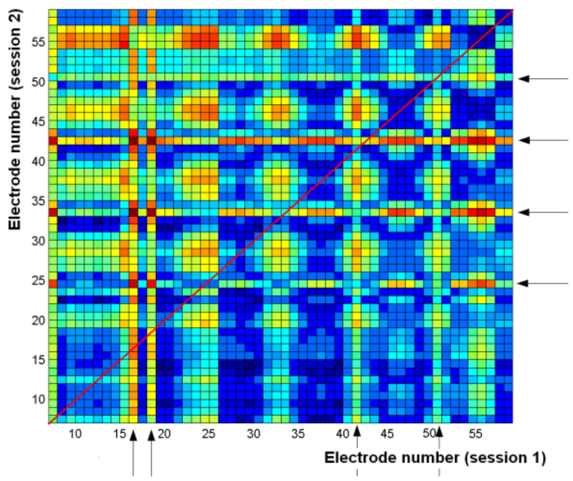

Figure 5: Matrix representing the differences in spectrograms from each electrode between both recording sessions. Note the lines indicating problematic electrodes marked with arrows.

Note that the matrix is not symmetric; although the same electrode numbers are on both axes, one time a spectrogram from the first session is compared to the all spectrograms of the second and vice versa. In other words, a bad electrode contact in the first session is manifested as vertical line (meaning it differs from all electrodes of the second session); while bad electrode contact in the second session is manifested as horizontal line as shown in Figure 5. These problematic electrodes were removed from both sessions.

We also did the check for misconnected electrodes by looking up for the lowest differences between the spectrograms. The lowest differences are placed on the diagonal marked by a red line as can be seen in Figure 5. This means that spectrograms of the same electrodes are the most similar. A shifted electrodes (e.g. mis-indexing from one or zero) would shift all the minima out of the diagonal while misconnected (e.g. swapped) electrodes would place only some minima out. The analysis of the spectrograms similarity did not reveal a change in the wirings.

EEG CLASSIFICATION

Spatial and subspace filtering

We use 8-neighbour Laplacian derivation [15] as a standard method to improve spatial resolution; we use the real electrode distances rather than a uniform mask to increase the accuracies. As Independent Component Analysis (ICA) is frequently being used with multi-channel recording and movement-related EEG classification [16], we apply it as well to see if it can improve our results. EFICA (Efficient FastICA) algorithm was used to compute independent components. Symmetric FastICA kernel of EFICA with *tanh* nonlinearity was used. More details on the EFICA algorithm can be found in [17]. Segments used to compute the ICA were selected from each block; the segments were checked for the presence of artifacts. The signal was filtered by a 5-40 Hz band pass filter prior to the ICA computation as the low frequencies are more influenced by biological artifacts and carry no useful information about the movement related activity in our case.

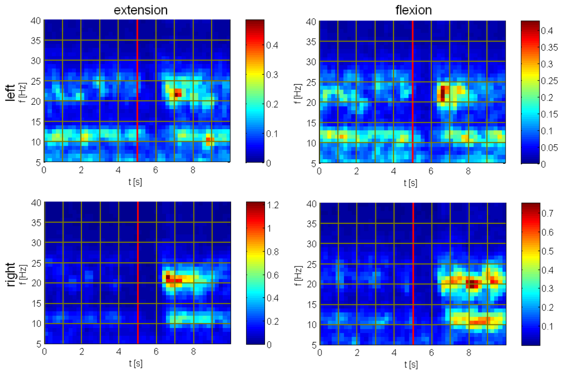

Figure 6: Short time spectral magnitude time development of EEG reconstructed from one movement-related component estimated from the merged sessions.

The components were visually sorted into movement-related and non-movement-related based on the visible ERD/ERS phenomena [18]. Roughly 15% of the components were marked as movement related. Examples of one movement related component for all the movement types are shown in Figure 6. The component presented in Figure 6 was computed using

both sessions, which indicates that the merge was successful. Components were also computed for each of the sessions alone using the same approach. The EEG was then reconstructed by combining only the movement related components.
Visual inspection of the data revealed more frequent and more visible ERS/ERD in the reconstructed EEG compared to the Laplacian filtered EEG [18].

Classification

The Hidden Markov Models (HMMs) classifier utilizing the EEG temporal context was used, see [4],[7] for more details on the classifier. FFT features computed from 1 sec long windows with 80% overlap at 256 Hz sampling rate were used, giving time resolution of 200 ms and frequency resolution of 1 Hz; same settings was used to analyze the data thus Figures 1 and 6 represent grand averages of the features fed to the classification system.
The classification results were validated using 16 fold *random* 50:50 training/testing set cross validation (Random in Table 3). *Sequential* training and testing set (training on the first recording session and testing on the second recording session) was used to asses the influence of long term changes (Sequential in Table 3). Referential classification results with *raw concatenation* of the sessions without using the merging method were done to show the benefits of the developed method. Random cross validation was used with raw concatenation of the sessions to obtain comparable results (Raw concatenation in Table 3).

Results

Movement detection scores averaged over all subjects are shown in Table 3. The ICA method gave slightly better results than Laplacian filtering, which can be contributed to the improved signal to noise ratio.
The scores achieved on the merged database are lower compared to the scores on the single recording session. This is not surprising as the database contains long-term changes. However, the classification is still possible with scores > 80% indicating that the movement-related activity is stable.

Tab. 3: Movement detection scores averaged over all subjects.

Session	*Methods*	*Score [%]*
1	ICA, random	94.1±04.7
1	Laplace, random	92.4±04.9
1+2	ICA, random	84.7±04.3
1+2	Laplace, random	80.9±06.0
1+2	ICA, sequential	72.0±07.7
1+2	Laplace, sequential	75.5±09.0
1+2	ICA, raw concatenation	64.6±11.0
1+2	Laplace, raw concatenation	62.3±13.0

One must keep in mind that a generative HMM classifier was intentionally selected to validate the merge due to the fact that each movement related activity and resting model alone is used to classify both recording sessions, i.e. only one data cluster is used with each model. Using Learning Vector Quantizer, nearest neighbor, or other classifier producing more clusters would not indicate validity of the merge as the classifier can find different clusters for each of the recording sessions.
Result with sequential training (first recording session) and testing (second recording second session) sets suggests that a previously trained classifier can be used; however a retraining of the classifier is necessary to achieve good performance.
The HMMs gave classification scores near chance level value when the sessions were just concatenated (see last rows in the table) justifying the need of the developed merging method.

CONCLUSIONS AND NEXT STEPS

We have proposed a method for merging of different EEG recordings obtained over a larger period of time and tested it on EEG recording sessions separated by a year period.
The method automatically detects various problems such as bad contact/noisy or misconnected electrodes which must be removed from the data prior to the classification experiments and provides a basic measure of experiment reproducibility.
We have performed classification experiments in order to validate the developed merging method. We have compared the results achieved using ICA and Laplacian filtering to find out that slightly better results can be achieved using the ICA due to it’s denoising abilities.
We have successfully repeated the recording procedure and classification experiments to find out that the movement related EEG responses are stable. Therefore we can move to investigation of long-term changes related to user training in future multi-channel online experiments using our developed real-time processing system [8],[9].
We shall also apply the method to exploit short-term changes such as to indicate the time instant when the EEG is settled down in the beginning of experiments or when re-training of the classifier should be applied.

ACKNOWLEDGEMENT

Jakub Šťastný has been supported by the research program Transdisciplinary Research in Biomedical Engineering II No. MSM 684 0770012 of the Czech University in Prague; Jaromír Doležal by the grant GACR 102/08/H008: Biological and Speech Signal Modeling and by the Grant Agency of the Czech Technical University in Prague, grant No. SGS10/178/OHK3/2T/13. Our gratitude belongs to doc. Jan Kremláček for his huge support during the EEG recording experiment at the Faculty of Medicine of the Charles University in Hradec Králové.

REFERENCES

[1] G. Pfurtscheller, C. Neuper, *Future prospects of ERD/ERS in the context of brain-computer interface (BCI) developments*, Progress in Brain Research, vol. 159, pp. 433-437, 2006.

[2] J. R Wolpaw, N. Birbaumer, *Brain-computer interfaces for communication and control,* Textbook of Neural Repair and Rehabilitation, vol. 1, pp. 602-614, 2006.

[3] A. Kübler, B. Kotchoubey, J. Kaiser, J. R.Wolpaw, N. Birbaumer, *Brain–computer communication: unlocking the locked in*, Psychological bulletin, vol. 127, pp. 358-375, 2001.

[4] J. Doležal, J. Šťastný and P. Sovka, *Recognition of Direction of Finger Movement From EEG Signal Using Markov Models*, Proceedings of the 3rd European Medical & Biological Engineering Conference (EMBEC '05), pp. 1492-1492, 2005.

[5] A. Stančák, *Event-related desynchronization of the μ rhythm in E/F finger movements*, Clinical Neurophysiology at the Beginning of the 21st Century, Supplements to Clinical Neurophysiology, vol. 53, pp. 210-214, 2000.

[6] A. Stančák, *The electroencephalographic β synchronization following extension and flexion finger movements in humans*, Neuroscience Letters, vol. 284, pp. 41-44, 2000.

[7] J. Doležal, J. Šťastný, P. Sovka, *Recording and recognition of movement related EEG signal,* Applied Electronics 2009, pp. 95-98, 2009.

[8] J. Šťastný, J. Doležal, V. Černý, J. Kubový, *Design of a modular brain-computer interface,* Electroscope 2010, vol. 2, 5 pages, 2010.

[9] J. Doležal, V. Černý and J. Šťastný, *Constructing a Brain-Computer Interface,* Applied Electronics 2011, pp. 99-102, 2011.

[10] T. Brismar, *The human EEG — Physiological and clinical studies*, Physiology & Behavior, vol. 92, pp. 141-147, 2007.

[11] C. Neuper, R. H. Grabner, A., Aljoscha C. Neubauer, *Long-term stability and consistency of EEG event-related (de-)synchronization across different cognitive tasks*, Clinical Neurophysiology, vol. 116, pp. 1681-1694, 2005.

[12] G. Pfurtscheller, T. Solisescalante, *Could the beta rebound in the EEG be suitable to realize a “brain switch”?,* Clinical Neurophysiology, vol. 120, pp. 24-29. 2009.

[13] C. Neuper, *Feedback-Regulated Mental Imagery in BCI Applications: Using Non-Invasive EEG and NIRS Signals*, BBCI Workshop, 2009.

[14] V. Černý, *Real time EEG signal processing*, Bachelor thesis, Czech Technical University in Prague, Faculty of Electrical Engineering, Department of Circuit Theory, 32 pages, 2010.

[15] J. Šťastný, P. Sovka, *The 3D approach to the surface Laplacian filtering with integrated sampling error compensation*, Elsevier Signal Processing, pp. 51-60, 2007.

[16] L. Ručkay, J. Šťastný, and P. Sovka, *Selection and Classification of EEG Movement-Related Independent Components*, in Analysis of Biomedical Signals and Images, 19-th Biennial International EURASIP Conference BIOSIGNAL 2008, 5 pages, 2008.

[17] Z. Koldovsky, P. Tichavsky and E. Oja, *Efficient Variant of Algorithm FastICA for Independent Component Analysis Attaining the Cramer-Rao Lower Bound*, IEEE Trans. on Neural Networks, vol. 17, no. 5, pp. 1265-1277, 2006.

[18] M. Švadlenka, *Aplikace analýzy nezávislých komponent v oblasti BCI systémů*, Masters thesis, Czech Technical University in Prague, Faculty of Electrical Engineering, Department of Circuit Theory, 66 pages, 2011.